ENGINEERING SCIENCE

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VOLUME I

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PREFACE

This book has been planned to cover the full study of the Engineering Science of the Junior Technical School, and at the same time to present the work in the more intensive form required by evening students taking the first and second year National Certificate Courses who have to dispense with some of the practical and graphical work.

More precisely, its scope is that of the Engineering Science Course of the Junior Technical School, the first and second years of the Ordinary National Certificate Courses in Mechanical Engineering, the Engineering Science of the Ordinary National Certificate in Electrical Engineering, and the introductory work for the Institutions of Mechanical and Electrical Engineers examinations and those of the Scottish Education Authorities affiliated to the Royal Technical College, Glasgow. Volumes I and II will also be found to cover the subject matter of many of the service and pre-service courses in engineering subjects and their application to engineering practice.

With very few exceptions, the approach to the subject is through detailed experimental work of either a demonstrative or individual character, and every effort has been made to bring the theory and examples into alignment with practice. The experimental work has been fully treated in the early chapters of the book, but as the student reacts to experimental methods, less attention is given to detail and more is required of the student conducting the experiment. The verification and applications of principles have been the primary aim throughout the book, and exercises and examples have been chosen to give a practical bias, and to form the student's equipment for a more advanced study of the subject with the design of machines and structures.

The presentation has been made with a full appreciation of the lack of mathematical training in the early stages, and it is confidently recommended that a student should commence his study at

the same time as he approaches algebraic processes. Providing the mathematical knowledge grows in parallel stages with the study of this subject, there should be no period at which the science is obscured by the complexity of the mathematical processes employed.

Graphical solutions have been adopted, generally, as an alternative to mathematical solutions, and the appreciation and treatment of vectors has been introduced at a very early stage. A chapter has been devoted to the testing of materials, and although this is, of necessity, brief, it will probably cover the immediate requirements of students at this stage of their work.

It has been found that the amount of work to be covered could not be treated in a single volume of reasonable proportions, and, after consideration of the requirements of the courses for which this book is intended to cater, a division has been made on the following lines. Vol. I to include the whole of the Mechanics and Hydraulics required in the course, and Vol. II the Heat, Heat Engines and Electro-technics. Where these sections of the subject generally classed as Engineering Science are taken as a composite course, it is believed that the work is so arranged that the two volumes can be used in conjunction without detriment to the presentation.

In the Heat, Heat Engines and Electro-technics section, the main principles have been summarized so far as possible, and the work treated mainly along descriptive lines to prepare the student for the more detailed and quantitative treatment demanded by the later study in the National Certificate Courses.

The authors have found that many students enrolling for National Certificate Courses have received, in Junior Technical, Secondary or Central Schools, instruction in the elementary physics leading up to the work in Heat Engines and Electro-technics. Some elementary experiments have, however, been added as a supplementary scheme to illustrate certain fundamental physical principles and to cater for students who would profit by their performance of these experiments.

In the majority of Junior Technical Schools the Engineering Science is reinforced by a course of General Science, so that where the

book is employed for Junior Technical Schools this supplementary scheme of experiments need only be taken at the discretion of the teacher.

It is hoped that this book will provide a progressive course along practical lines, and will find its place among books suitable to the work for which it is planned.

The thanks of the authors are due to the firms who have been good enough to loan blocks and supply data concerning their manufactures. Among these may be quoted Messrs Cussons Ltd. of Manchester; Messrs Morris Ltd., of Loughborough; Messrs A. Macklow Smith, of Westminster; Messrs Babcock and Wilcox Ltd.; Messrs C. A. Parsons Ltd., of Newcastle-on-Tyne; Messrs Petters Ltd., of Yeovil; The Zenith Carburettor Co. Ltd., of Stanmore; Messrs Dobbie McInnes Ltd., of Glasgow; The National Gas and Oil Engine Co. Ltd., of Ashton-under-Lyne; Messrs Hopkinson Ltd., of Huddersfield; Messrs Dewrance & Co., of London; Messrs Hick Hargreaves & Co. of Bolton; The Premier Gas Engine Co. Ltd., of Nottingham; The Standard Motor Co. Ltd., of Coventry, Messrs Philip Harris Ltd., of Birmingham and Messrs Skefco Ltd. of Luton.

Particular appreciation is due to Sir Richard Gregory, who has ungrudgingly lent his wide experience to the effective production of the book, and many suggestions of his have been adopted with very happy results.

Grateful acknowledgement is also gladly made to Mr. A. J. V. Gale, M.A., and Mr. F. G. W. Brown, M.Sc., for kindly reading through the proofs and making many helpful and valuable suggestions. The authors also wish to thank the publishers and authors concerned for permission to include tables of logarithms and diagrams from their publications, especially from Castle's *Practical Mathematics for Beginners*, Duncan and Starling's *Text-Book of Physics*, Duncan's *Steam and Other Engines*, and Gregory and Hadley's *Class-Book of Physics*.

Permission has been kindly given by Messrs Edward Arnold & Co. to include data from their *Callendar's Abridged Steam Tables*, and acknowledgement is gladly accorded to the following authorities for

permission to include a selection of their examination questions: The Union of Lancashire and Cheshire Institutes; The Union of Educational Institutions; The Institutions of Mechanical and Electrical Engineers; The Scottish Educational Authorities affiliated with the Royal Technical College, Glasgow.

The authors would be very glad to receive, from teachers and others, notification of any errors which may have escaped detection.

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SCHEME OF EXPERIMENTAL WORK

THE experimental work performed by a student must be largely determined by the scope provided in the laboratory to which he has access, and the time which he can reasonably afford to spend in this manner. In the Junior Technical School, considerably greater time can be devoted to practical work than in the part-time National Certificate classes, and thus, this type of student can, with advantage, obtain experimental verification of all the leading principles of the subject. At the same time, the approach to a large amount of the work attempted in this volume is best made through a systematic course of experiments; some of which may be demonstrated, in order to save time, and the others carried out by the student, or small groups of students, and conclusions drawn from the results.

Careful attention has been given to the process of "logging" this experimental work, and every effort should be made to convince the student of the necessity of careful systematic work in this direction.

In setting out the book, some experiments are treated fully, specimen observations and derived results given, and the possible conclusions shown. Other experiments are in skeleton form, with the object of providing the student with a certain amount of information which will enable him to conduct the experiment. The remaining experiments are left to be executed by the student after he has become acquainted with the objects and the apparatus to be used.

The authors have found that by varying the amount of information available to the student, before commencing the experiment, it is possible to check periodically any tendency towards slackness which may develop as he becomes familiar with experimental work. This is particularly noticeable in the processes of logging results, preparation of graphs and deduction of conclusions.

Certain work in which the Mathematics may present difficulty is approached experimentally, particularly in the sections dealing with deflections of beams, torsion and the pendulum. The more detailed mathematical treatment in this work may reasonably be left to a later reading of the subject. A suggested scheme of experimental work is given, which may be varied, in subject and order, according to the apparatus available and the presentation selected.

COURSE OF EXPERIMENTS, VOL. I

1. A weighted lever to verify the Principle of Moments.
2. The moment board to verify the Principle of Moments.
3. Reaction of beam supports.
4. The use of force boards to verify the Laws of Coplanar Forces.
5. A supported cord, or the board experiment to verify the funicular polygon.
6. Construction of a simple vernier slide gauge to read to $\frac{1}{100}$ inch.
7. Practice in the use of the micrometer and vernier for the measurement of the dimensions of solids and the use of the planimeter.
8. Use of the dial test indicator.
9. An experimental roof truss or simple crane with spring balances incorporated in the members.
10. Experiment to verify the laws of plane friction with sleighs of different materials or facings.
11. To investigate the forces acting upon a body which is supported upon an inclined plane.
12. Experiment with a Simple Pulley of unity velocity ratio. Introduction to velocity ratio, mechanical advantage, effect of friction and efficiency in a machine.
13. Efficiency tests on certain of the following machines. These machines should be selected to provide illustrations of (*a*) Wheel and Rope machines, (*b*) Toothed wheel machines, (*c*) Screw machines, and comparative conclusions and relative efficiencies obtained.

(1) Pulley systems and tackles.

(2) Wheel and Axle.

(3) Wheel and Differential Axle.
(4) Weston's Differential Block.
(5) Worm and Worm Wheel.
(6) Screw Jack.
(7) Wall Crane or hoist.
(8) Jib Crane with Single and Double Purchase Winch.

14. Arrangements of wheel trains to give a desired velocity ratio. Investigation of directions of rotation and use of an idle wheel.

15. Practice on a lathe quadrant. Setting up wheels to cut a certain thread pitch from a leading screw of known pitch.

16. Determination of the position of the centroids of laminae. Centroid of triangle, trapezoid and certain well-known beam sections, for example, Angle, Channel and I Section.

17. Position of the centre of gravity of a bicycle.

18. Properties of materials—Ductility, malleability, plasticity and hardness. Elasticity and methods of failure.

19. Verification of Hooke's Law for a loaded spring. Calculation of stiffness.

20. Modulus of elasticity for india-rubber.

21. Modulus of elasticity for steel and non-ferrous wire by direct loading. Verification of Hooke's Law.

22. A simple tension, compression and shearing test of suitable specimens.

23. Experimental work on the deflection of beams and cantilevers.

24. Test to destruction of a series of wooden beams. Determination of modulus of rupture.

25. A simple torsion test to verify the relation between angle of twist and torque, unsupported length and the diameter of circular shafts.

26. Demonstration of the Fletcher Trolley to show that a force produces an acceleration on a mass. This should include cutting the strip to show that the increment of increase of velocity is constant in uniform acceleration for equal increments of time.

27. Atwood machine experiment to verify the dynamics formulae and obtain a value for "g".

28. Fletcher Trolley experiment, varying the mass moved, to show the relation between mass moved, force and acceleration.

29. Determination of "g" by the falling ball and oscillating bar method.

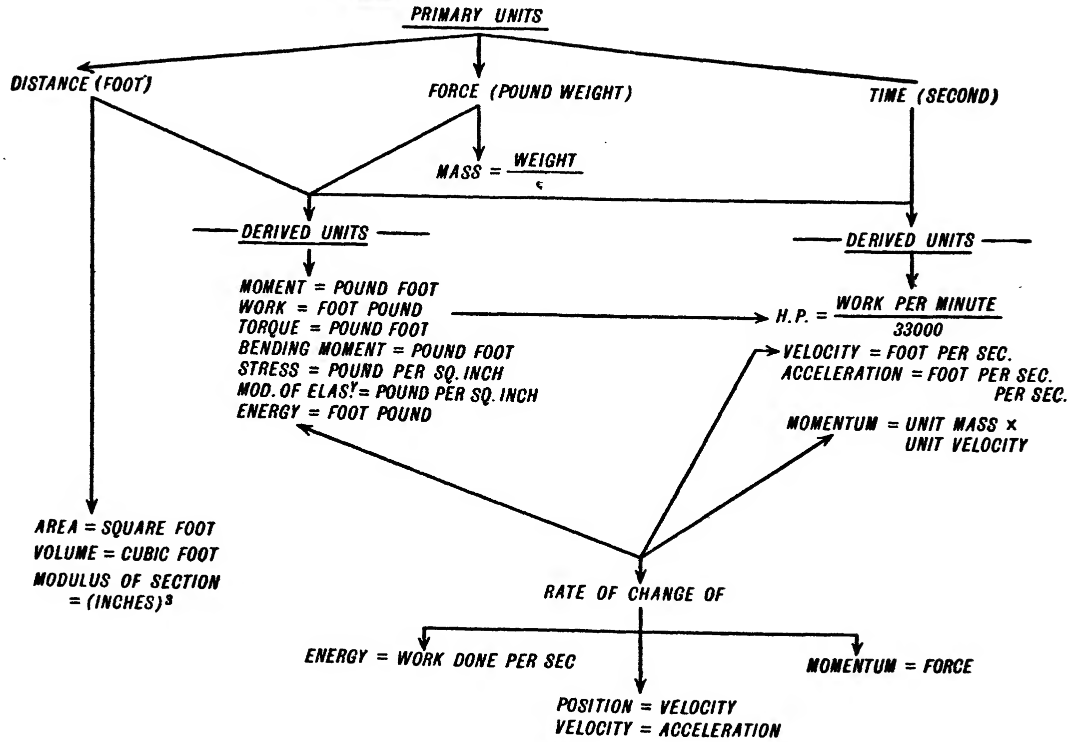
30. Experiment to find the law connecting the time of oscillation and the length of a simple pendulum. Determination of "g" from this law.

31. Experiment on centre of percussion and equivalent simple pendulum.

32. Experiment with a flywheel.

33. Flow of water through orifices of various shapes with varied head.

ENGINEERS UNITS



CHAPTER I

FORCE AND ITS MEASUREMENT—NATURE OF A FORCE—REPRESENTATION OF FORCES—VECTOR AND SCALAR QUANTITIES—STIFFNESS OF SPRINGS

Matter and mass. It is common to refer to any substance, or material which we know to exist, as **matter**; and any quantity of this matter, bounded by known limits, as a **body**. The quantity of matter contained in such a body is spoken of as the **mass** of the body.

The mass of the body is the *measure* of the actual quantity of matter it contains, and is not affected by change of shape, volume, or state; that is, a body may change its state, and exist as solid, liquid or gas without alteration to its mass. For example, the mass of a body of water will remain the same whether the water is hot or cold, changed to ice or steam, in a kettle or transported to Jamaica or Peru.

The mass of a body is measured by comparison with that of a standard body. In British engineering practice the standard used is the Imperial Standard Pound, which is described later in this chapter.

Force. Force can; perhaps, best be described as an influence which acts upon a body; and in order to study forces, it is convenient to pay attention to their effects, and thus to understand the nature of a force. For example, if a bicycle is travelling at a steady or *uniform* speed in a straight line, and then comes under the action of a force, the force may produce any of the following results, or a combination of them:

The motion of the cycle may

(*a*) be slowed or *retarded*,
(*b*) *cease*,
(*c*) be *increased* or *accelerated*,
(*d*) suffer a *change of direction*.

Further, if the cycle were at rest the force might cause it to *move.*

Thus a force can be defined as follows: **A force is that which, by acting upon a body, alters, or tends to alter, its state of rest, or its uniform motion in a straight line.**

Distinction between forces. Bodies may exert forces without being in contact, as, for example, when a magnet attracts a steel nail. When two bodies, not in contact, are drawn together by such a force, the force is called a **force of attraction**. If the bodies tend to separate under the action of the force, the force is called a **force of repulsion**. These forces are often termed attractive and repulsive forces respectively. The earth revolves around the sun in an almost circular path, and it is the attractive force between the earth and the sun that prevents the earth from leaving the sun, and continuing in a straight line through space, as when a stone is released from a catapult. Other bodies in the solar system also exert attractive forces, and their combined action produces the results with which we are familiar. Likewise the attraction between the earth and bodies on, or near, its surface provide, perhaps, the commonest illustration of force. This force is known as the **force of gravity,** and its strength depends upon the distance between the body and the earth's centre. *The greater the distance the less will be the force.* The relation between the force and the distance follows what is known as the **inverse square law**; for example, if the distance is doubled the force is reduced to one-quarter, and if the distance is trebled the force is reduced to one-ninth.

A force may also be transmitted to a body by the process of impact or collision with another body. In such cases contact may only occur for a very short interval of time. The striking of a ball with a bat, and the impact of a forging hammer with the metal, afford illustrations of this method of applying a force.

Force of gravity and weight. The force of gravity is exerted by the earth upon all bodies on, or near, its surface and tends to pull such bodies towards the earth's centre. The **weight** of a body is a measure of the amount of the earth's pull upon the body.

Imperial standard pound. In the Standards Office of the Board of Trade in London a piece of pure platinum is kept and the pull of the earth upon this piece of platinum is called the Imperial

Standard Pound (Fig. 1). This provides the standard by which forces and weights can be measured. Platinum is chosen for this standard because it is a metal which strongly resists corrosion at ordinary temperatures and thus the standard pound does not gain mass by rusting or other causes.

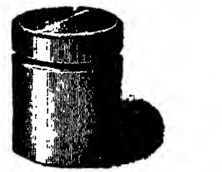


FIG. 1. Imperial standard pound: one-half actual size.

If a weight is dropped from a height, it can be easily seen that the weight gains speed. Such a gain of speed is termed an **acceleration.** The force of gravity produces an acceleration on a body which is allowed to fall. This acceleration is directed towards the earth's centre and is only prevented when the medium through which the body tends to fall offers a greater resistance than the earth's pull.

The magnitude or size of a force, such as a load on a spring, is indicated by stating that it is so many pounds weight; a force of 20 lb. weight is equivalent to the force with which gravity acts on a mass of 20 lb. at sea-level. In engineering practice, the word "weight" is omitted for brevity, but it should be understood that when reference is made to a force of "10 lb.", a force of "10 lb. weight" is meant.

The spring balance. EXPT. 1.

OBJECTS. *(a) To compare the extension of a coil or helical spring with the load producing it. (b) To find the load required to produce an extension of one inch, that is, the* **stiffness** *of the spring.*

METHOD OF PROCEDURE. Carefully measure the original length of the spring before any load is applied, and while it is hanging vertically from a suitable support. Hang a series of increasing weights upon the lower end of the spring and for each loading measure the extended length of the spring (Fig. 2, *a*). Tabulate the results and calculate the extension of the spring for each loading by subtracting the original length from the length after loading. At the end of the experiment all load should be taken from the spring and its length measured. If the result is the same as the original length, before loading, it is a proof that the spring has not been loaded beyond its power of recovery. Plot a graph of *Load against Extension* from the observations obtained. When performing this experiment and the extensions are small, as in the case of stiff springs, a vernier attachment (see p. 14) should be used for accurate measurement.

OBSERVATIONS. Length of spring before loading = 7·4 in.

Load in pounds - - -	0	1	2	3	4
Length in inches - - -	7·4	7·9	8·4	8·9	9·4
Extension in inches - -	0	0·5	1·0	1·5	2·0
Length after unloading -	7·4	7·9	8·4	8·9	—

GRAPH OF OBSERVATIONS.

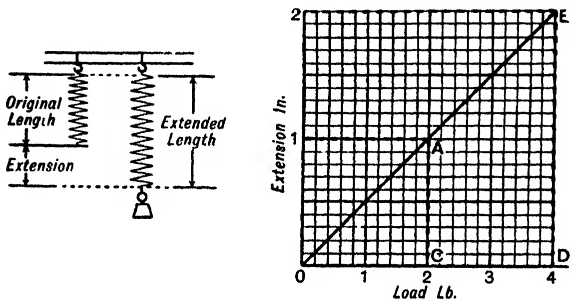


FIG. 2. (*b*)

DEFINITION. **The stiffness of a spring is the load required to produce an extension or contraction of one inch.**

For example, a spring may have a stiffness of

30 pounds per inch of contraction.

To determine the stiffness. Take two points on the extension axis of the graph which are one inch apart to the scale selected, and draw two horizontal lines to meet the graph at A and B. From A and B draw two vertical lines to meet the load axis at C and D. Read off the load value at D and at C; the difference between the readings at C and D will give the stiffness of the spring.

CONCLUSION. The graph of load against extension (Fig. 2, *b*) will be found to be a straight line passing through the origin, that is, the point where both load and extension are zero. Such a graph indicates that the two quantities, load and extension, are in proportion; that is, **the extension of a spring is proportional to the load which produces it.**

At this stage it is advisable to perform a further experiment with a short spring of relatively large coil diameter and arrange the load to produce compression. It will then be possible to show whether the compression of a spring is also in proportion to the load producing it. NOTE.—*The compression spring must be of sufficient diameter, in its coils, to prevent it buckling under the load, and the coils must not touch when loaded.*

Practical applications.

Example 1. *A spring is required which will extend a distance of* 0·8 *in. under a load of* 16 *lb. Calculate the stiffness.*

Let x lb. be the required stiffness, or load required to stretch the spring 1 in.; then, since the extension is proportional to the load producing it,

$$\overbrace{16 : x}^{\text{Loads}} = \overbrace{0{\cdot}8 : 1}^{\text{Extensions}},$$

and

$$0{\cdot}8 \times x = 16 \times 1;$$

$$x = \frac{16}{0{\cdot}8} = 20. \quad \textbf{Ans. 20 lb. per inch.}$$

Example 2. *The return spring of a valve is known to have a stiffness of* 85 *lb. per inch. The valve lifts* 0·65 *in. when working. Calculate the force which operates the valve.*

Let F lb. be the force, then

$$\overbrace{F : 85}^{\text{Loads}} = \overbrace{0{\cdot}65 : 1}^{\text{Compression}},$$

and

$$F \times 1 = 85 \times 0{\cdot}65;$$

$$F = 55{\cdot}25. \quad \textbf{Ans. 55·25 lb.}$$

Example 3. *The spring of an engine governor is one having a stiffness of* 28 *lb. per in. of compression. Calculate the lift of the governor if the lifting force is* 17 *lb.*

Let x in. be the lift, then:

$$28 : 17 = 1 : x,$$

and

$$28x = 17.$$

$$x = \tfrac{17}{28} = 0{\cdot}607. \quad \textbf{Ans. 0·607 in.}$$

The spring balance. This is a piece of apparatus designed to measure forces and weights. It makes use of the fact that the

extension of a spring is proportional to the load producing it, and consists of a spring, Fig. 3, attached by hooks, or pins, to two tubes, one sliding within the other. When a load is placed upon the hook attached to the inside tube and the outside tube is secured, the load tends to draw the two tubes apart against the action of the spring which connects them. An indicator is fitted to the inner tube, and the outer tube is graduated, not in inches, but in pounds, according to the stiffness of the spring employed. The outer tube is slotted to expose the indicator which makes contact with the scale on the outer tube.

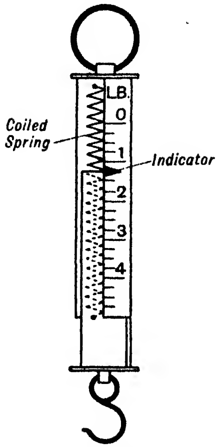


FIG. 3. Sportsman type spring balance.

This type of balance will give, within practical limits, a true reading for the measurement of forces, although a mass hung upon it will be affected by nearness to the earth's centre. Thus a spring balance will record variations in gravitational force on a given mass. This variation can generally be ignored in engineering calculations but it must be remembered that it exists.

Example 4. *What force is measured by a spring balance in which the indicator moves* 0·6 *in. when the spring has a stiffness of* 25 *lb. per inch ?*

Let F be the force in lb.

then
$$F : 25 = 0{\cdot}6 : 1,$$
$$F = 25 \times 0{\cdot}6 = 15. \quad \textbf{Ans. 15 lb.}$$

Trade scales and chemical balance. In the use of this type of balance (Fig. 5), the force of gravity is equally exerted upon the contents of each scale pan, and the balance therefore gives the true mass of a body in any part of the world.

Materials under the action of forces. The effects of a force, when the force is acting upon a material, may now be considered. For example, the compression of a block of india-rubber, or the bending of

a piece of wire, can be brought about by the application of a force. This alteration of size or shape produced by a force is called **strain** and the force is said to have a **straining action.**

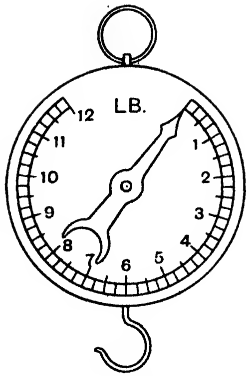


Fig. 4. Salter spring balance (Dial type).

Now, what happens in the material in order to resist this straining action? If the material is *elastic* a force will be exerted within the material, to oppose the straining force. **Elasticity** is thus the property a body possesses of resisting internally the straining effect of an external force.

When the material can no longer provide the necessary resisting force, the elasticity is said to have *broken down,* and the straining proceeds until fracture occurs. It follows that, while a body remains elastic, it will completely, or partially, recover its original shape and size after the straining force is removed.

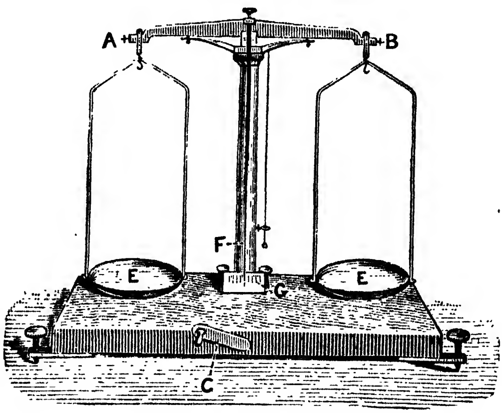


Fig. 5. Chemical balance.

Note.—*The failure of the elasticity of a material is not, generally, sudden and complete, but the material remains partially elastic after the elasticity appears to have failed.*

Load is the name given to the external force, or forces, which tend to strain or distort the material.

A tensile load is a load which is acting upon a body in the nature of a *direct pull.* A material is said to be in **tension** when it is internally resisting an external pull, Fig. 6.

Fig. 6.

A compressive load is a load which is acting upon a body in the nature of a *direct thrust.* A material is said to be in **compression** when it is internally resisting an external thrust, Fig. 7.

Fig. 7.

Compressive and tensile loads are described as *direct loads or forces.* They produce the simplest straining actions to which a body can be subjected. A stick, when subjected to a compressive or a tensile load, is receiving a force *directed* along its axis. Examples of bodies in tension and compression follow :

Tension: (*a*) the chain of a bicycle when the rider is pedalling,
(*b*) a tug-of-war rope in use,
(*c*) a rope supporting a weight.

Compression : (*a*) a chair leg,
(*b*) the foundation of a building,
(*c*) the saddle springs of a bicycle which is carrying a rider.

Necessity for an opposite force. Tension, or compression, cannot be produced unless there is an opposing force of the same size, or *magnitude,* as it is called. This is another way of saying that to every *action* there must be an equal and opposite *reaction.* For example, a loose rope, if pulled at one end, would not be in tension unless an equal and opposite force were applied. If the force applied to the end of the loose rope (Fig. 8) were 5 lb., there would be no tension in the

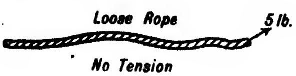


Fig. 8.

rope, except that due to its weight, but the rope would tend to move in the direction of the force. Now if a pull of 5 lb. were applied to the other end of the rope (Fig. 9), the rope would be in tension and the amount of the tension would be 5 lb. *not* 10 *lb.*, because there would be no tension until the second force be applied. If the rope is tied to a tree, the tree provides the opposite force and produces tension in the rope. NOTE.—*In this example the weight of the rope has been neglected.*

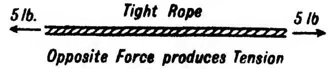


FIG. 9.

Example 1. *What is the tension in the rope when a truck offering a resistance of 220 lb. is pulled along a level track without gain of speed ?*

Ans. 220 lb.

Example 2. *Find the compressive force in a spring which has a stiffness of 20 lb. per inch of compression if it is compressed, under load, a distance of 1·4 in.*

Compressive force $= 1{\cdot}4 \times 20 = 28$ lb.

Ans. 28 lb.

Specification and representation of forces. It is often necessary to specify, or represent, a force, and before this can be done it is necessary to understand the meaning of two terms, namely, scalar and vector quantities.

A **scalar quantity** is a quantity which possesses size or magnitude only.

A **vector quantity** is a quantity which possesses *both* size, or magnitude, *and* direction.

For example, a wind blowing with a speed of 10 miles per hour is a *scalar* quantity, because its direction is not specified ; but, if the wind is of 10 miles per hour from the south-west, this quantity becomes a *vector* quantity because a *direction* is specified.

Forces are vector quantities.

Examples of scalar quantities.

(*a*) 6 feet, (*b*) 30 shillings, (*c*) 20 miles per hour.

Examples of vector quantities.

(*a*) A force of 10 lb. acting vertically downwards, that is, a weight of 10 lb.,

(*b*) 50 miles along a road from north to south,

(*c*) 20 knots due east.

NOTE.—A knot is a speed of 1 nautical mile per hour, that is, about 6080 ft. per hour.

A ship which is sailing at 15 knots is, therefore, moving at 15 nautical miles an hour. It would be incorrect to refer to the speed as 15 *knots an hour.*

Sense of direction. A wind may be blowing from the east towards the west, or from the west towards the east; its sense of direction is the indication of whether it comes from the east or from the west. For example, a force may act downwards as a weight, but the resistance of the floor, on which it rests, would be upward and these two forces would have opposite senses of direction.

To represent a force it is necessary to know its

(*a*) **point of application,**
(*b*) size, or **magnitude,**
(*c*) **direction,**
(*d*) **sense of direction.**

Example. *Represent a force of 30 lb. acting on a point* A *in a direction 30 degrees above the horizontal. Scale* 1 *in.* = 10 *lb.* Fig. 10.

METHOD. Draw a horizontal line AD of any length and from A draw AB at 30° to AD. Make AB 3 in. in length, that is 30 lb. to a scale of 1 in. = 10 lb. Place an arrow on AB pointing towards A, then BA represents the force, and the force is called the **force BA.** If the arrow pointed away from A the force would be referred to as the **force AB.**

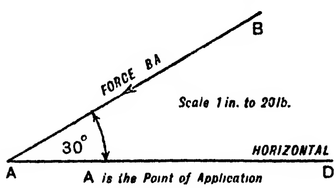


FIG. 10.

In this method of representing forces the line AB is of a length equal to the force's magnitude to scale, A is the point of application; the direction is shown by the line AB and the arrow indicates the sense of direction. Thus the force is represented by the line BA.

Example. *A boat is pulled by two ropes* A *and* B, A *with a force of* 30 *lb. at* 45° *to the boat's axis, and* B *with a force of* 40 *lb. at* 30° *to the axis on the opposite side. Represent these forces to a scale of* 1 *in.* = 20 *lb.*

Space diagram is the name given to the diagram on the left, a diagram which shows the direction and sense of the forces, and indicates their magnitude, but is not drawn to scale.

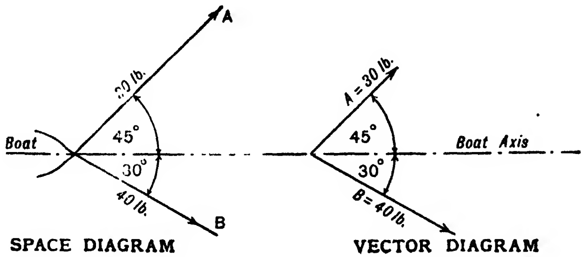


FIG. 11.

Vector diagram is the name given to the diagram on the right, a diagram in which the forces are represented in magnitude and direction and drawn to scale. It is a representation of each force as a vector showing its point of application, magnitude, direction and sense.

EXERCISES ON CHAPTER I

1. What is meant by the terms Matter, Mass and a Body ?

2. State a few alterations which may be made to a body without altering its mass.

3. What do you understand by a force ? Give several examples of forces and state, in each case, the effect of these forces upon the body on which they act.

4. Give three examples of each of the following effects of a force : (*a*) Retardation, (*b*) Acceleration, (*c*) Alteration of the direction of motion.

5. What is the meaning of Weight, and what causes a body to possess weight ?

6. What is the difference between the mass of a body and its weight ; which of these quantities is variable, and what causes the variation ?

7. Explain the terms : Strain, Tension, Compression, Elasticity and Load.

8. Some forces act without the bodies being in contact, while other forces are the result of collision or contact. Give examples of each type of force and distinguish between attractive and repulsive forces.

9. Draw two diagrams to illustrate the action of the internal resisting forces, (*a*) when a bar of steel is in tension, (*b*) when the bar is in compression.

10. Explain why a delicate spring balance could be used to detect variations in the force of gravity, or the weight of the same body.

11. State what distinguishes a scalar from a vector quantity, and show in what way they are alike.

12. Give three examples, of a general character, of things, or parts of things, you know to be in (*a*) tension, (*b*) compression.

13. A piece of metal is pulled at one end with a force of 300 lb. What is necessary in order to produce a tension in the metal, and what would happen if this were not provided ? State the value of the tension set up when the necessary conditions are fulfilled.

14. The two tug-of-war teams in a match are each exerting forces of 1000 lb. State the magnitude of the tension in the rope between the teams. What do you think would happen if the force exerted by one team were increased to 1100 lb. ?

15. Make a list of the forces dealt with in this chapter under the headings of forces due to (1) contact, or collision, (2) to interaction, without contact or collision.

16. Write down three scalar quantities and three vector quantities. How could a speed of 30 miles per hour be expressed as a vector quantity ? Represent your answer by a straight line to a scale of 1 in. = 10 miles per hour.

17. State the information required in order that a force may be represented by a straight line.

18. Represent to a scale of 1 in. = 30 lb. a force of 75 lb. acting away from a point B in a direction 30° North of East. Is this a vector quantity, if so, why ?

19. Three forces, respectively 8, 10 and 12 lb. in magnitude, act at 120° to each other on a point O. Represent these forces to a scale of 1 in. = 4 lb.

20. What is meant by the stiffness of a spring ? A spring compresses 2 in. under a load of 27 lb. State the stiffness of the spring.

21. If a spring is loaded with increasing loads from 0 to 10 lb. and the extension for each load measured, the graph of load against extension is a straight line passing through the origin. What does this graph show ?

22. What stiffness of spring will be required for a spring balance with an indicator which moves 7·6 in. when the load is 45 lb. ?

23. What stiffness of spring is required, for a machine, to produce a return force of 800 lb. if the movement is $2\frac{1}{2}$ in. ?

24. The return spring of a valve has a stiffness of 45 lb. per inch of compression. How far will the valve open if the opening force is 19 lb. ?

25. The following are the corresponding loads and extensions for a number of different springs. Calculate the stiffness, in each case, and arrange the springs in descending order of stiffness.

(*a*)	L lb.	0	7·4	(*b*)	L lb.	0	3·6	(*c*)	L lb.	0	6·1
	E in.	0	1·3		E in.	0	0·9		E in.	0	0·6

26. The following are the loads and corresponding extensions for the same spring. Plot a graph of load against extension and read off the load for an extension of 3·10 in. and the extension for a load of 5·5 lb.

Load lb. -	0	2	4	6	7·5
Exten. in. -	0	1·6	3·2	4·8	6·0

CHAPTER II

INSTRUMENTS FOR ACCURATE MEASUREMENT

THE engineer uses, for the general purposes of measurement, two types of instruments, (*a*) **The Vernier Slide Gauge** and (*b*) **The Micrometer Screw Gauge.** In addition, for certain purposes, an instrument known as a Dial Test Indicator is frequently employed. Where exceptional accuracy is demanded **measuring machines** are employed, but the use of this type of machine is limited to the manufacture of gauges and similar accurate details, and demands certain conditions of temperature and absence of vibration for its effective use.

Limits of accuracy usually associated with measuring instruments

Instrument	Degree of Accuracy obtained in use
(*a*) Vernier Slide Gauge.	Readings correct to 0·01 or 0·005 in.
(*b*) Micrometer Screw Gauge.	Readings correct to 0·001 in. or to 0·0001 in. with vernier attachment.
(*c*) Dial Test Indicator.	Readings correct to 0·001 in. or to 0·0001 in. with suitable lever attachment.
(*d*) Measuring Machine.	Readings correct to 0·00001 in. or 0·000001 in. in special circumstances.

Both the vernier and micrometer are to be found fitted to various types of measuring instruments, and the student is advised to consult the catalogues of standard tool makers in order to make himself familiar with the applications of these measuring devices.

The Vernier. This instrument was invented by a Frenchman, Pierre Vernier (1580-1637) and is, essentially, in the form of two slides, one working in contact with the other, and both graduated upon a definite principle.

EXPT. 2. **Construction of an experimental vernier.** This vernier is to read to $\frac{1}{100}$ of an inch, that is, to 0·01 inch, but for the purpose of

the experiment it is better to construct it to a scale of twice full size. Suitable materials for carrying out this construction are drawing paper, cardboard or ply-wood (Fig. 13).

The fixed scale. Set out the frame A, externally 5 inches by 1½ inches, internally 3½ inches by ¾ inch.

Scales G and H. Graduate these scales very carefully, as shown in the diagram, making each division 0·2 inch in length.

The sliding scale. Set out the sliding scale B, 5½ inches, or longer, in length and 1 inch in width. Inside this, equally spaced, draw two lines ¾ inch apart.

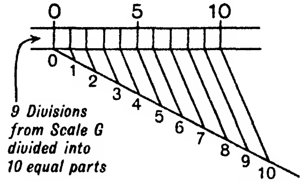


FIG. 12. Construction for the division of a line into 10 equal parts.

Scale E. To construct the scale E, take 9 divisions from the scale G and carefully divide the total length of 9 divisions into 10 equal parts, employing the geometrical construction shown in Fig. 12.

Scale F. To construct the scale F, take 11 divisions from scale G, or H, and divide the total length of 11 divisions into 10 equal parts, again employing the construction shown for scale E.

The backing. Prepare two strips C and D just under ¼ inch in width and 5 inches in length.

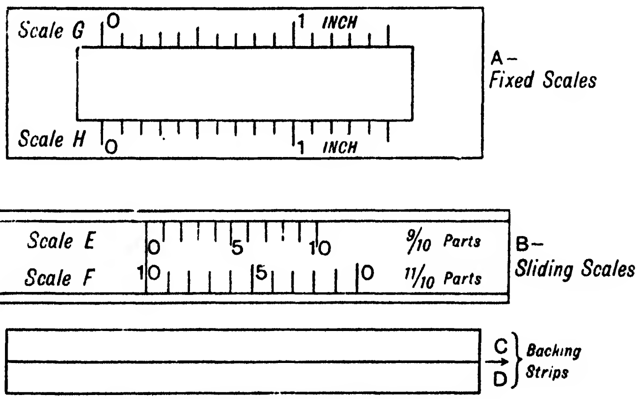


FIG. 13. Details of experimental vernier.

Assembly. Cut out the figures A, B, C, D and P and paste them together according to the method illustrated in the isometric

sketch (Fig. 14). The strip B should slide freely under A between the strips C and D and over P, which is the same size as A. P is pasted to C and D only, thus C and D provide packing pieces which allow B to slide between A and P.

This experimental vernier can be used to obtain practice in the use of the vernier, and illustrates the construction of the two types of vernier in general use.

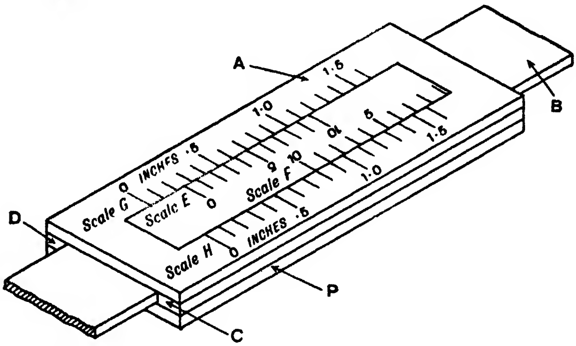


FIG. 14. Assembly of an experimental vernier to read to $\frac{1}{100}$ inch.

Use of the vernier. Suppose that the sliding scale E (Fig. 15) is moved to the position shown, then the distance x measured on scale G is the vernier reading.

Method of reading scales G and E (Fig. 15). (*a*) Note upon the scale G the number of complete $\frac{1}{10}$ in. spaces contained in the distance

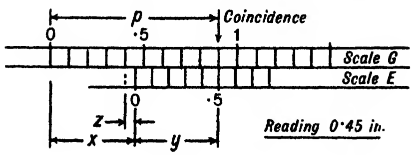


FIG. 15.

x. In the diagram it is 4. Therefore x is equal to 0·4 in. $+z$ where z is the number of $\frac{1}{100}$ in.

(*b*) Look along scale E for a line which exactly coincides with a line on scale G. In the diagram it is the fifth line. Then 5 is the number of $\frac{1}{100}$ in. or $z = 0{\cdot}05$ in.

The reading is $x = 0{\cdot}4 + 0{\cdot}05 = 0{\cdot}45$ in.

THEORY. Referring to the diagram, the vernier reading $x = p - y$

and the scale E divisions are each equal to $\frac{9}{10}$ of the scale G divisions. that is, 0·09 in., so that

$$\begin{aligned} x &= p - y \\ &= 9 \times \tfrac{1}{10} - 5 \times 0{\cdot}09 \\ &= 0{\cdot}9 - 0{\cdot}45 = 0{\cdot}45 \text{ in.} \end{aligned}$$

Method of reading scales H and F (Fig. 16). Follow the method outlined for scales G and E; the reading x is made up of 1·4 in. in

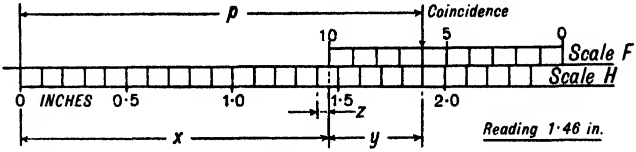


FIG. 16.

complete $\frac{1}{10}$ divisions together with a distance z. Coincidence occurs at 6 on scale F, so that the reading is

$$\mathbf{x} = 1{\cdot}4 \text{ in.} + 0{\cdot}06 \text{ in.} = 1{\cdot}46 \text{ in.}$$

THEORY. *Note that the sliding scale* F *is numbered in the opposite direction to scale* E.

Again $x = p - y$ and one division F $= 1{\cdot}1 \times$ one division on H, and

$$\begin{aligned} x &= 1{\cdot}9 - 4 \times 0{\cdot}11 \\ &= 1{\cdot}9 - 0{\cdot}44 = 1{\cdot}46 \text{ in.} \end{aligned}$$

NOTE.—*y is made up of four divisions of scale* F.

A vernier to read to $\frac{1}{64}$ in. (Fig. 17). Verniers are sometimes constructed to read to $\frac{1}{64}$ in., in which case the fixed scale is graduated in $\frac{1}{16}$ in. spaces and the sliding scale is made by dividing three of these $\frac{1}{16}$ in. spaces into four parts.

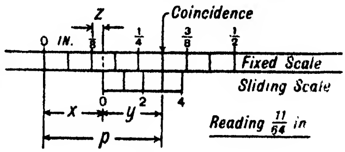


FIG. 17.

READING.—x is the vernier reading and is made up of two complete $\frac{1}{16}$ in. spaces together with z, which is $\frac{3}{64}$ in. Coincidence occurs at 3 on the sliding scale. Thus, $x = \frac{2}{16} + \frac{3}{64} = \frac{11}{64}$ in.

THEORY.—One division on the fixed scale $= \frac{1}{16}$ in. and one division on the sliding scale $= \frac{1}{4}$ of $\frac{3}{16}$ in. $= \frac{3}{64}$ in.

$$\begin{aligned} x &= p - y \\ x &= 5 \times \tfrac{1}{16} - 3 \times \tfrac{3}{64} \\ x &= \tfrac{5}{16} - \tfrac{9}{64} = \tfrac{11}{64} \text{ in.} \end{aligned}$$

NOTE.—*The graduation of a vernier reading to fractions of an inch is falling into disuse because engineers are adopting as a general practice the use of the decimal system in production.*

A vernier to read to $\frac{1}{200}$ in. (Fig. 18). In this case the fixed scale is divided into divisions each of $\frac{1}{10}$ in. and 19 of these divisions are

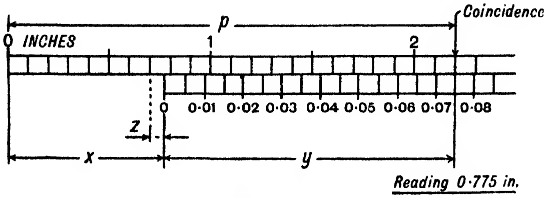


FIG. 18.

taken and subdivided into 20 parts to form the sliding scale. The reading is taken similarly to the method used in the previous examples, but each division on the sliding scale is $\frac{19}{20}$ of a division on the fixed scale, or $\frac{19}{200}$ in.

THEORY.

$$\begin{aligned} x &= p - y \\ &= 22 \times \tfrac{1}{10} - 15 \times \tfrac{19}{200} \\ &= 2{\cdot}2 - 1{\cdot}425 = \mathbf{0{\cdot}775\ in.} \end{aligned}$$

The micrometer screw gauge. This instrument, in varied forms, finds extensive use in engineering production, both of mass and detail character. It is commonly known as a **micrometer** or, even more commonly as a **mike** ; and it is, perhaps, desirable to acquaint the student with its workshop designation.

The micrometer is fitted to calipers, depth gauges, internal gauges and many other tools designed for accurate measurement, and in its commonest form will give readings correct to **0·001 inch** or $\frac{1}{1000}$ **inch**. For greater accuracy the hub is provided with a vernier scale in which 9 of the $\frac{1}{1000}$ in. divisions on the thimble is divided into 10 equal parts, thus extending the reading to $\frac{1}{10000}$ **inch** or **0·0001 inch.**

The diagram is of micrometer calipers designed to measure diameters, or lengths, between 0 in. and 1 in. It consists of :

(*a*) A steel **frame**.

(*b*) A steel **hub** graduated as shown, in $\frac{1}{40}$ in. divisions.

(*c*) A **leading screw of 40 threads per inch** attached to a knurled **thimble.** The end of the thimble is tapered and divided, circumferentially, into 25 equal parts.

(*d*) The leading screw is made to work in a suitable **nut** at the top of the hub, so that the top and bottom **anvils** constitute two jaws in which the work to be measured may be held.

(*e*) The anvils are of hardened steel, ground and lapped to, as near as possible, perfectly flat parallel surfaces.

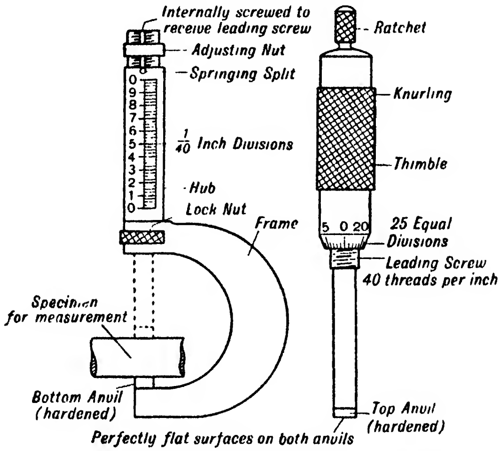


FIG. 19. 0 to 1 inch micrometer screw gauge.

(*f*) An **adjusting nut** is fitted to the top of the hub, which is **sprung** by two, or more, slots, and this serves to tighten, or loosen the fit of the leading screw in the hub.

(*g*) Provision of a **lock nut** allows the micrometer to be locked at any desired setting.

(*h*) A **ratchet stop and spring pawl** is often fitted to prevent excessive pressure being applied by the operator. This allows the ratchet to slip past the pawl when excessive pressure is applied, thus preventing further turning of the leading screw.

Theory and use of the micrometer. As the knurled thimble is rotated, the leading screw works in its nut and opens, or closes, the jaws

of the instrument. The object to be measured is inserted between the jaws and a light contact between the specimen to be measured and the anvils is secured by turning the thimble. *This contact should be firm, but in no circumstances should the micrometer be forced.* The reading is then taken according to the method outlined in the specimen reading.

THEORY. One complete revolution of the thimble will cause the jaws to open or close $\frac{1}{40}$ in., because the leading screw is 40 threads per in.

The division of the thimble into 25 equal parts allows the operator to rotate the thimble any number of $\frac{1}{25}$ parts of a revolution and every $\frac{1}{25}$ of a revolution opens, or closes, the jaws $\frac{1}{25} \times \frac{1}{40}$ in.

Thus, **1 revolution** causes $\frac{1}{40}$ **in. or 0·025 in.** opening or closing of the jaws,

$\frac{1}{25}$ **revolution** causes $\frac{1}{25} \times \frac{1}{40}$ **or** $\frac{1}{1000}$ **in. or 0·001 in.** opening or closing of the jaws.

A specimen reading (Fig. 20). In taking a micrometer reading the following method should be followed: (*a*) Examine the hub and note the number of graduations uncovered by the thimble; in the example this is 0·8 in. together with $\frac{3}{40}$ in. which, expressed as a decimal, is 0·875 in.

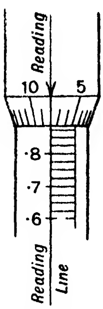


FIG. 20.

(*b*) Examine the thimble and note the reading opposite to the reading line; in the example this is 8 and indicates an additional $\frac{8}{1000}$ in. or 0·008 in. to the 0·875 in. already recorded.

(*c*) The actual micrometer reading is

$$0{\cdot}875 \text{ in.} + 0{\cdot}008 \text{ in.} = \mathbf{0{\cdot}883 \text{ in.}}$$

NOTE.—*It is possible to estimate fractions of* $\frac{1}{1000}$ *in., thus employing the micrometer for reading to* $\frac{1}{2000}$ *in., or even* $\frac{1}{4000}$ *in.*

The Dial Test Indicator. This is an instrument which has found considerable use in both works and laboratories in cases where a visible indication of a small alteration in a dimension, or a small movement of a subject under test is required. For example, this instrument (Fig. 21) will measure correct to $\frac{1}{1000}$ in. the deflection of a beam, or the amount of eccentricity in a rotating shaft, or it

may be applied to indicate the amount of the vibration of a structure under certain loading conditions.

The diagram outlines the standard type of instrument supplied by small tool manufacturers in which the dial of the instrument is graduated to read to $\frac{1}{1000}$ in. The contact piece may be brought

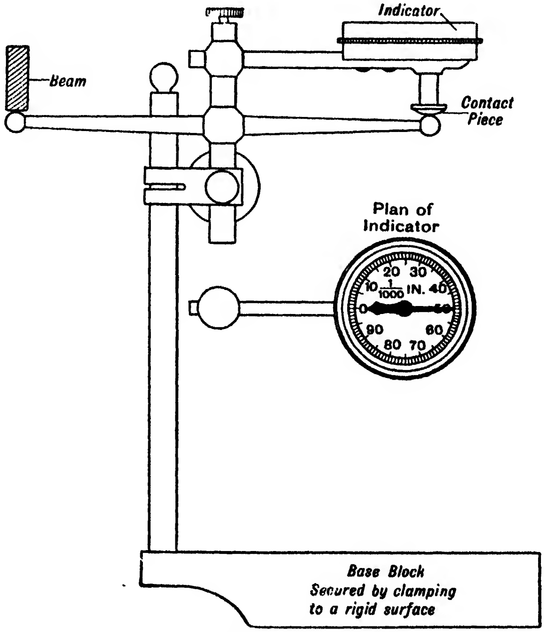


FIG. 21. Dial test indicator arranged to indicate the deflection of a loaded beam.

into contact with the work directly, or the lever system shown may be employed to measure a movement at a distance from the indicator. The dial can be rotated so that any initial reading may be regarded as the zero from which measurement is taken. The lever system can be utilised to allow the indicator to read to a greater degree of accuracy by making the lever arms unequal. Further arrangements of the apparatus are shown (Figs. 22 and 23), and illustrate some of the many methods by which the instrument may

be employed to obtain indications of small movements of the contact piece.

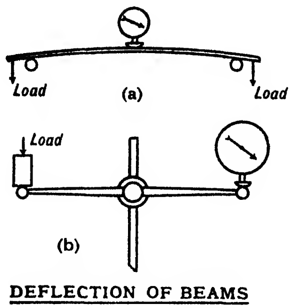


FIG. 22. Use of a dial test indicator.

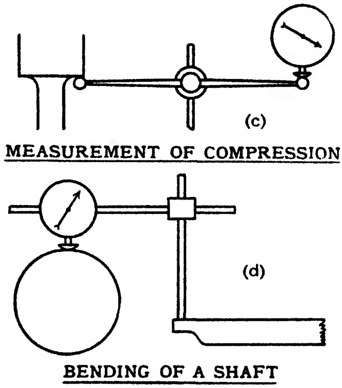


FIG. 23. Use of a dial test indicator.

This apparatus can be obtained in the alternative form shown, a form which in some respects is more convenient for use in an engineering laboratory (Fig. 24).

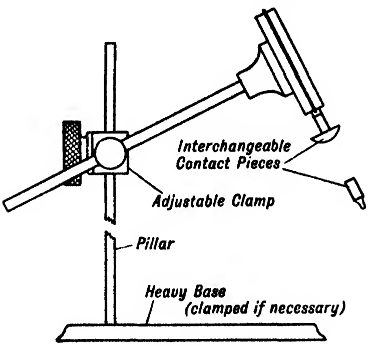


FIG. 24.

The planimeter. The planimeter is an instrument used for measuring, approximately, the areas of irregular plane figures. One of the best known forms is the **Amsler planimeter**. This polar planimeter was invented by Jacob Amsler in 1854. A simple form of this type of instrument is shown in Fig. 25, and it consists of two arms AB and BC pivoted together at B. In using the instrument the point *s* is kept fixed in a convenient position outside the area to be measured by weighting the end of the arm A with a cylindrical weight, while the tracing point T is guided carefully round the outline of the figure. Usually a small foot rest (not shown) is supplied near the point T, which assists in preventing the point from digging

into the paper and thus generally adds to the stability of the instrument. The arm BC carries a wheel D accurately made, the periphery of which is usually divided into 100 equal parts. Thus when the instrument is in use the point T and the foot rest, the rim of D and the fixed point *s* are in contact with the paper. The shaft of wheel D is supported in pivots at *a* and *b* and a small worm F, cut on the shaft, enables the motion of D to be communicated to the dial W in such a way that one revolution of D turns W through $\frac{1}{10}$th of a revolution.

A vernier V is fixed to the frame of the instrument and nine small divisions of the scale D are divided into ten equal parts on the vernier. Readings on the dial W are made by means of the pointer shown in

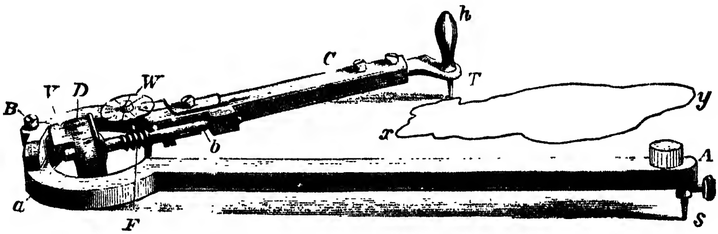


FIG. 25. The Amsler planimeter.

Fig. 25. If one division on W is taken as the unit, the small divisions on D will represent $\frac{1}{100}$th unit, while the vernier enables a reading to $\frac{1}{1000}$th unit or three decimal places to be made.

In finding the area of a figure such as *xy* the reading of the dial and wheel is first noted by taking the lowest figure near the dial pointer, then the lowest figure and subdivision near the zero vernier division and finally the vernier reading itself. The area is then traced and the instrument read again. By taking the difference between the initial and final readings, and multiplying by a constant, the area of the figure is obtained. This constant depends upon the distance between TB, which, in more expensive instruments, is made adjustable to adapt them for variations in scales on plans and maps. Should the value of the constant be not known, it can be found easily by finding the instrument reading corresponding to the area of a square or rectangle of known dimensions. Then the constant equals

$$\frac{\text{Area of square or rectangle}}{\text{Difference of instrument readings for square or rectangle}}.$$

For greater accuracy, the average of two readings may be taken, one reading by tracing the area clockwise and one anti-clockwise.

Example. For xy (Fig. 25) the readings are :—

	Readings of Instrument	Difference	Mean Reading	Area
Initial - - -	8·317			0·517 × 10 = 5·17 sq. in.
After clockwise tracing - - -	8·832	0·515	0·517	
After anti-clockwise tracing - - -	8·313	0·519		

Constant of instrument = 10.

Where portions of the area are bounded by straight lines it may be convenient to use a straight edge or ruler to guide the pointer T over these portions. It is important to place the fixed point s in such a position that the area can be traced freely and wheel D moves only over a clean and uniform surface so that slipping is prevented as far as possible.

EXERCISES ON CHAPTER II

1. A vernier is to read to $\frac{1}{80}$ inch and the fixed scale is divided into $\frac{1}{10}$ inch parts. Set out small portions of the fixed and sliding scales to indicate readings $\frac{65}{80}$ and $1\frac{5}{80}$ in. in length.

2. Read the examples of the vernier settings shown in Fig. 26.

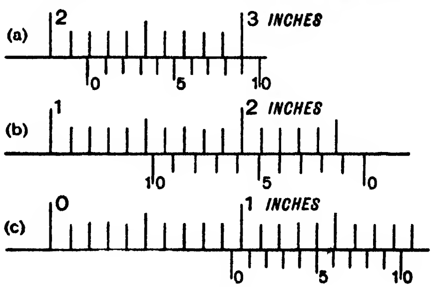


FIG. 26.

3. Sketch a vernier setting which reads 1·26 in. and construct a vernier to read to $\frac{1}{10}$ millimetre.

4. Write down the reading of the micrometer in the setting shown (Fig. 27).

5. Make a sketch of a micrometer set to a reading of 0·3855 in.

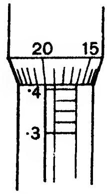


Fig. 27.

6. A micrometer is to read to $\frac{1}{100}$ millimetre, and the leading screw is 25 threads per centimetre. How will the hub be divided and how many divisions will appear on the thimble ?

7. Consult a manufacturer's catalogue, or attempt to see the micrometers in a workshop, and sketch micrometers suitable for the following purposes :

(1) Measuring the internal diameter of motor-car cylinders.
(2) Measuring the depth of a flat-bottomed hole in a piece of metal.
(3) Measuring the thickness of thin plates.
(4) Measuring the core, or bottom of the thread, diameter of a screw thread.

8. Sketch an arrangement of a dial test indicator for each of the following purposes :

(1) To measure the amount by which a small flywheel is not truly set upon its shaft.
(2) To measure the compression of a very stiff spring under load.
(3) To record accurately the lift of a mushroom valve in a small petrol engine.
(4) To record the transverse variation in diameter of a small shaft in a lathe.

9. Describe with sketches an instrument for measuring the areas of irregular plane figures.

10. Describe how the constant for a planimeter may be obtained and how the instrument may be read.

CHAPTER III

ROTATING EFFECT OF A FORCE—MOMENTS AND LEVERS —THE PRINCIPLE OF MOMENTS AND ITS APPLICATIONS

Equilibrium. **When a body is at rest under the action of external forces it is said to be in equilibrium.**

Rotating effect of a force. Among the effects of a force which have not previously been considered is the rotating effect. For example, the force exerted by the rider of a cycle upon the pedal causes the crank to rotate about its axis, or, a door is opened by a force which causes the door to rotate about its hinges.

The fulcrum. The point about which rotation occurs is known as the fulcrum.

Consider a wheel (Fig. 28) turned by a force of 2 lb. and free to rotate about an axle O. It is easy to see from a simple experiment that the wheel will turn much more readily when the force is applied at B, than at A. It thus appears that the ease of turning depends on the distance of the point of application of the force from the axle, or fulcrum. In other words, the 2 lb. force has a greater turning effect the greater the distance of its line of action from the axle, and is said to have a greater **moment** at B than at A. The term **moment** in this case means *importance*, and the force of 2 lb. at B is more important from the rotating point of view than when it is applied at A.

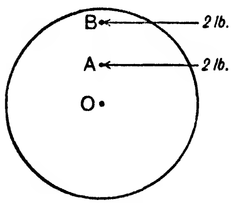


Fig. 28.

Measurement of moment. The moment of a force depends upon two quantities :

(*a*) **the magnitude, or size of the force,**

(*b*) **the perpendicular distance of its line of action from the fulcrum.**

It seems reasonable, therefore, to measure moment as the product

of the magnitude of the force and the perpendicular distance of its line of action from the fulcrum.

Moment of a force = Force × perpendicular distance of its line of action from the fulcrum.

Unit of moment. The unit of moment generally used is the **pound foot**—a unit in which the force is 1 pound and the distance or arm 1 foot.

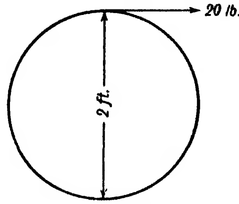


Fig. 29.

Example 1. *A wheel 2 ft. in diameter* (Fig. 29) *is turned about its axle by a force of 20 lb., applied at the rim. Find the moment of this force.*

Force = 20 lb.
Distance = Radius of wheel = 1 ft.
Moment = 20 × 1 = **20 lb. ft.**

Example 2. *Find the moment in the example shown* (Fig. 30).

Force = 6 lb.
Arm or Distance = 4 ft.
Moment = 6 × 4 lb. ft. = **24 lb. ft.**

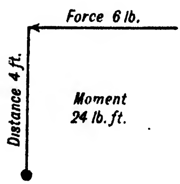


Fig. 30.

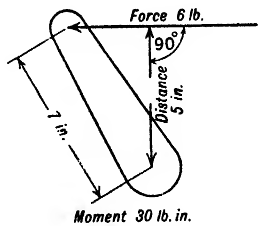


Fig. 31.

Example 3. *Find the moment of the force acting on the crank shown in Fig.* 31.

Force = 6 lb.

Note.—*Distance must be the perpendicular distance.*

Distance = 5 in. *not* 7 in.
Moment = 6 × 5 lb. in. = **30 lb. in.** or **2·5 lb. ft.**

Classification of moments. For convenience, it is customary to classify moments under two headings: (1) those moments having

a tendency to rotate the body in the direction in which the hands of a clock move, or **clockwise moments**, and (2) those moments having a tendency to rotate the body against the direction of the clock-hands, or **anti-clockwise moments** (Fig. 32).

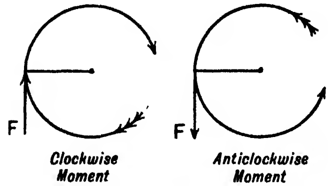
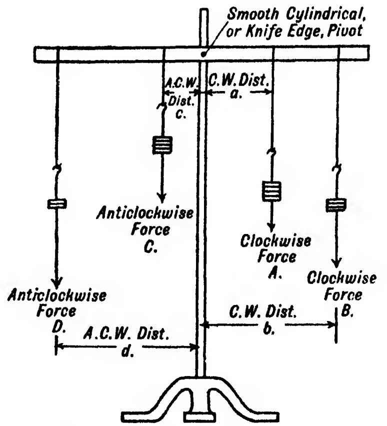


Fig. 32.

Expt. 3.—Object. *To show that if a pivoted body is in equilibrium under the action of a system of forces, then the sum of the clockwise moments, about the pivot, is equal to the sum of the anti-clockwise moments, also taken about the pivot.*

Apparatus. A suitable stand, a pivot, a stick of about 3 feet in length, of uniform material depth and thickness, a scale, suitable weights and hangers.

Method of Procedure. The stick is pivoted about its centre on a small circular pivot or a knife-edged fulcrum, so that it is free to rotate, but is balanced about its pivot. Cotton loops are made to hang upon the scale, and to be capable of sliding freely along it. To the lower ends of these loops, hangers with weights, chosen at random, are attached. In the first two experiments, one hanger is placed on each side of the fulcrum and the position of the loops adjusted to balance the lever, or to produce equilibrium. After deciding which of the two forces is clockwise in its effect the distance from the fulcrum to the loop supporting each load is measured. The results, forces and corresponding distances are then entered in a table and the clockwise moment calculated by multiplying the clockwise force by the clockwise distance. Similarly, the anti-clockwise moment is obtained by using the anti-clockwise force and distance.



Sum of Clockwise Moments = $A \times a + B \times b$ lb. in.
Sum of Anticlockwise Moments = $C \times c + D \times d$ lb. in.

Fig. 33.

In the later experiments a larger number of forces are employed and the *sums* of the clockwise and anti-clockwise moments obtained

separately. To satisfy the object of the experiment, the clockwise moment or the sum of the clockwise moments must be compared with the anti-clockwise moment or the sum of the anti-clockwise moments, that is, the result in column C. with that in column A.C.

OBSERVATIONS AND RESULTS.

					C.	A.C.
Expt.	Clockwise Force	Clockwise Distance	Anti-clockwise Force	Anti-clockwise Distance	Clockwise Moment	Anti-clockwise Moment
1	0·5 lb.	9·4 in.	0·3 lb.	15·6 in.	4·7 lb. in.	4·68 lb. in.
2	0·2 lb.	19·0 in.	0·6 lb.	6·3 in.	3·8 lb. in.	3·78 lb. in.
3	0·3 lb. 0·4 lb.	3·9 in. 10·9 in.	0·8 lb.	6·8 in.	5·53 lb. in.	5·44 lb. in.
4	0·5 lb. 0·3 lb.	15·3 in. 3·2 in.	0·5 lb. 0·2 lb.	10·2 in. 17·6 in.	8·61 lb. in.	8·62 lb. in.
5	0·7 lb. 0·5 lb.	15·2 in. 6·4 in.	0·5 lb. 0·5 lb.	10·2 in. 17·6 in.	13·84 lb. in.	13·9 lb. in.

CONCLUSION. When the stick is in equilibrium under the action of forces, the sum of the clockwise moments is, in each case, very nearly equal to the sum of the anti-clockwise moments, which verifies what is known as the **Principle of Moments.**

As an alternative experiment the moment board can be used. The object of the experiment is unaltered, but the apparatus (Fig. 34) consists of an irregular board of uniform thickness pivoted at its centre of gravity, that is, a point where the weight of the board may be considered to be concentrated. In the first instance loads are hung from the cords A, B, C, and D and the board allowed to rotate until its equilibrium position is reached. Then the distances w, x, y, and z are measured. An analysis of the arrangement gives the following:

Forces: clockwise, C and D lb.
anti-clockwise, A and B lb.

Distances: clockwise, y and z in.
anti-clockwise, w and x in.

Moments: clockwise, $C \times y$ and $D \times z$ lb. in.
anti-clockwise, $A \times w$ and $B \times x$ lb. in.

In the second arrangement (Fig. 35) some of the forces may be directed by pulleys, to act in directions other than vertical. The distances must be measured at right angles to the lines of action of the forces, when the analysis shown above will still be true.

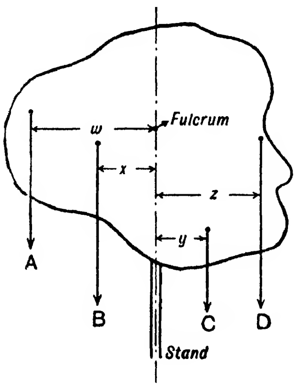


FIG. 34. Moment board (1).

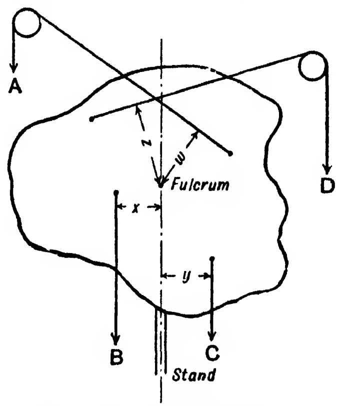


FIG. 35. Moment board (2).

The observations for these experiments should be tabulated in the manner shown for the pivoted lever experiment, and the conclusions drawn from the calculated moments.

Principle of moments. The conclusions to be drawn from these experiments verify what is known as the Principle of Moments, which has extensive application in the treatment of levers and practical problems generally. The principle is :

If any number of forces, in one plane, act upon a body free to rotate about a fixed point, and produce equilibrium, then the sum of the moments of the clockwise effect forces is equal to the sum of the moments of the anti-clockwise effect forces when the moments are taken about the fixed point, or fulcrum.

NOTE.—As the forces keep the body at rest and there is no rotation, any fixed point in the plane of the forces could be chosen for fulcrum so long as it was on the body or rigidly connected to it. Then the moments of all the forces must be taken, including forces of gravity.

Levers. A lever is a rigid bar free to, or able to rotate about a fixed point, known as the **fulcrum**, which may also be regarded as the point about which the lever is sustained.

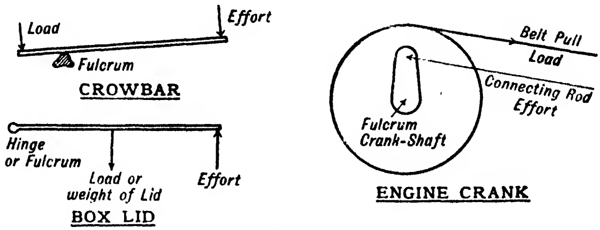


FIG. 36.

Common examples of levers. (1) **The crowbar.** In this case the fulcrum is, possibly, a block of wood placed near to the load (Fig. 36).

(2) **Boxlid.** In this case the fulcrum is at the hinge.

(3) **Engine crank.** In this case the fulcrum is at the crankshaft.

The load is the name given to the resistance overcome by the lever.

The effort is the name given to the force used to operate the lever and overcome the resistance.

The reaction at the fulcrum is the pressure, or force, existing between the fulcrum and its point of contact with the lever when the lever is working.

The skeleton lever. In the treatment of levers it is convenient to reduce the lever to its skeleton form; for example, a wheelbarrow may be reduced to a skeleton diagram (Fig. 37).

This can be treated by the principle of moments in order to find the unknown effort E lb. Let E lb. be the effort.

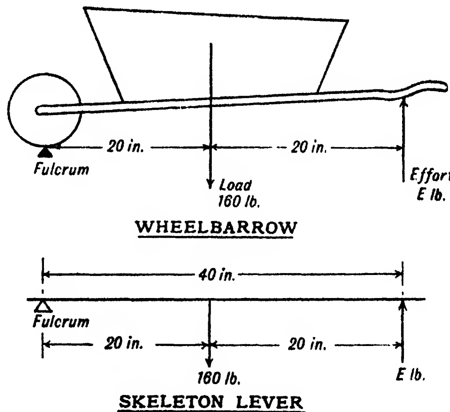


FIG. 37.

Then, the **clockwise moment** $= \text{Load} \times 20$
$= 160 \times 20 = 3200$ lb. in.,

and the **anti-clockwise moment** $= \text{Effort} \times 40$
$= E \times 40 = 40E$ lb. in.

By the **principle of moments**

$$40E = 3200 \text{ for equilibrium,}$$

and $$E = 80. \quad \textbf{Ans. 80 lb.}$$

NOTE.—*The student should realise that the effort found in this example is the effort required to produce equilibrium, and any additional effort, when applied, will produce motion in the direction of the effort.*

This method may be extended to levers having several loads.

Example. Signal control lever. *To find the effort E,* Fig. 38.

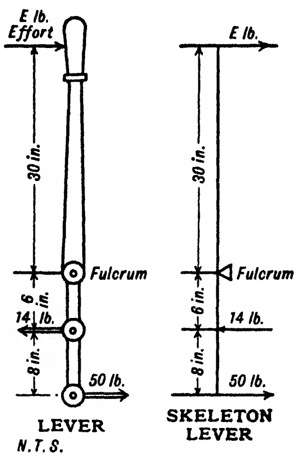


FIG. 38.

The clockwise effect forces are the effort and the force of 14 lb., whereas the force of anti-clockwise effect is 50 lb.

Sum of clockwise moments
$= E \times 30 + 14 \times 6 = 30E + 84$ lb. in.
Anti-clockwise moment
$= 50 \times 14 = 700$ lb. in.

NOTE.—*The distance is measured to the fulcrum and is* 14 *in., not* 8 *in.*

Then, by the principle of moments,

$$30E + 84 = 700,$$
$$30E = 700 - 84 = 616,$$
$$E = \tfrac{616}{30} = 20{\cdot}53. \quad \textbf{Ans. 20·53 lb.}$$

To find the reaction at the fulcrum. The fulcrum must exert a reaction sufficient to balance the thrust of the forces upon the lever. If the lines of action of the loads and effort are parallel, it will be equal to the algebraic sum of these forces, that is, the loads and effort, and opposite in its direction of action. In the previous example, the fulcrum must supply a pressure acting against

the combined loads and effort, sufficient to prevent it moving in the direction of their application. Thus :

(A) The sum of the forces acting from left to right = effort + 50 lb.
(B) The force acting from right to left = 14 lb.

∴ Reaction at the fulcrum $= A - B$

$$= (20{\cdot}53 + 50) - 14$$
$$= 70{\cdot}53 - 14 = 56{\cdot}53 \text{ lb.}$$

in a direction right to left, that is opposite to the combined forces A.

Example. *Find the effort and the pressure on the fulcrum in the lever shown* (Fig. 39).

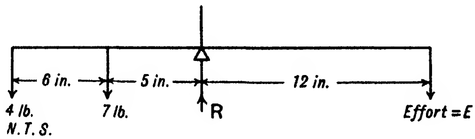


FIG. 39.

Let E be the effort and R the reaction at the fulcrum.

Clockwise moments $= 12E$ lb. in.
Anti-clockwise moments $= 7 \times 5 + 4 \times 11 = 79$ lb. in.

By the principle of moments $12E = 79$

$$E = 6{\cdot}58 \text{ lb.}$$

Pressure on the fulcrum $= R =$ algebraic sum of loads and effort.

$$R = 4 + 7 + 6{\cdot}58 = 17{\cdot}58 \text{ lb.}$$

NOTE.—*If one of these forces were acting upwards it would reduce the pressure on the fulcrum.*

Bent and curved levers. The principle of moments is used in the treatment of these types of levers, and a few worked examples will illustrate this treatment. This type of lever enables a resistance to be overcome in a direction other than that in which the effort is applied. In consequence they are frequently employed in the construction of engines and machinery.

It is particularly important to realise that all distances taken for the calculation of the moment must be measured at right angles to the line of action of the force.

Example 1. *To find the effort E* (Fig. 40).

Clockwise moment $= 10E$ lb. in.
Anti-clockwise moment $= 60 \times 8$ lb. in.

then $$480 = 10E,$$
$$E = 48.$$ **Ans. 48 lb.**

The distance 8 in. is taken at right angles to the force of 60 lb. and as the lever turns the distance alters.

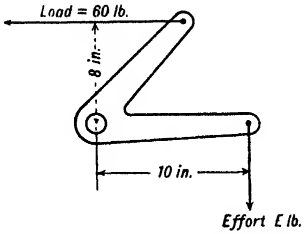


FIG. 40. Bell crank lever.

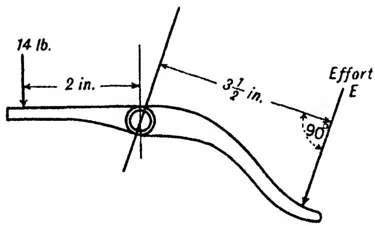


FIG. 41. Rocker bar.

Example 2. *To find the effort E* (Fig. 41).

Clockwise moment $= 3\frac{1}{2}E$ lb. in.
Anti-clockwise moment $= 14 \times 2$ lb. in.

$$3\tfrac{1}{2}E = 28,$$
$$E = 8.$$ **Ans. 8 lb.**

The distance $3\frac{1}{2}$ in. is measured at right angles to the line of action of the effort.

In the following examples it is necessary to find quantities other than the effort.

Example 1. *Find the position of the fulcrum when a bar* 25 *in. in length is to lift* 40 *lb. with an effort of* 14 *lb.*

Apply the principle of moments and let the fulcrum be at a distance x in. from the load.

Then clockwise moment = anti-clockwise moment,

$$14 \times (25 - x) \text{ lb. in.} = 40x \text{ lb. in.},$$
$$350 - 14x = 40x,$$
$$54x = 350, \; x = 6 \cdot 48.$$ **Ans. 6·48 in. from the load.**

Example 2. *A bar carries two loads A and B of magnitude 6 and 4 lb., situated respectively at 10 in. and 3 in. on one side of the fulcrum. Find the position of an effort of 3 lb. which would operate the lever* (Fig. 42).

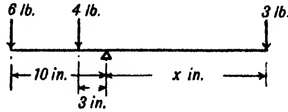


FIG. 42.

Let the effort be applied at a point x in. from the fulcrum, then the clockwise moments = anti-clockwise moments.

$$3x \text{ lb. in.} = (4 \times 3) + (6 \times 10) \text{ lb. in.}$$

$3x = 72$, $\mathbf{x} = 24$. **Ans. 24 in. from the fulcrum.**

Example 3. A structural problem. *A section of a Warren girder is shown in Fig. 43. Find the force F necessary to produce equilibrium if all the joints are rigidly connected.*

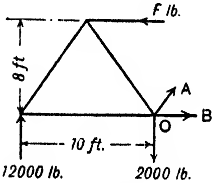


FIG. 43.

NOTE.—Before attempting this example it is necessary to understand that the forces 12,000 lb., 2000 lb., A, B and F keep the body, that is, the portion of the girder shown, in equilibrium. A and B are not known, so that it is necessary to select a point about which to take moments in order that A and B shall have *no moment* about this point. For this reason it is convenient to take the point O as the imaginary point, or fulcrum.

Moments about O: Clockwise $= 12{,}000 \times 10$ lb. ft.

Anti-clockwise $= F \times 8$ lb. ft.,

then $8F = 120{,}000$ and

$\mathbf{F} = 15{,}000$. **Ans. 15,000 lb.**

Example 4. A complex system of levers (Fig. 44). *Find the force F required to operate a remote control for a hydraulic accumulator.*

(1) Consider the lever A,

then the force in C $\times 3 = 16 \times 4$,

force in C $= \frac{64}{3} = 21\frac{1}{3}$ **lb**

(2) Consider the lever B,

then the force in C $\times 7 = F \times 8$.

$$F = \frac{21\frac{1}{3} \times 7}{8} \text{ lb.} = \mathbf{18{\cdot}66 \text{ lb.}}$$

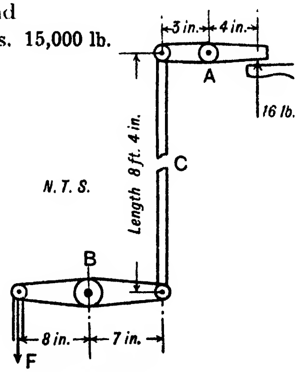


FIG. 44. Remote control.

Example 5. *Find the position of the footstep in order to operate the lever shown* (Fig. 45).

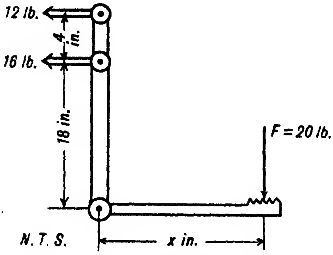


FIG. 45. Footstep control.

Let x be the distance in inches from the fulcrum.

Clockwise moment

$$=20x \text{ lb. in.}$$

Anti-clockwise moment

$$=18 \times 16 + 22 \times 12$$

$$=552 \text{ lb. in.}$$

$$20x = 552.$$

$x = 27{\cdot}6$. **Ans. 27·6 in.**

Moment of a couple. A couple is a combination of two equal and opposite forces in the same plane, which together tend to produce rotation, that is, act along parallel straight lines (Fig. 46).

The distance $2r$ is known as the **arm of the couple.**

Moment of the couple $= 2Pr$

$= 2Pr$ lb. ft. if r is in feet and P in pounds.

PROOF. Taking moments about A, the anti-clockwise moment $= 2r \times P$ lb. ft., and P has no moment when acting through A.

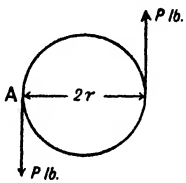


FIG. 46. Couple.

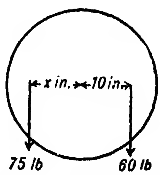


FIG. 47. Face plate.

Balancing.

Example 1. *A face plate carries a job weighing* 60 *lb. situated* 10 *in. from its centre. Find the position of a* 75 *lb. weight which a workman proposes to use to balance the plate* (Fig. 47).

The weight will be diametrically opposite the 60 lb. load at a distance x in. from the centre, so that

$$60 \times 10 \text{ lb. in.} = 75 \times x \text{ lb. in.}$$

$75x = 600$, $x = 8$. **Ans. 8 in. from the centre.**

Example 2. *An overhung crank weighs 35 lb., and this weight is concentrated at a point 6 in. from the centre of the crankshaft. What weight must be placed 4 in. from the crankshaft centre, on the other side, in order to balance the crank* (Fig. 48) ?

Let the weight be W lb.,

then
$$6 \times 35 \text{ lb in.} = 4 \times W,$$
$$210 = 4W,$$
$$W = 52{\cdot}5. \quad \textbf{Ans. 52·5 lb.}$$

This static balance is also the condition for running balance.

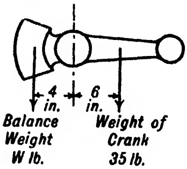


FIG. 48.

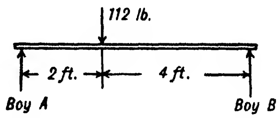


FIG. 49.

Reaction of beam supports. An important application of the principle of moments is found in the determination of the reactions, or upward pressures, exerted by the supports of horizontal beams with vertical loading.

Example 1. *If two boys,* A *and* B, *carry a load of* 1 *cwt. placed upon a pole, as shown, find the proportion of the weight carried by each boy. Neglect the weight of the pole* (Fig. 49).

Suppose the boy B removed his effort, then the pole would fall, rotating about the boy A in falling. Therefore if A is considered as a fulcrum, and moments taken about A, the following is obtained :

Clockwise moment $= 112 \times 2 = 224$ lb. in.

Anti-clockwise moment $= 6 \times$ force exerted by B $= 6B$ lb. in.

$$6B = 224,$$
$$B = 37\tfrac{1}{3}.$$

Similarly, if the boy A were to remove his effort the pole would revolve about the boy B, and taking B as the fulcrum,

Clockwise moment $= 6 \times$ force exerted by A $= 6A$ lb. in.

Anti-clockwise moment $= 112 \times 4 = 448$ lb. in.,

therefore,
$$6A = 448,$$
and
$$A = 74\tfrac{2}{3}.$$

It will be noticed that the loads carried by A and B together make up the total load of 1 cwt., that is, $37\frac{1}{3} + 74\frac{2}{3} = 112$ lb.

Ans. A carries $74\frac{2}{3}$ lb., B carries $37\frac{1}{3}$ lb.

Example 2. *Consider a beam loaded as shown. Find the reactions of the supports P and Q* (Fig. 50).

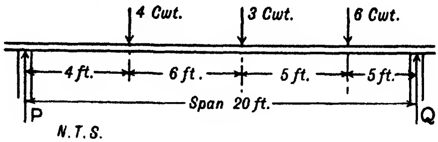


FIG. 50.

Since both P and Q are unknown take moments about P.

$$\begin{aligned}
\text{Clockwise moment} &= 4 \times 4 + 10 \times 3 + 15 \times 6 \\
&= 16 + 30 + 90 \\
&= 136 \text{ cwt. ft.} \\
\text{Anti-clockwise moment} &= Q \times 20 = 20Q \text{ cwt. ft.}
\end{aligned}$$

then
$$20Q = 136,$$
$$Q = 6{\cdot}8.$$

Now
$$\begin{aligned}
P &= \text{total load} - Q \\
&= 4 + 3 + 6 - Q = 13 - 6{\cdot}8 \\
P &= 6{\cdot}2.
\end{aligned}$$

Ans. Reactions P = 6·2 cwt. Q = 6·8 cwt.

EXPT. 4. OBJECT.—*To compare the calculated and observed reactions for the supports of a beam or bar, supported horizontally and carrying vertical loading.*

APPARATUS. A beam supported at its ends by two spring balances, each of which can be adjusted to maintain the loaded beam in a horizontal position. A variety of weights, several shackles and a scale.

METHOD OF PROCEDURE. Find the weight of each shackle, and load the beam with the loads A and B (Fig. 51) which include the weight of the shackle in each case. Then adjust the flynuts at the top of the balances P and Q in order to set the beam horizontal. Read the spring balances P and Q and enter into the table the observations required, that is, the loads A and B, including the weight of the shackles, the weight of the beam W and the distances a and b. Read the spring balances, P and Q, and enter the readings in the table; these will be the observed reactions. Now take moments about a point in the line of action of P: the loads A and B, together with the weight of the

beam, which acts at mid-span, constitute the forces with clockwise effects, while the upward force, or reaction Q, has an anti-clockwise effect. Calculate the clockwise moments X, Y and Z, which are respectively $A \times a$, $B \times b$ and the weight of the beam $\times \frac{1}{2}$ span. Next calculate the anti-clockwise moment, which is $Q \times$ span, then by the

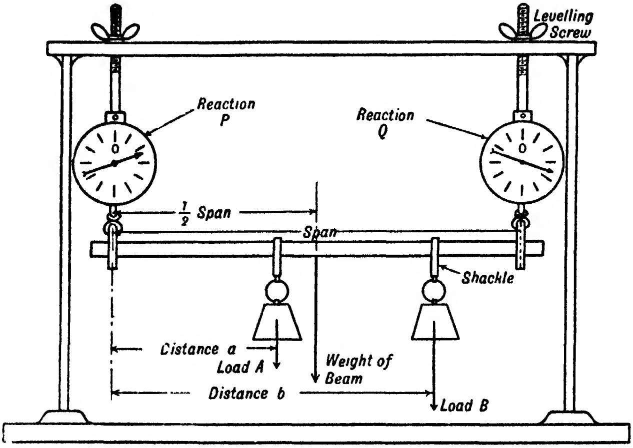


FIG. 51. Apparatus for reaction of beam supports.

principle of moments the clockwise moments are equal to the anti-clockwise moments, that is,

$$Q \times \text{span} = X + Y + Z,$$

and
$$\mathbf{Q} = \frac{X + Y + Z}{\text{span}} \text{ or } \frac{\mathbf{R}}{\textbf{span}}.$$

To find the reaction P, subtract from the total load $A + B + W$ the reaction Q, or alternatively, take moments about a point in the line of action of Q in a manner similar to the above and thus find P.

Repeat the experiments for several different loadings, and in each case compare the observed with the calculated or derived reactions.

In every case when the beam is horizontal and subjected to a series of vertical loads, including the weight of the beam, it will be found that :

(1) the sum of the moments of all the loads about a support is equal to the moment of a beam reaction about that support,

(2) the sum of the upward vertical supporting forces or reactions is equal to the sum of the loads.

OBSERVATIONS. Span, or distance between supports = 3 ft.
Weight of each shackle = 0·75 lb.
Weight of the beam = 1·5 lb. = W.

Experiment No.	Load A lb.	Load B lb.	Distance a in.	Distance b in.	Observed Reaction P lb.	Observed Reaction Q lb.
1	6·75	—	20·0	—	3·75	4·5
2	—	8·75	—	30·0	2·0	8·0
3	6·75	5·75	10·0	20·0	8·0	6·0
4	10·75	4·75	20·0	10·0	9·0	8·0
5	12·75	7·75	15·0	25·0	10·5	11·5

DERIVED RESULTS. Moments about P.

	X	Y	Z	R	S	P	Q
Experiment No.	Moment of A $A \times a$ lb. in.	Moment of B $B \times b$ lb. in.	Moment of Beam $W \times \frac{\text{Span}}{2}$ lb. in.	Total Moment $X+Y+Z$ lb. in.	Moment of Q $Q \times \text{span}$ lb. in.	$P = A+B+W-Q$ lb.	$Q = \frac{R}{\text{Span}}$ lb.
1	135	—	27	162	162	3·75	4·5
2	—	262·5	27	289·5	288	2·21	8·04
3	67·5	115	27	209·5	216	8·2	5·8
4	215	47·5	27	289·5	288	8·96	8·04
5	191·2	193·8	27	412	414	10·6	11·4

The platform weighing machine (Fig. 52) is designed to register the correct weight of a body placed in any position upon the platform. The lever system under the platform consists of a long lever, pivoted at F and fitted to the effort rod at the other end. The weight on the platform is divided into two forces W_1 and W_2, of which the latter is made to act directly on to the long lever, while W_1 actuates a short lever pivoted at G and transmits through this lever a force W_3

to the long lever. The effort rod operates the weigh bar, which is a simple lever with a fulcrum at H.

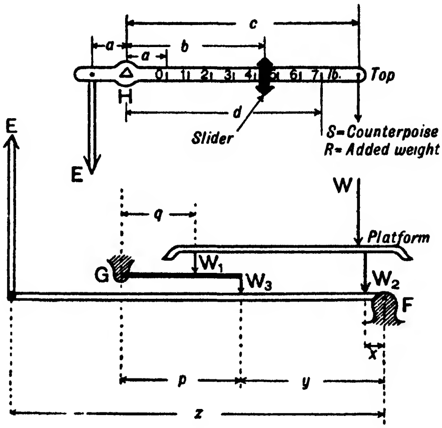


Fig. 52.

Principle of the platform lever system.

Taking moments about F, for equilibrium,

(a) $$Ez = W_3y + W_2x,$$

and moments about G, since the reaction to W_3 acts upwards,

$$W_1q = W_3p,$$

or $$W_3 = \frac{W_1q}{p}.$$

Substituting in (a) above,

$$Ez = \frac{W_1qy}{p} + W_2x.$$

Let $\mathbf{x} = \frac{\mathbf{qy}}{\mathbf{p}}$, then $Ez = W_1x + W_2x = x(W_1 + W_2)$,

but $$W = W_1 + W_2,$$

(b) therefore, $$\mathbf{Ez} = \mathbf{Wx} \text{ or } \mathbf{E} = \frac{\mathbf{Wx}}{\mathbf{z}}.$$

Thus the condition for a registration of the true weight of W placed anywhere on the platform is $\mathbf{x} = \frac{\mathbf{qy}}{\mathbf{p}}$.

Principle of the top lever system. This is a simple lever pivoted at H (Fig. 52) and counterpoised with a weight S which balances the lever when the slider is at O. Weights R are added to the counterpoise to produce equilibrium when W is applied to the platform. These weights are arranged so that

$$R \times c = E \times a,$$

or
$$R = \frac{Ea}{c} \text{ where } E = \frac{Wx}{z} \text{ (see (}b\text{) above).}$$

Thus
$$R = \frac{Wxa}{zc} = \left(\frac{xa}{zc}\right) W.$$

For the finer readings of the weight W, the slider is used. The weigh arm is graduated so that the full movement of the slider along the arm produces the same effect as adding the smallest unit to R.

Suppose the slider weighs m lb., then

$$m \times d = (\text{smallest value of } R) \times c.$$

This determines the distance d, which can be subdivided according to the magnitude of the smallest value of R. Usually this is 7 lb., so that the distance d can be divided into 7 parts and again subdivided as required.

Motion aspect of a lever. Consider a lever simply loaded as shown (Fig. 53).

The load L will move a distance x in the same time as it takes the effort E to move the distance y. Now, from a consideration of *similar triangles* or *sectors*,

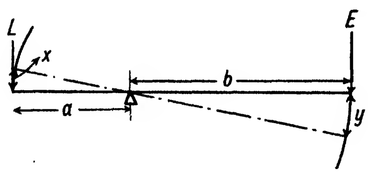


FIG. 53.

$$x : y :: a : b \text{ or } \frac{x}{y} = \frac{a}{b}.$$

From the principle of moments,

$$L \times a = E \times b \text{ and } \frac{E}{L} = \frac{a}{b}.$$

Combining these results,

$$\frac{E}{L} = \frac{x}{y} \text{ or } \frac{L}{E} = \frac{y}{x}.$$

Thus, if the effort be less than the load the effort will have to move a distance proportionally greater than the load ; in other words, **what is gained on the force is lost on the distance.**

This is an important relation which has a considerable bearing upon the treatment of machines in a later chapter.

The ratio $\frac{\text{load}}{\text{effort}}$ is called the **mechanical advantage** of this simple lever.

The ratio $\frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$ in the same time is called the **velocity ratio** of this simple lever.

When there is no friction **the mechanical advantage is equal to the velocity ratio.**

EXERCISES ON CHAPTER III

1. State the Principle of Moments and explain the meaning of the term equilibrium.

2. A crowbar 6 ft. in length is used to raise a load of 70 lb. Find the effort, if the fulcrum is placed 8 in. from the load.

3. In a pair of metal plate shears the force required to shear the metal at A is 126 lb. Find the pressure necessary at P (Fig. 54).

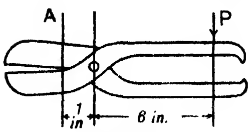


FIG. 54.

4. Solve the skeleton levers drawn below, finding the unknown quantity and the reaction at the fulcrum, in each case (Fig. 55).

5. A mast is rigged as shown. Find the force in the tie wire joining the mast to the brickwork (Fig. 56).

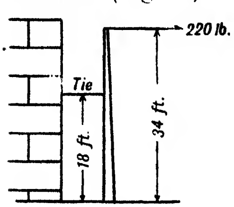


FIG. 56.

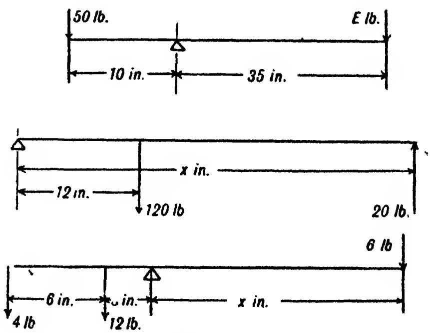


FIG. 55.

6. Draw the figure (Fig. 57), which shows a ladder resting against a wall, to a scale of 1 in. = 10 ft. If the weight of the ladder acts from a point 12 ft. from the base, measure any distances you require and calculate the reaction of the wall, which is assumed to be smooth.

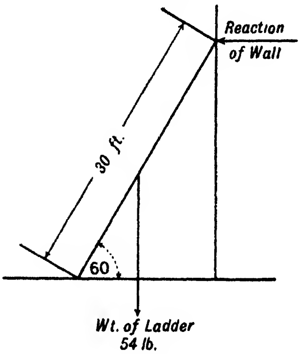


FIG. 57.

7. A special key spanner is made to the sketch shown. Find the force exerted at the pins if the effort at the ends of each handle is 10 lb. (Fig. 58).

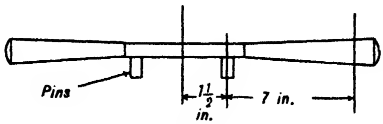


FIG. 58.

8. The diagram of a nail extractor is shown (Fig. 59). Find the force P on the nail if the pressure on the handle is 22 lb.

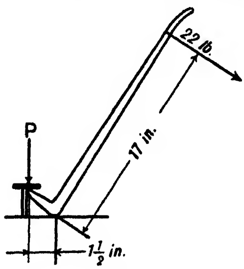


FIG. 59.

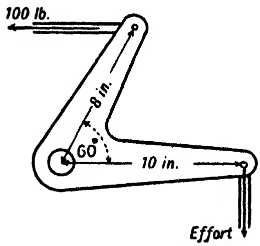


FIG. 60.

9. Draw to a scale of 1 in. = 4 in. the skeleton diagram for the bell-crank lever. Measure any distances you require and calculate the effort required to operate the lever in the position shown (Fig. 60).

10. A face plate has a load of 32 lb. placed 3·5 in. out of centre. Find the position of a balancing weight of 14 lb. placed diametrically opposite to the load.

11. A handle drives a wheel and belt. Find the pull in the belt if the force on the handle is 17 lb. and the diameter of the pulley is 24 in. (Fig. 61).

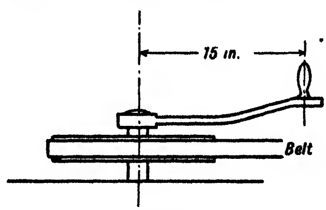


FIG. 61.

12. A small steam whistle is to operate at a pressure which corresponds to a load of 100 lb. on a spring loaded lever pivoted at A. Find the stiffness of the necessary spring, if the whistle operates when the spring is extended through 1 in. (Fig. 62).

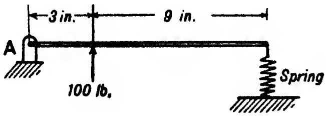


FIG. 62.

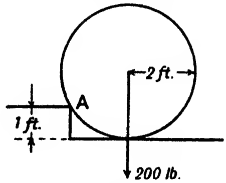


FIG. 63.

13. Find the least force necessary to roll the pulley shown up a step 1 foot in height. Take A as the fulcrum (Fig. 63).

14. In a single lever testing machine of capacity 10 tons, find the position of the 500 lb. balance weight when 10 tons is the load applied to the specimen (Fig. 64).

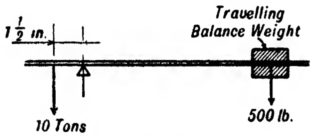


FIG. 64.

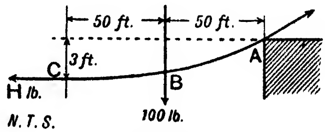


FIG. 65.

15. Estimate the magnitude of the horizontal force H necessary to maintain the equilibrium of the wire ABC, which is fixed at A and loaded at B. Neglect the weight of the wire (Fig. 65).

16. A rope passes over a pulley 2 ft. 6 in. in diameter. One end carries a load of 35 lb. and the other, one of 40 lb. Find the resultant turning moment on the pulley if the rope does not slip and the friction at the axle is neglected.

17. In the governor ball and arm shown (Fig. 66), find the total clockwise moments about the support, where 1 lb. is the weight of the arm and 6 lb. the weight of the ball. Hence determine the force F necessary to produce equilibrium.

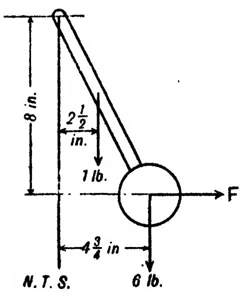


FIG. 66.

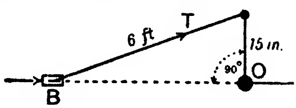


FIG. 67.

18. Find the force T acting along the connecting rod in order to provide a turning moment of 2000 lb. ft. on the shaft at O (Fig. 67).

19. If ABCD (Fig. 68) represents the section of a vehicle weighing 4 tons, find the horizontal force necessary to overturn it, about A, if the force is applied 4 ft. above the ground.

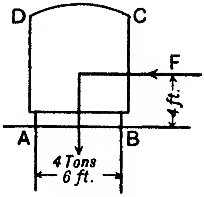


FIG. 68.

20. Find the magnitude of the overturning moment acting on the retaining wall tending to overturn it about A (Fig. 69). What must be the weight of the wall acting through B in order to produce equilibrium ?

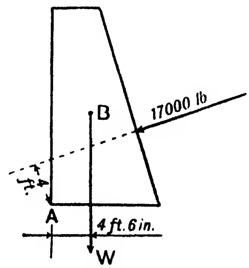


FIG. 69.

21. The twisting moment on a shaft is applied by two equal and opposite forces of 240 lb. each, applied to the circumference of a pulley 24 in. in diameter. Find the moment of the couple. What force would be necessary in order to produce the same twisting moment if applied to a pulley 15 in. in diameter ?

22. Find the reactions at the supports in each of the beam loadings shown (Fig. 70). In the second case the weight of the beam is 1·5 cwt., and acts at the centre of the span.

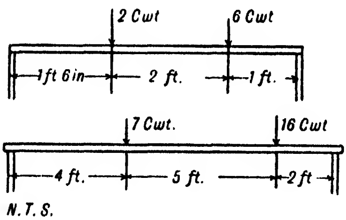


FIG. 70.

23. Two spring supports, at opposite ends of a beam 3 ft. span carrying a load of 120 lb. one foot from the left-hand support, are to compress equally a distance of 1 in. under the load. Find the stiffnesses of the two springs.

24. Find the force F which is transmitted by the transmission bar shown (Fig. 71). The bar is operated by a

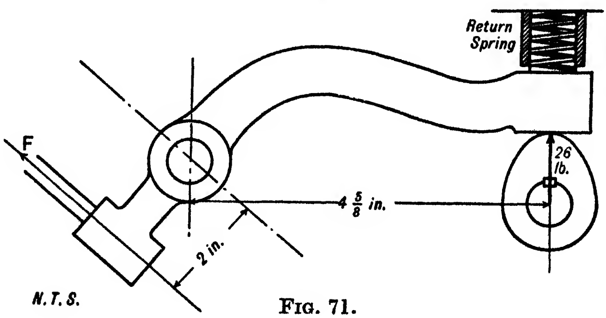


FIG. 71.

cam which, at this period, exerts a force of 26 lb. What will be the stiffness of the return spring if its maximum compression is 0·85 in. ?

25. The diagram of a brake mechanism is shown (Fig. 72). Find the force F required to produce equilibrium. Point A is the fulcrum or pivot.

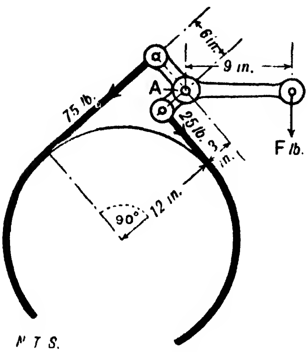


FIG. 72.

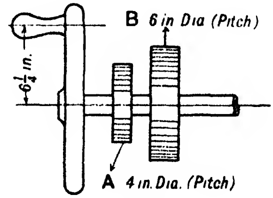


FIG. 73.

26. Find the force necessary to operate the handwheel, if the inter-tooth pressure on wheel A = 21 lb. and on wheel B = 14 lb. (Fig. 73).

27. The diagram shows the front and end elevations of a wheel and differential axle. Find the pull in the rope E for equilibrium (Fig. 74).

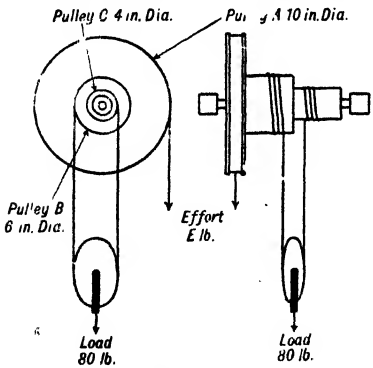


FIG. 74.

28. Find, by taking moments about D, the force H necessary to maintain the equilibrium of that portion of the framed girder shown (Fig. 75).

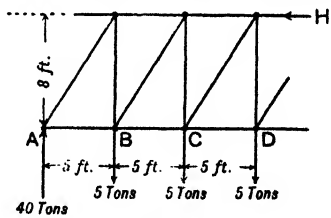


FIG. 75.

29. A horizontal girder weighing 21 lb. per ft. length rests on supports 20 ft. apart. Vertical loads of 1 ton and 2 tons are supported by the girder at points 5 ft. and 14 ft. respectively from the left end. Find the reactions of the supports. (U.L.C.I.)

30. A horizontal beam of uniform section and 18 ft. long rests on supports at its ends. The beam weighs 540 lb. and carries a load of

2 tons at a point 4 ft. from one end and a load of 1 ton at a point 5 ft. from the other end. Find the reactions of the supports. (U.L.C.I).

31. A weight of 10 lb. is suspended by a string AB from a fixed point A. A horizontal force F acts on the weight and moves it into such a position that the angle between the horizontal force F and the string is 120°. Find the magnitude of F. (U.L.C.I.)

32. A rod of uniform section, 6 ft. long and weighing 20 lb., rests on a fulcrum at one end and is supported in a horizontal position by a vertical force F acting at the other end. Determine the magnitude of F when the rod supports a weight of 40 lb. at a point 2 ft. from the fulcrum. Also find the pressure on the fulcrum. (U.L.C.I.)

CHAPTER IV

WORK AND POWER—HORSE POWER AND POWER OF ENGINES

Work. It is found that when a body moves, there is some *resistance*, however slight, to its motion. This resistance may be exerted by the medium through which the body is moving, as, for example, the air or water; or by a solid surface over which the body is moving. In each case a force known as the **effect of friction** offers the resistance to the motion. For example, when a locomotive is moving along a track, the motion of the locomotive is resisted by the air through which it is travelling and the frictional resistance offered by its track. **When a force overcomes a resistance and causes a body to move, work is said to be done by the force.**

If there is (*a*) no resistance, or (*b*) no motion, there can be no work done.

Measurement of work. The work done by a force is measured by the **product of the distance D moved and the resistance R overcome,** which, expressed algebraically, is $\mathbf{W} = \mathbf{D} \times \mathbf{R}$.

If either D or R is zero, the work done must be zero.

Unit of work. The unit by which work is measured is known as the **foot pound**, and is the amount of work done when a resistance of one pound is overcome through a distance of one foot measured in the direction in which the force acts.

This unit should not be confused with the pound foot, which is the unit of moment.

Examples. *Find the work done in each of the following cases :*

(*a*) *To raise a load of* 100 *lb. a distance of* 20 *ft.*

(*b*) *To drive a car a distance of* 100 *ft. when the resistance to the motion is* 65 *lb.*

(*c*) *For a man. of weight* 160 *lb., to carry a cycle, of weight* 30 *lb., up* 20 *stairs, each of rise* 6 *in.*

(*a*) Resistance = gravitational pull, or weight = 100 lb.
Distance = 20 ft.
Work done = $20 \times 100 =$ **2000 foot pounds.**

(*b*) Resistance = 65 lb.
Distance = 100 ft.
Work done = $100 \times 65 =$ **6500 foot pounds.**

(*c*) Resistance = weight of man + weight of cycle
= $160 + 30 = 190$ lb.
Distance = $20 \times$ rise $= 20 \times 6$ in. $= 10$ ft.
Work done = $190 \times 10 =$ **1900 foot pounds.**

Secondary units. Where the resistance is large, the work done may be expressed in foot tons, or foot cwt., in which cases 1 foot ton = 2240 foot lb. and 1 foot cwt. = 112 foot lb.

Work done in climbing an incline. In problems in which the work is done against gravity, the work done is always the product of the weight raised and the **vertical distance** through which it is moved, or overcome (Fig. 76). The same amount of work is done in raising the weight W to A as in pulling it up the incline to B, that is, if the friction between the incline and the weight is neglected. The work done is then, in each case $\mathbf{W} \times \mathbf{h} = \mathbf{Wh}$ **foot pounds.**

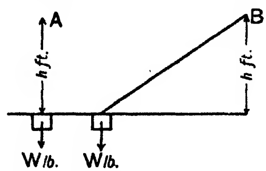


Fig. 76. Equal quantities of work.

This is a particular case of the general rule, namely, *work done = resistance, or force × distance measured in the direction in which the resistance acts.*

Power. It has been seen that work depends upon two conditions, (*a*) resistance overcome, (*b*) distance moved, and does not depend on the time taken.

For example, one crane may raise 5 cwt. through 20 feet in one minute, while another crane may take half a minute to raise the same load through the same distance. The work done would be the same in each case, namely $5 \times 20 = $ **100 ft. cwt.**, but the second crane is doing the same work in less time and is said to exert a greater power.

Thus, **power is the rate of performing work.**

Unit of power. Power is measured in horse power, and **one horse power is a rate of working of 33,000 foot pounds per minute.**

Example. *A crane raises* 8000 *lb. through a distance of* 15 *ft. in half a minute. Calculate the horse power of the lifting mechanism.*

Resistance $= 8000$ lb.
Distance per $\frac{1}{2}$ minute $= 15$ ft.
Distance per 1 minute $= 30$ ft.
Work done per minute $= 30 \times 8000$ ft. lb.
$= 240000$ ft. lb.

Since 1 horse power $= 33000$ ft. lb. per minute,

$$\text{the horse power} = \frac{2400\cancel{0}\cancel{0}}{330\cancel{0}\cancel{0}} = \frac{240}{33} = \frac{80}{11}.$$

H.P. $= 7 \cdot 27$.

The abbreviation used for horse power is H.P.

Origin of the term horse power. The unit of power is due to James Watt and dates from the early days of the steam engine. The term was introduced in order to rate early steam engines, which were called upon to do work previously performed by horses. It is possible that, in response to a demand for an engine to replace a certain number of horses, an experiment was made on these lines, using strong draught horses. Suppose that a horse raised a weight of 2 cwt. by means of a rope passing over a pulley placed at the top of a shaft (Fig. 77). If the horse raised this 2 cwt. at a steady speed of 100 ft. per minute the work done would be $224 \times 100 = 22{,}400$ ft. lb. per minute. It would then be natural to increase this figure by about 50 per cent. in order to ensure that the

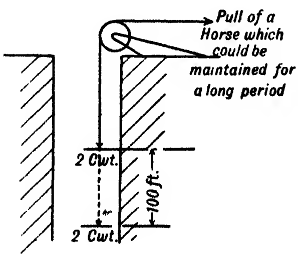


Fig. 77.

engines would satisfactorily replace horses. Hence the value for a horse power would be 33,000 ft. lb. per minute. Doubt exists whether a horse could maintain 33,000 ft. lb. per minute over a day's work, but it could do this rate of working for short periods, and possibly this knowledge led to engines being rated at the higher value.

Graphical representation of work. Since work done against a resistance is the product of resistance and distance, and the area of a rectangle is its length times its breadth, work done may be represented by the area of a rectangle, one side of which represents the resistance and the other side the distance. Examples of this representation follow.

Uniform resistance. *The work done when a machine raises 200 lb. through 15 ft.* (Fig. 78).

Total work done $= 200 \times 15 =$ **3000 ft. lb. and is represented by area ABCO.**

Also, work done after 5 feet (**area ADEO**) $= 5 \times 200 =$ **1000 ft. lb.,**

and, work done after 10 feet (**area AFGO**) $= 10 \times 200 =$ **2000 ft. lb.**

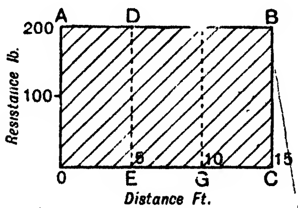


FIG. 78.

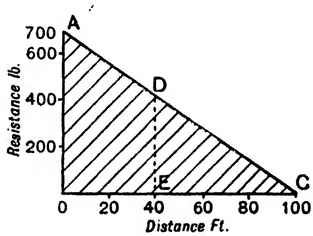


FIG. 79.

Uniformly varying resistance. *A chain weighing 7 lb. per foot is being hauled from a shaft and wound on to a drum 100 ft. above its free end* (Fig. 79).

$$\textbf{Total work done} = \frac{700 \times 100}{2} = 35{,}000 \textbf{ ft. lb. and is represented by area AOC.}$$

Work done while 40 ft. is hauled in is represented by

$$\text{area ADEO} = 40 \times \frac{700 + 420}{2} = 40 \times 560$$

$$= \textbf{22,400 ft. lb.};$$

and the work left to be done is represented by **area DEC**

$$= 60 \times \frac{420 + 0}{2} = \textbf{12,600 ft. lb.}$$

For the first foot approximately 700 lb. weight of chain has to be lifted, this decreases until at the last foot only 7 lb. weight of chain has to be raised. It follows that this problem resolves itself into one where an average resistance has to be overcome throughout the distance, and this average resistance is one half of the weight of the chain, that is, 350 lb. Similarly, the resistance to be considered when 40 feet of the chain has to be hauled in is the average of 100 feet of chain and 60 feet of chain, that is $\frac{1}{2}(100 \times 7 + 60 \times 7)$ or 560 lb.

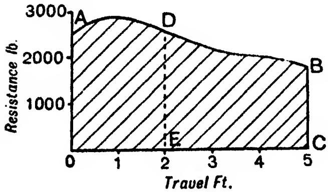


Fig. 80.

Variable resistance (not uniformly variable). *The following table gives the resistance offered to rolling in a rolling mill, taken at every* 1 *foot of a* 5 *foot travel* (Fig. 80).

Travel, ft. -	0	1	2	3	4	5
Resistance, lb.	2400	2800	2600	2200	2000	1700

The area ABCO represents the total work done.

This area may be found by means of the **mid-ordinate method** or **Simpson's Rule**, or for general purposes,

the work done = **average resistance** × **travel.**

$$\text{Total work done} = (2400 + 2800 + 2600 + 2200 + 2000 + 1700)\tfrac{1}{6} \times \text{travel.}$$
$$= 2283 \text{ lb.} \times 5 \text{ ft.}$$
$$= \mathbf{11{,}415 \text{ ft. lb.}}$$

$$\text{Work done during 2 ft. of travel} = \tfrac{1}{3}(2400 + 2800 + 2600) \times 2$$
$$= 2600 \times 2 = \mathbf{5200 \text{ ft. lb. (area ADOE).}}$$

Indicator diagrams. These diagrams are taken by means of an instrument known as an **indicator** and show, in the form of a work diagram, the actual work done on the engine piston during each stroke, measured upon each square inch of the piston area. The diagram shown (Fig. 81) is the type of indicator diagram obtained for a steam engine in which A shows the conditions at *admission* of steam, C at *cut off*, B at *release* and D at *compression*. The period AC is the admission period, CB the expansion period, BD the exhaust and DG the compression period.

The area shaded, that is **ACBD**, represents the work done on the piston, but the portion under the curve **GDBEF** is **negative work.** The steam has to be *compressed* by the engine, which has to do work upon this steam, in order to form a cushion of compressed steam to allow the piston to finish its return stroke without shock. This process of pulling up the moving parts against a cushion of steam is known as **cushioning.**

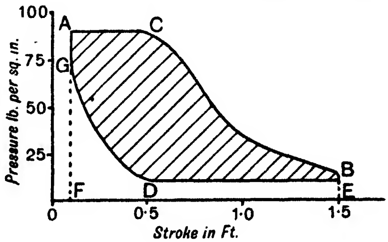


FIG. 81.

NOTE.—A more complete explanation of the above terms will be found in the section on Heat Engines.

Energy and work. When a body is capable of doing work, either by virtue of its position, or some quality it possesses, it is said to possess energy. That is, **energy is the capacity a body possesses for doing work.**

For example, a hammer at a height above a piece of metal may do work by striking the metal, and is said to possess energy.

Energy may exist in various forms, such as chemical energy, electrical energy, heat energy and mechanical energy, which means that a body possesses one of these types of energy which under favourable conditions can be used to do work.

A later chapter of this book is devoted to the study of energy, and at this stage the student need only realise that energy is the capacity for doing work.

Efficiency. Every machine, whether it be an engine, a hydraulic press, a dynamo or a motor, receives a certain amount of energy and, for reasons to be studied later, it gives out a less quantity of work or energy than it receives. **The relation between the energy received by a machine and the work, or energy, given out by it is called its efficiency.**

Mechanical efficiency is the relationship between the mechanical energy supplied and the actual useful work obtained, and may be written thus :

$$\textbf{mechanical efficiency} = \frac{\textbf{work taken out}}{\textbf{work put in}} \text{ or } \frac{\textbf{work output}}{\textbf{work input}},$$

which is always less than unity.

Horse power of engines. In the determination of the horse power of an engine, two values are obtained, (1) **the indicated horse power** or **I.H.P.**, (2) **the brake horse power** or **B.H.P.**

The indicated horse power is the horse power actually produced in the engine cylinders.

The brake horse power is the horse power available for useful work after all mechanical and other losses have been accounted for, and this horse power is always less than the indicated horse power by the amount of such losses.

Mechanical efficiency is the name applied to the ratio between brake and indicated horse powers, that is,

$$\textbf{mechanical efficiency} = \frac{\textbf{B.H.P.}}{\textbf{I.H.P.}}.$$

Measurement of I.H.P. It has been mentioned already that an engine may be fitted with an instrument known as an indicator which will produce a work diagram for each stroke. The action of this indicator will be found in the section of the book devoted to heat engines, but the calculation of the I.H.P. can be taken at this stage.

Mean effective pressure (M.E.P.) is the name given to the average or mean pressure, in lb. per sq. in., acting upon the piston throughout the stroke. This is found by obtaining the average height of the diagram, which represents the mean effective pressure during the stroke to a scale dependent upon the stiffness of the indicator spring used. There are several methods of reaching the desired result, the first of which is to obtain the area of the indicator diagram in sq. in. by means of a planimeter (p. 22), or by the mid-ordinate or other rule. The area is then divided by the length of the diagram in inches, and in this way the height of a rectangle equal in area to the indicator diagram is found. This height, when converted according to the scale of pressure used in the indicator diagram, gives the mean effective pressure.

A second method is to divide the diagram into a number of equal intervals by ordinates drawn at right angles to the atmospheric line (Fig. 82). The ordinates touching each end of the diagram are called **terminal ordinates,** and by adding the lengths of half the ter-

minal ordinates to those of the intermediate ones and dividing by the number of intervals taken, a value representative of the average height of the diagram is obtained, which can be converted to mean effective pressure by the use of the pressure scale.

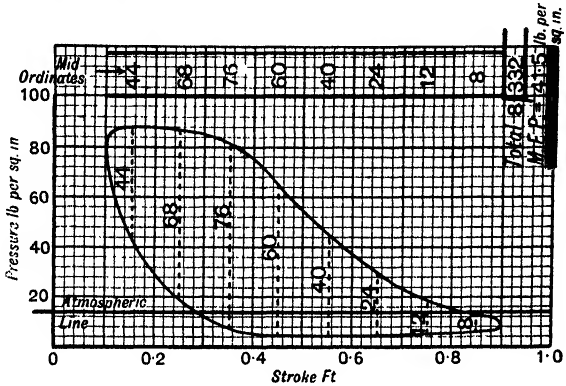


FIG. 82.

A third method, which is shown in the diagram (Fig. 82), is the method generally used to find the mean effective pressure. It consists of dividing the extreme length of the diagram into eight, or ten, equal intervals and erecting a mid-ordinate at the middle point of each interval at right angles to the atmospheric line. The average length of these mid-ordinates, when converted to the pressure scale, is the mean effective pressure.

Calculation of I.H.P. The diagram (Fig. 83) shows the action of the steam, or other pressure, on the piston of the engine. This pressure varies throughout the stroke, and its mean value is the mean effective pressure.

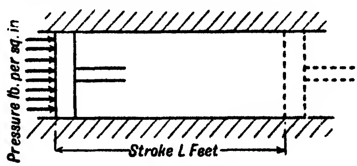


FIG. 83.

Let the area of the piston subjected to pressure be **A sq. in.** and the stroke of the piston be **L ft.**

Then if the mean effective pressure is **P lb. per sq. in.**

the total load on the piston $= P \times A$ lb.

and work done per stroke $= P \times A \times L$ ft. lb.

Now, if the piston makes N working strokes per minute,

$$\text{the work done per minute} = \mathbf{P} \times \mathbf{A} \times \mathbf{L} \times \mathbf{N} \text{ ft. lb.}$$

$$\text{and the I.H.P.} = \frac{\mathbf{P} \times \mathbf{A} \times \mathbf{L} \times \mathbf{N}}{33000}.$$

This formula is generally written $\mathbf{I.H.P.} = \dfrac{\mathbf{P.L.A.N}}{33000}$.

When working questions on I.H.P. it is better to work through the processes leading to the horse power rather than substitute values into the formula.

Number of strokes per minute. This varies with the type of engine ; for example, in a single-acting engine the number of strokes per minute is equal to the number of revolutions per minute of the engine ; whereas in a double-acting engine the number of strokes is double the number of revolutions per minute, the engine making two working strokes to a revolution.

In an internal combustion engine working on a four-stroke cycle, only one stroke in four is a working stroke, and thus the number of working strokes per minute is half the number of revolutions per minute.

NOTE.—*If the governing of this engine is on the "hit and miss" principle, the number of explosions per minute must be taken.*

Effect of the piston rod in double-acting engines. In a double-acting engine, the piston rod reduces the effective area of the piston on the return stroke (Fig. 84), when the pressure is acting on the back, or crank end, of the piston. The I.H.P. is best worked out for each side of the piston, taking account of this reduction of area on the crank end. The sum of these results will be the I.H.P. of the engine.

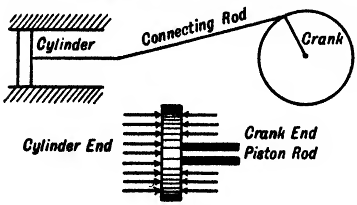


FIG. 84.

Efficiency of an engine. A machine is incapable of usefully employing the whole of the work it receives, and the output is always less than the input when they are measured in work units. Similarly, the work performed in an engine cylinder cannot be all transferred to the crankshaft to do useful work ; there will be losses due to the friction in the mechanism and to other causes. In consequence,

the indicated horse power of an engine is not the useful horse power available, and a further quantity known as the brake horse power has to be obtained, a quantity which is measured after the engine losses have taken effect. **Mechanical efficiency** has already been defined (p. 53) as the ratio

$$\frac{\text{brake horse power}}{\text{indicated horse power}},$$

which is another way of expressing the ratio for the efficiency of any machine, that is :

$$\text{efficiency} = \frac{\text{work output}}{\text{work input}}.$$

The methods of measuring the brake horse power, or actual useful horse power, are outlined later in this chapter, where examples are taken to illustrate the relation between brake and indicated horse power.

Example 1. *Find the I.H.P. of a single-acting steam engine if the piston is 10 in. diameter, stroke 1 ft. 9 in., engine speed 95 revolutions per minute and the mean effective pressure 47·1 lb. per sq. in.*

$$\mathsf{P} = 47{\cdot}1 \quad \mathsf{L} = 1{\cdot}75, \quad \mathsf{N} = 95.$$

Area of Piston A $= \pi \times 5^2 = 25\pi$ (sq. in.).

Load on piston $= \mathsf{P} \times \mathsf{A} = 47{\cdot}1 \times 25\pi$ (lb.).

Work done per stroke $= \mathsf{P} \times \mathsf{A} \times \mathsf{L} = 47{\cdot}1 \times 25\pi \times 1{\cdot}75$ (ft. lb.).

Work done per minute $= \mathsf{P} \times \mathsf{A} \times \mathsf{L} \times \mathsf{N}$

$= 47{\cdot}1 \times 25\pi \times 1{\cdot}75 \times 95$ (ft. lb.).

$$\text{I.H.P.} = \frac{47{\cdot}1 \times 25\pi \times 1{\cdot}75 \times 95}{33000}$$

$$= \mathbf{18{\cdot}62\ H.P.}$$

Example 2. *If the engine of example* (1) *were double-acting and the piston rod 2 in. in diameter, find the I.H.P. if the M.E.P. at the cylinder end was* 47·1 *lb. per sq. in. and at the crank end* 49·4 *per sq. in.*

Cylinder end.

Load on piston $= \pi \times 25 \times 47{\cdot}1$ lb.

Work done per stroke $= \pi \times 25 \times 47{\cdot}1 \times 1{\cdot}75$ ft. lb.

$$\text{I.H.P.} = \frac{\pi \times 25 \times 47{\cdot}1 \times 1{\cdot}75 \times 95}{33000}$$

$= \mathbf{18{\cdot}62}$ **H.P. as for a S.A. engine.**

Crank end.

Area of piston − area of rod $= 25\pi - \pi = 24\pi$ sq. in.,

or Area effective to steam $= \pi \times 5^2 - \pi \times 1^2$ sq. in.

Load on piston $= \pi \times 24 \times 49{\cdot}4$ lb.

Work done per stroke $= \pi \times 24 \times 49{\cdot}4 \times 1{\cdot}75$ ft. lb.

$$\text{I.H.P.} = \frac{\pi \times 24 \times 49{\cdot}4 \times 1{\cdot}75 \times 95}{33000}$$

$$= \mathbf{18{\cdot}75\ H.P.}$$

$$\text{Total I.H.P.} = 18{\cdot}62 + 18{\cdot}75$$

$$= \mathbf{37{\cdot}37\ H.P.}$$

Example 3. *A petrol engine working on a four-stroke cycle makes* 2500 *revolutions per minute. The cylinder is* 4 *in. diameter, stroke* 6 *in. and the M.E.P.* 88 *lb. per sq. in. Find the I.H.P.*

Data. P = 88. L = 0·5.

A = area of piston $= \pi \times 2^2 = 4\pi = 12{\cdot}56$ (sq. in.).

N = number of explosions per minute $= \frac{2500}{2} = 1250$.

Load on piston $= 88 \times 12{\cdot}56$ lb.

Work done per stroke $= 88 \times 12{\cdot}56 \times 0{\cdot}5$ ft. lb.

$$\text{I.H.P.} = \frac{88 \times 12{\cdot}56 \times 0{\cdot}5 \times 1250}{33000}$$

$$= \mathbf{20{\cdot}9\ H.P.}$$

Example 4. *A single cylinder internal combustion engine develops an I.H.P. of* 16·1. *The stroke is* 6 *in. and the piston* $4\frac{1}{2}$ *in. diameter. Find the mean pressure if the engine makes* 1050 *explosions, or working strokes per minute.*

Data. I.H.P. = 16·1, L = 0·5, N = 1050,

A = area of piston $= \pi \times (2\frac{1}{4})^2$ (sq. in.).

Then $$\text{I.H.P.} = \frac{\text{P.L.A.N}}{33000}$$

and $$\text{P} = \frac{33000 \times \text{I.H.P.}}{\text{L.A.N}}$$

$$= \frac{33000 \times 16{\cdot}1}{0{\cdot}5 \times \pi \times (2\frac{1}{4})^2 \times 1050}$$

Mean Pressure = 63·75 lb. per sq. in.

Example 5. *A four cylinder, 4 stroke cycle, internal combustion engine is to be designed to work on the following data: speed 2000 revolutions per minute, mean pressure 80 lb. per sq. in. Calculate suitable cylinder dimensions to develop an I.H.P. of 25.*

Data. H.P. required = 6·25 per cylinder.

$$\mathsf{P} = 80, \ \mathsf{N} = 1000 \text{ explosions per minute.}$$

$$\text{I.H.P.} = \frac{\mathsf{P.L.A.N}}{33000}.$$

and isolating $\mathsf{L.A}$ from this formula,

$$\mathsf{L.A} = \frac{\text{I.H.P.} \times 33000}{\mathsf{P} \times \mathsf{N}}$$

$$= \frac{6{\cdot}25 \times 33\cancel{000}}{80 \times 1\cancel{000}} = 2{\cdot}578.$$

It is now necessary to obtain suitable values of L and A, that is, length of stroke and area of piston, in order that $\mathsf{L.A} = 2{\cdot}578$.

Suppose the length of stroke to be $1\frac{1}{2}$ diameter, then

$$\mathsf{L} = \frac{1\frac{1}{2}d}{12} = \frac{d}{8} \text{ (feet),}$$

and
$$\mathsf{A} = \frac{\pi d^2}{4}.$$

Therefore
$$\mathsf{L.A} = \frac{\pi d^3}{32} = 2{\cdot}578.$$

$$d^3 = \frac{32 \times 2{\cdot}578}{\pi} = 26{\cdot}3.$$

$$d = \sqrt[3]{26{\cdot}3 \text{ in.}} = 2{\cdot}97 \textbf{ inches.}$$

Then suitable dimensions would be, diameter of cylinder 3 in. and stroke $4\frac{1}{2}$ in.

Piston diameter 3 in., stroke $4\frac{1}{2}$ in.

(These dimensions are often given in millimetres.)

Measurement of brake horse power (B.H.P.) The brake horse power of an engine is generally measured by means of an absorption dynamometer, which is an apparatus for converting the work output of the engine into heat by means of some type of friction brake. Before the operation of a dynamometer can be understood it is necessary to study work performed during rotation.

Work done during rotation. Consider a pulley of radius R feet which is making N revolutions per minute against a resistance of P lb.

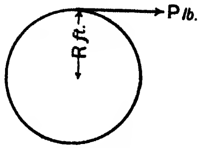


FIG. 85.

Resistance $= P$ lb.

Distance per revolution $= 2\pi R$ ft. or the circumference of the pulley.

Work done per revolution $= 2\pi R.P$ ft. lb.

Work done per minute $= 2\pi R.P.N$ ft. lb.

$$\text{H.P.} = \frac{2\pi R.P.N}{33000}.$$

NOTE.—*In this theory the resistance is considered as acting at the rim of the pulley.*

Example. *Find the H.P. transmitted by a belt which has an effective pull of* 140 *lb. on a pulley* 4 *feet in diameter making* 240 *revolutions per minute.*

Force, or resistance $= 140$ lb.

Distance per revolution $= \pi \times 4$ ft.

Work done per revolution $= 140 \times \pi \times 4$ ft. lb.

Work done per minute $= 140 \times \pi \times 4 \times 240$ ft. lb.

$$\text{H.P.} = \frac{140 \times \pi \times 4 \times 240}{33000}$$
$$= 12{\cdot}8 \text{ H.P.}$$

The Prony brake. This type of brake (Fig. 86) can be used for small pulleys, or directly on to a shaft transmitting power. It consists of

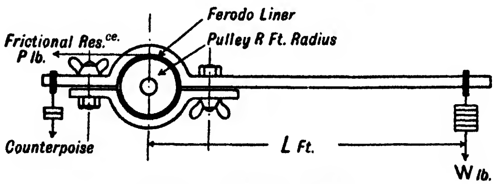


FIG. 86. Prony brake.

two steel straps, each lined with ferodo, a material used for lining brakes, or it may consist of two blocks of wood arranged so that they may be pressed on to the pulley rim by means of bolts and fly nuts. The two straps are fitted with bolts and nuts which allow the pressure on the pulley to be regulated at will, and one strap is extended to form an arm from which weights may be hung. To the other end of

this strap counterpoise weights are added to steady the brake against the rotation. Weights are added to the arm of the brake and the brake tightened until the speed of the pulley remains constant, due to the frictional load. It follows that the moment of the weight added to the arm is equal to the moment of the resistance offered by the brake on the pulley ; otherwise the brake would rotate with the pulley, and the force at the end of the arm is sufficient to prevent this rotation happening.

Let the frictional resistance between the pulley and the brake be P lb. and the radius of the pulley R ft.

Then the anti-clockwise moment $= P\mathrm{R}$ lb. ft.,

and the clockwise moment $= W\mathrm{L}$ lb. ft.

Hence, $$P\mathrm{R} = W\mathrm{L},$$

$$\mathrm{P} = \frac{\mathrm{WL}}{\mathrm{R}}.$$

If there are N revolutions of the pulley per minute, the work done per minute against friction

$$= P \times 2\pi\mathrm{R} \times \mathrm{N} \text{ ft. lb.}$$

but $$P = \frac{W\mathrm{L}}{\mathrm{R}},$$

$$\therefore \text{ work done per minute} = \left(\frac{W\mathrm{L}}{\mathrm{R}}\right) 2\pi\mathrm{R} \,.\, \mathrm{N} \text{ ft. lb.}$$

$$\text{B.H.P.} = \frac{2\pi\mathrm{WLN}}{33000}.$$

NOTE.—*This type of brake is only satisfactory at relatively slow speeds.*

The rope brake. This brake (Fig. 87) is very generally used to measure the output, or brake horse power of engines. It consists of a rope, looped, and wound double around the fly wheel of the engine, with one of its free ends attached to a spring balance suspended from a beam above the flywheel. The other end of the rope carries the dead load L.

When the flywheel is not shrouded, the rope is held in position, on the rim, by several wooden blocks placed at intervals around the circumference of the flywheel. The rope is generally wired into

position with thin copper wire. The blocks are slotted to receive the rim of the flywheel and thus prevent the ropes sliding from the rim even if the pulley, or flywheel rim, is slightly cambered for belt driving. The brake is loaded with weights until the engine maintains a constant speed, that is, is receiving a load due to the brake sufficient

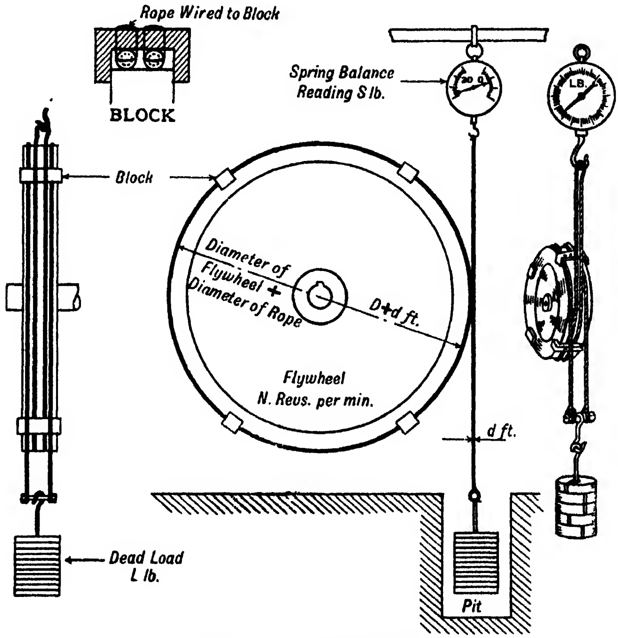


FIG. 87. Rope brake.

to prevent acceleration of the flywheel. Dead load **L** and spring balance reading **S** are taken and the B.H.P. is calculated as follows where **N** is the speed in revolutions per minute.

Effective load or resistance $= (\mathbf{L} - \mathbf{S})$ lb.

Distance per revolution $= \pi(\mathbf{D} + \mathbf{d})$ ft.

Work done per revolution $= \pi(\mathbf{D} + \mathbf{d})(\mathbf{L} - \mathbf{S})$ ft. lb.

Work done per minute $= \pi\mathbf{N}(\mathbf{D} + \mathbf{d})(\mathbf{L} - \mathbf{S})$ ft. lb.

$$\text{B.H.P.} = \frac{\pi\mathbf{N}(\mathbf{D} + \mathbf{d})(\mathbf{L} - \mathbf{S})}{33000}.$$

NOTE.—*The ropes are often specially treated with graphite to ensure an even resisting torque on the brake.*

Example 1. *Find the B.H.P. of an engine, which when using a rope brake gave the following observations: diameter of flywheel* $=3$ *ft.* 4 *in., diameter of rope* $=\frac{5}{8}$ *in., speed* 300 *revolutions per minute, dead load,* $L=110$ *lb., spring balance reading* $S=16$ *lb.*

$$D+d=3 \text{ ft. } 4 \text{ in. } +\tfrac{5}{8} \text{ in. } =3\cdot385 \text{ ft.}$$

$$L-S=110-16=94 \text{ lb.}$$

$$N=300 \text{ revolutions per min.}$$

$$\text{B.H.P.}=\frac{\pi N(L-S)(D+d)}{33000}$$

$$=\frac{\pi\times300\times94\times3\cdot385}{33000}.$$

B.H.P. $=9\cdot1$ **(nearly).**

Example 2. *Find the B.H.P. of an engine of* 78 *per cent. mechanical efficiency and I.H.P.* 23·4.

$$\text{Mechanical efficiency}=\frac{\text{B.H.P.}}{\text{I.H.P.}}=\frac{78}{100}.$$

$$\frac{\text{B.H.P.}}{23\cdot4}=\frac{78}{100}.$$

$$\text{B.H.P.}=\frac{78\times23\cdot4}{100}.$$

B.H.P. $=18\cdot25$.

Example 3. *The I.H.P. of an engine is* 11·42 *and the B.H.P.* 9·49. *Find the mechanical efficiency.*

$$\text{Efficiency}=\frac{\text{B.H.P.}}{\text{I.H.P.}} \text{ or } \frac{9\cdot49}{11\cdot42}.$$

Efficiency $=83\cdot0\%$.

Slope, gradient and incline. A slope or gradient is measured by the ratio

$$\frac{\text{rise}}{\text{horizontal distance}},$$

but an incline is often defined by the ratio

$$\frac{\text{rise}}{\text{distance along the slope}}.$$

Work done in ascending an incline. In the examples which follow an incline is said to be 1 in 10 when a vertical rise of 1 ft. occurs in every 10 ft. measured along the incline (Fig. 88). Thus the vertical height of any point on the incline measured from the datum level, or a level to which reference is made, is equal to $\frac{1}{10}$ of the distance measured along the incline.

In order to raise the weight W lb. from A to the position B, the total work done is, at constant speed, made up of (a) the work done

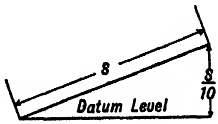


Fig. 88.

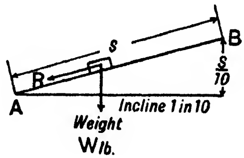


Fig. 89.

against gravity, and (b) the work done against the resistance of the incline.

Note.—*If acceleration takes place, further work must be done to accelerate the mass (see Chap. XII).*

(a) Work against gravity (Fig. 89) $= W \times \frac{S}{10}$ ft. lb.

(b) Work against resistance of the incline (Fig. 89) $= R \times S$ ft. lb.

$$\textbf{Total work done} = \frac{W \cdot S}{10} + R \cdot S \text{ ft. lb.}$$

Example 1. *A tramcar of weight* 5 *tons is driven up an incline of* 1 *in* 40 *against a track resistance of* 74 *lb. without gain or loss of speed. Find the H.P. required at a speed of* 10 *miles per hour.*

Distance per minute $= \frac{1}{6}$ mile $= 880$ ft.

(a) **Work against gravity** $= \frac{880}{40} \times 5$ ft. tons $= 110$ ft. tons per min.

(b) **Work against resistance** $= 74 \times 880$ ft. lb. $= 65120$ ft. lb. per min.

Total work done per minute $= (a) + (b)$

$= 110 \times 2240 + 65120$ ft. lb.

$= 311520$ ft. lb. per min.

$$\text{H.P.} = \frac{311520}{33000}.$$

$$\textbf{H.P.} = 9{\cdot}44.$$

Example 2. *A roller weighing 3 cwt.* (Fig. 90) *is pushed up an incline of 30 degrees to the horizontal by an effort which is kept horizontal. Find the magnitude of the effort if the resistance to the motion of the roller along the incline is 25 lb.*

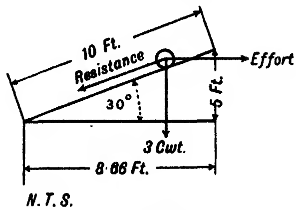


FIG. 90.

Suppose the roller to move 10 feet up the incline, then the work done against gravity

$$=3 \times 5 \text{ ft. cwt.},$$

and the work done against resistance

$$=10 \times 25 \text{ ft. lb.}$$

which together equal the work done by the effort

$$= E \times 8{\cdot}66 \text{ ft. lb.}$$

$$3 \times 5 \times 112 + 10 \times 25 = 8{\cdot}66E.$$

$$1680 + 250 = 8{\cdot}66E.$$

$$E = \frac{1930}{8{\cdot}66} = 223 \quad \therefore \text{ Effort} = 223 \text{ lb.}$$

NOTE.—*The distances through which the effort and weight move can be determined by actual scale drawing, or by trigonometry.*

Descent of an incline. A body free to descend an incline is assisted by gravity and if there is no resistance will gain speed during the descent. In other words, the body possesses energy at the top of the incline and is capable of doing work in descending.

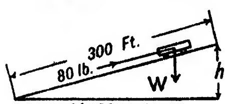


FIG. 91.

Example. *A truck at the top of an incline of* 1 *in* 30 *and* 300 *feet in length* (Fig. 91) *is free to move down the incline against a resistance. Find the* **least** *possible weight of the truck in order that it may descend against a resistance of* 80 *lb. steady force without gain of speed.*

Let W lb. be the weight of the truck, then work in ft. lb., against gravity, required to raise it to the height h of 10 ft.

$$= Wh = \frac{W \times 300}{30} = W \times 10.$$

Then the truck possesses $10W$ ft. lb. of energy at the top.

This is used to overcome the resistance coming down the incline, which is 300×80 ft lb

Therefore $$300 \times 80 = W \times 10,$$

and $$W = \frac{300 \times 80}{10} = 2400.$$ **Ans. 2400 lb.**

NOTE.—*Any energy possessed by the truck in addition to that required to overcome the resistance will be used to accelerate the truck during the descent.*

EXERCISES ON CHAPTER IV

1. Find the work done in each of the following cases :
   (*a*) A load of 700 lb. raised a vertical distance of 25 ft.
   (*b*) 5000 gallons of water pumped to a height of 300 ft.
   (*c*) A car of weight 1500 lb. driven up a hill to a height of 320 ft.

2. A hydraulic ram takes a force of 7000 lb. and makes a stroke of 1 ft. 3 in. If the diameter of the ram is 4 in., find (*a*) the pressure on the ram end in lb. per sq. in., and (*b*) the work done per stroke.

3. A barrel and its contents, weighing together 220 lb., are hauled up an incline of 1 in 10 for a distance of 35 ft. Find the work done against gravity.

4. A train weighing 500 tons is pulled along a level track against a resistance of 14 lb. per ton of its weight. Find the work done in a distance of 100 ft.

5. A lift is hoisted up an incline of 1 in 6 against a steady track resistance of 54 lb. Find, (*a*) the work done against gravity per minute, (*b*) the work done against the track resistance per minute, (*c*) the total work done per minute, if the lift moves with a constant speed of $7\frac{1}{2}$ miles per hr. and weighs 1050 lb.

6. Find the H.P. of a crane which raises a load of 2 tons a distance of 22 ft. in 45 seconds.

7. A car of weight 1940 lb. including the load travels along a level road at 30 miles per hr., constant speed, against a road and wind resistance of 45 lb. Find the H.P. required.

8. What additional H.P. would be needed for the car in Question 7 to ascend an incline of 1 in 30 at this speed, assuming constant resistance on level and incline.

9. A retarding force of 850 lb. is supplied by means of the brakes of a car. Find the work absorbed in heat if the car is brought to rest in 20 ft. under the combined action of brakes and a track resistance of 44 lb. How far would this car travel on a level track, before coming to rest, without the action of the brakes ?

10. A screw jack has a screw of $\frac{1}{2}$ in. pitch. Find the work done in 1 revolution in raising a load of 2000 lb. How many gallons of water could be raised to a height of 6 ft. with this amount of work ?

11. 20,000 gallons of water are pumped to a height of 250 ft. every hour. Find the work done per minute and the least H.P. of the pumps.

12. The dead load, including the weight of the ram, in a hydraulic accumulator is 120 tons. Find the useful H.P. of a pump which will raise the ram at a speed of $2\frac{1}{2}$ ft. per minute.

13. Two tugs are pushing a large liner broadside on to a quay against a direct wind resistance of 30 lb. per sq. ft. If the area of the liner exposed to wind is 40,250 sq. ft. and the tugs move at 20 ft. per min., find the useful H.P. developed by each tug in overcoming the wind resistance.

14. If a locomotive develops 1200 B.H.P. measured at the draw bar, find its draw bar pull at a speed of 60 miles per hr., assuming an efficiency of 65 per cent.

15. Calculate the work done in emptying a shaft 15 ft. diameter, 150 ft. in depth, which is full of water. How long, assuming no losses, will it take to empty if a pump of 5 H.P. is employed? 1 cu. ft. of water weighs 62·3 lb.

16. Find the average H.P. required for a winding engine to lift a cage of weight 5 tons a distance of 500 ft. to the surface at a speed of 2 ft. per second, if the lifting cable weighs 7 lb. per foot.

17. Find the average resistance due to wind and water if a warship developing 100,000 H.P. is steaming at a speed of 30 knots. 1 knot = 6080 ft. per hour.

18. A pillar of cast iron 8 in. in diameter, 15 ft. in length and supporting a load of 100 tons expands to a length of 15·02 ft. What work has been done by the expansion in foot-pounds?

19. A conveyor belt raises 8 tons of material through a vertical distance of 20 ft. in 1 hour. What H.P. has to be supplied to the power pulleys if the mechanical efficiency is 63 per cent.?

20. A hydraulic buffer with a ram 6 in. in diameter is forced backward a distance of 3 ft. 7 in. If the pressure behind the buffer increases uniformly from 20 lb. to 270 lb. per sq. in. during the process, find the work done. What is the average H.P. absorbed if the time taken is 2·8 seconds?

21. The tup, or hammer, of a pile driver weighs 1200 lb. What work is done in lifting it through a distance of 9 ft. 6 in., if there is a resistance to its upward motion of 37 lb. in addition to its weight? Find the least H.P. of the lifting machinery if the time taken is $7\frac{1}{2}$ seconds.

22. If the tractive resistance to the motion of a train, weighing 550 tons including the locomotive, is 14 lb. per ton of its weight, find the work done in travelling 8000 ft. up an incline of 1 in 80. What uniform speed could be reached on this incline if the effective H.P. of the locomotive is 1000?

23. A trolley bus weighing 7 tons, with brakes off, runs freely down an incline of 1 in 20 without gain of speed, at 10 miles per hr. If the frictional resistance remains constant, what H.P. is required to drive the bus up the incline at the same speed ?

24. A hydraulic press is to exert a force of 500 tons in a stroke of length 9 in. Calculate the work done per minute if the ram makes 2 strokes per minute. Find the H.P. available from this press and the necessary diameter of the ram if the water pressure is to be 700 lb. per sq. in.

25. A lever AB, 33 in. in length, is pivoted at a point C 7 in. from A. Find the force required at B in order to exert a pressure of 95 lb. at A. If A makes 240 upward strokes per minute and each stroke is 4 in. in length, find the H.P. required.

26. A wheel of weight W cwt. is rolled a distance of 30 ft. up an incline at 30° to the horizontal. Find the work done if the rolling force is maintained horizontal and has a magnitude of 50 lb. If this work is performed by two men of united H.P. 0·3, find the time taken and the weight of the wheel. Neglect friction and the effect of rotation.

27. A belt drives a pulley 26 in. in diameter with an effective pull of 230 lb. Find the H.P. transmitted at 350 revolutions per minute.

28. During the process of turning a shaft 6 in. in diameter the tool pressure is 516 lb. tangential to the work. Find the H.P. required if the speed of the lathe is 16 revs. per minute.

29. The load on the punch in a power press, and the corresponding travel of the punch in inches are given in the following table :

Load, lb. -	1500	2000	3500	4500	6500	7900	8600	8700	8000	7000	6100
Travel, in.	0	0·02	0·05	0·06	0·07	0·09	0·13	0·19	0·26	0·33	0·4

Plot the work diagram, that is, a graph of load against travel, and find the total work done during a stroke.

30. The tractive force T lb. exerted by the motors of an electric train at different distances S ft. measured from when the train was at rest are :

Tractive force, T lb.	13470	13400	13000	3200	2400	2400	2400	2400	2400	2400	0
Distance, S feet	0	300	660	700	1000	1500	2000	2500	3000	3300	3500

Plot a graph of T against S, with T vertically, and determine the area under the graph, that is, the total work done. If the train takes 1½ minutes to cover the distance of 3500 ft., determine the average H.P. developed.

31. The following observations, of the extension and corresponding load, were taken during the test of a steel specimen to fracture by tension :

Load (tons) - -	0	5	10	12	13	14	15	15·2	12·8
Extension (inches)	0	0·12	0·24	0·50	0·61	0·82	1·7	2·1	2·8

Plot a graph of load (vertically) against extension, and from this graph determine the work done in order to fracture the specimen.

32. A single-acting steam engine has a mean effective pressure in the cylinder of 41 lb. per sq. in. Find the I.H.P. if the stroke is 1 ft. 9 in. piston area 110 sq. in., and speed 80 revs. per minute. If the mechanical efficiency is 83 per cent., what is the B.H.P. ?

33. An internal combustion engine working on a four-stroke cycle makes 2500 revs. per minute. Calculate the I.H.P. of four cylinders if the cylinder diameter is 4 in., stroke $6\frac{1}{2}$ in., and mean effective pressure 92 lb. per sq. in.

34. A two stroke nine cylinder double-acting Diesel engine for the Hamburg Electricity Works has the following dimensions : diameter of pistons 33·86 in., stroke 59·05 in. Calculate the I.H.P. if the mean effective pressure is 88·25 lb. per sq. in. and the speed 94 revs. per minute (one impulse per stroke).

35. A locomotive has four equal cylinders in each of which the mean effective pressure is 41·5 lb. per sq. in. Find the I.H.P. if the stroke is 28 in., the piston area of each piston 208 sq. in. and the engine is travelling at 61·7 miles per hour, the driving wheels being 78 in. diameter.

36. A six cylinder petrol engine has the following dimensions : stroke 4·18 in., piston diameter 2·87 in. Calculate the I.H.P. on a four-stroke cycle if the M.E.P. is 96 lb. per sq. in. and each cylinder registers 1500 explosions per minute.

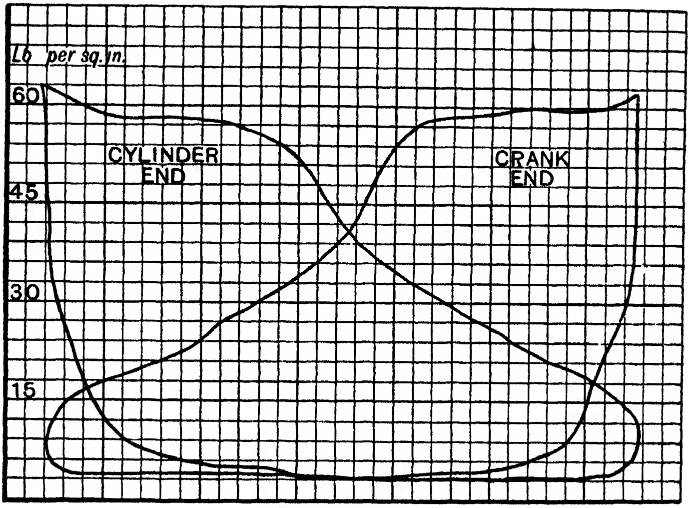
37. The following observations were taken during the brake test of an engine : diameter of brake wheel 2 ft. $10\frac{1}{2}$ in., thickness of brake belt, $\frac{3}{8}$ in., dead load 87 lb., spring balance reading 10 lb., speed 357 revs. per minute. Find the B.H.P. of the engine. What I.H.P. would you expect if the mechanical efficiency is known to be 81 per cent. ?

38. The following readings were taken during a brake test on a large 4 stroke Diesel engine : net load on brake arm 1870 lb., radius of brake arm 10·2 ft., revs. per minute 200. Calculate the B.H.P. If the indicated horse power of this engine at the time was 950, find its mechanical efficiency.

39. Find a suitable dimension for the cylinder of a steam engine to develop, as a single acting engine, 20 I.H.P. The stroke is to be 18 in.

in length, speed 100 revs. per minute, and the probable M.E.P. 50 lb. per sq. in.

40. Trace the indicator cards shown (Fig. 92), which were obtained from a double-acting engine, and calculate the mean effective pressure on each of the crank and cylinder end diagrams. Find the total I.H.P. if the piston is 18·4 in. in diameter, piston rod 2·25 in. diameter and the speed 95 revs. per minute.



Zero Pressure
Stroke = 21 in.

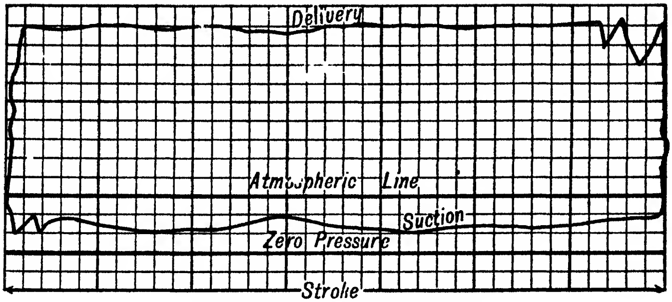
FIG. 92.

41. The diagram (Fig. 93) is one taken from a reciprocating pump of stroke $8\frac{1}{4}$ in., diameter of ram $4\frac{1}{2}$ in. and making 126 cycles, or revolutions, per minute. Trace the diagram and calculate the mean pressure on the ram during the suction and delivery strokes, the effective work done per revolution and the H.P. of the pump.

42. A chain 500 ft. long weighing 8 lb. per ft. run, hanging vertically, is wound up. Draw a graph having as ordinates the force required to draw it up against the lengths drawn up from 0 to 500 ft. as abscissae.

From this graph, calculate the work done in winding up (*a*) the first 150 ft. of the chain, (*b*) the whole chain. (U.L.C.I.)

43. An electric coaling crane raises 32 tons to a height of 50 ft. in 40 sec. Calculate the number of ft. lb. of work done per minute and the horse power expended. (U.L.C.I.)



Pressure Scale 1" = 50 lb. per sq. in.

FIG. 93.

44. What is the "unit of work"?

In the table, F is the force acting on a body and x is the distance of the body from a fixed point at corresponding instants.

F lb.	6	8	10	12	9	6
x ft.	0	1	2	3	4	5

Plot F and x and determine the amount of work done in moving the body 5 ft. from the fixed point. (U.L.C.I.)

45. The cutting stroke of a planing machine, which cuts in both directions, is 8 ft. The number of single strokes made per hour is 170, and the average resistance to cutting is 420 lb. Find the horse power absorbed in cutting. (U.L.C.I.)

CHAPTER V

EQUILIBRIUM OF A BODY UNDER THE ACTION OF FORCES, AND GRAPHIC STATICS APPLIED TO FRAMED STRUCTURES

Concurrent, co-planar forces. It has been shown that a force may be represented by a straight line (Fig. 94), in which the length of the line is the magnitude of the force to scale, and the direction of the line corresponds to that of the force, an arrow indicating the **sense of direction.**

It is now necessary to study the properties of a system in which several forces act at the same point, and are all contained in the same plane; that is, a system of concurrent, co-planar forces.

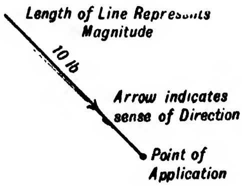


FIG. 94.

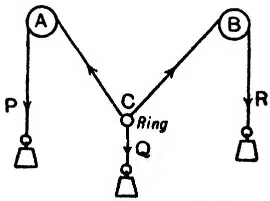


FIG. 95. Space diagram.

Concurrent means acting upon, or through, the **same point.**

Co-planar means contained in the **same plane surface.**

The construction used for obtaining the resultant of two forces can now be verified and extended to a number of forces.

The conclusions obtained from a series of experiments show the conditions which must be fulfilled in order that a body may be in equilibrium under the action of concurrent, co-planar forces.

EXPT. 5. OBJECTS.—(*a*) *To verify the law of the parallelogram of forces.* (*b*) *To verify the law of the triangle of forces.*

APPARATUS. Two pulleys A and B attached to a vertical board and running freely upon their bearings. A small ring at C to which are looped three cords, arranged as shown (Fig. 95) to support the weights *P*. *Q*, and *R*.

Method of Procedure. Arrange the three weights, P, Q and R, and, if necessary, adjust their magnitude until the ring C is kept in equilibrium under their combined action. Place a sheet of drawing paper behind the cords and transfer to it the position of C and the direction of each cord.

Note.—*The eye must be kept on a level with the cord when transferring the direction.*

The diagram so obtained is known as the **space diagram**. To obtain the **vector diagram** it is necessary to represent the forces as **vectors**, or in other words, in magnitude and direction.

Verification of the law of the parallelogram of forces. Represent the forces P and R to scale along their appropriate directions by the lines CE and CD where C is the point of application (Fig. 96). Construct on CE and CD the parallelogram CDFE and draw its diagonal to C. Measure the diagonal FC to the scale adopted for the lines CE and CD and note its direction. If the transferring has been carefully done the following conclusion may be drawn.

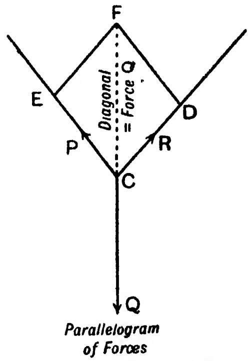


Fig. 96. Vector diagram.

Conclusion (a). The diagonal of the parallelogram represents in magnitude, and is in the same direction as, the third force Q.

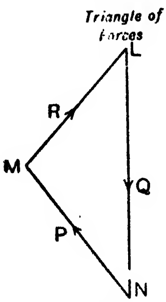


Fig. 97. Vector diagram.

Verification of the law of the triangle of forces. Draw the straight line MN parallel to the direction of the force P (Fig. 97) and make its length represent P to scale. From M draw ML parallel to the force R, and of length representing R to scale. Complete the triangle LMN and measure and note the direction of the closing line LN, when the following conclusion may be drawn.

Conclusion (b). The closing line of the triangle represents, in magnitude, and is in the same direction as, the third force Q.

Note.—*In order to make these experiments effective the student should now vary the weights, thus producing a different configuration for the space diagram, and work the experiment under different conditions to obtain the same conclusions.*

EXPT. 6. OBJECT.—*To verify the law of the polygon of forces.*

APPARATUS. For this experiment a similar apparatus is employed with the addition of several pulleys and a corresponding addition to the number of forces (Fig. 98).

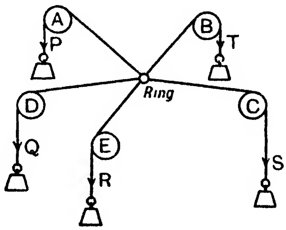


FIG. 98.

METHOD OF PROCEDURE. Arrange the forces P, Q, R, S and T to produce equilibrium of the ring O, and transfer the space diagram to a sheet of drawing paper in the manner described in the previous experiment.

Verification of the law of the polygon of forces. Represent the forces Q, P, T and S *in order*, either clockwise or anti-clockwise, by the straight lines AB, BC, CD and DE which form a polygon ABCDE (Fig. 100) when the closing line AE is drawn. Measure the closing line AE to the scale employed for the forces Q, P, T and S and note its direction, when the following conclusion may be drawn.

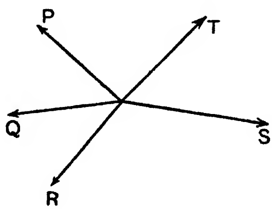


FIG. 99. Space diagram.

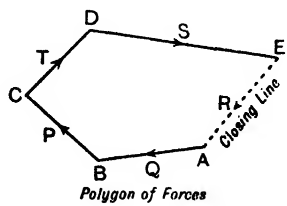


FIG. 100. Vector diagram.

CONCLUSION. The closing line will be in the same direction and represents in magnitude the last force R.

NOTE 1.—*It is important, in following this construction, that the forces be taken in order, either clockwise or anti-clockwise, around the space diagram, in the sequence Q, P, T, S and R or Q, R, S, T and P.*

NOTE 2.—*The order in which the forces are drawn does not affect the validity of the law, for forces acting at a point; but, in view of the use of Bow's Notation in later work the student is advised to follow the procedure in Note 1, which simplifies the construction.*

Laws of concurrent, co-planar forces. (1) **Parallelogram of forces.** If three forces, in one plane, act at a point and produce equilibrium, and a parallelogram be drawn so that two adjacent sides represent two of the forces in magnitude and direction, then the diagonal of the parallelogram drawn through the angular point formed by the adjacent sides will represent the third force in magnitude and direction. The arrows denoting the senses of the forces represented by the adjacent sides must both point towards or away from the angular point.

(2) **Triangle of forces.** If three forces, in one plane, act at a point and produce equilibrium, then these three forces may be represented in magnitude and direction by the sides of a triangle, *taken in order.*

(3) **Polygon of forces.** If any number of forces in one plane act at a point and produce equilibrium, then these forces may be represented in magnitude and direction by the sides of a closed polygon, *taken in order.*

Resultant and equilibrant. The single force which can be used to replace, in every respect, two or more forces is called the **resultant** of these forces. The **equilibrant** of any number of forces is the single force which will balance these forces, or produce equilibrium when used in conjunction with these forces.

NOTE.—*The equilibrant is equal in magnitude but opposite in sense of direction to the resultant.*

Example 1. In Experiment 5 the force Q is the *equilibrant* of the forces P and R, whereas the *resultant* is equal to Q, but acting vertically upwards.

Example 2. In Experiment 6, R, the closing force in the polygon, is the *equilibrant* of the forces P, Q, S and T, whereas the *resultant* is equal to R, but acting from A towards E.

Example 3. *Find the equilibrant and resultant of the two forces A and B.*

Draw the straight line PO to scale to represent 10 lb. (Fig. 101).

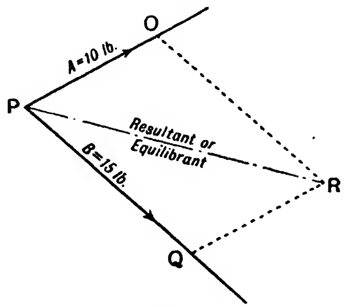


FIG. 101.

and the straight line

PQ to scale to represent 15 lb.

in the specified directions from P.

Complete the parallelogram OPQR and draw the diagonal PR.

Measure

PR to scale; this is **23 lb.**

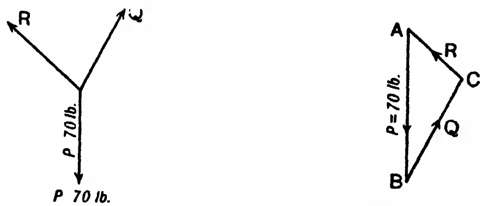
then,

Resultant is 23 lb. from P towards R,

and

Equilibrant is 23 lb. from R towards P.

Example 4. *Three forces P, Q* and *R act at a point in the directions shown. P is known to be* 70 *lb. Find the magnitude of Q and R if the point is kept in equilibrium under their action.*



Space diagram. Vector diagram.

FIG. 102.

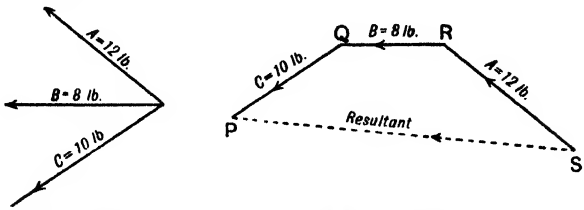
Construct the vector diagram (Fig. 102) by drawing a load line AB $=P=70$ lb. to scale, and from B draw BC parallel to the force *Q*. Complete the triangle by drawing AC from A parallel to the force *R*. The resulting triangle is the triangle of forces for the three forces *P*, *Q* and *R*, and the sides CA and BC represent, respectively, the forces *R* and *Q*.

R = 35 lb., Q = 52 lb. Ans.

Example 5. *Find the resultant of the three forces* $A=12$ *lb.,* $B=8$ *lb.,* $C=10$ *lb., acting in the directions shown.*

Draw the straight line QP (Fig. 103) parallel to the force *C* and representing it to scale.

From Q draw RQ representing the force B to scale and from R draw SR representing the force A to scale.



Space diagram. Vector diagram.

FIG. 103.

Close the polygon of forces with the line SP, which represents the resultant of the forces when the force acts from S *towards* P. Notice that its sense is opposite to that determined by following anti-clockwise the forces represented by the other sides of the polygon.

NOTE.—*An equal and opposite force, that is, one from* P *towards* S, *would be the equilibrant.*

Resultant = 26 lb. in the direction indicated by the line SP.

Treatment of simple framed structures. A framed structure is made up of a series of bars, or **members** as they are called, riveted, pin jointed or otherwise secured together to form the structure. In the simple forms of a framed structure, each joint or **node**, is the point of application of a series of forces, loads or forces in members, which, together, keep the node in equilibrium. It follows that the forces acting at each node may be represented to form some type of polygon of forces.

NOTE.—*The forces are assumed to act at a point at each node, but in the case of riveted joints this is not so; nevertheless, it is generally possible to ignore the effect of the riveted joint because the forces calculated are on the large side, and a structure designed to these forces would be over strong.*

Bow's notation. This is a scheme of lettering used in problems involving forces and members of structures. The method is to letter every space so that each force or member, which separates two spaces, can be referred to by the letters of its two adjacent spaces. In the example shown (Fig. 104) the force $AB = 10$ lb., $BC = 7$ lb., $CD = 12$ lb. and $AD = 4$ lb.

Bow's notation applied to a roof truss is illustrated in Fig. 105. The load of 1000 lb is called AB, or BA, the reactions of 500 lb.

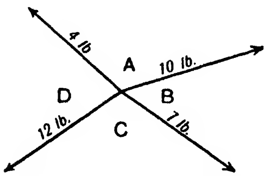


FIG. 104. Bow's notation.

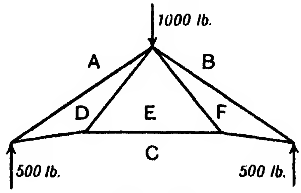


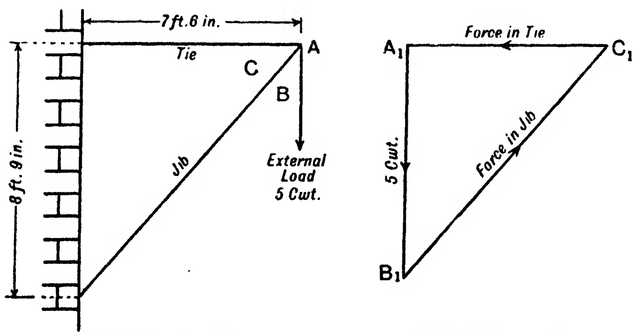
FIG. 105.

each are referred to as AC and BC and the members as AD, DE, DC, EC, EF, FC, and FB.

Thus, every force and every member is specified according to the letters assigned to the two spaces which it separates.

Forces in the members of a framed structure.

Example 1. A warehouse crane. *To find the force in each member of the structure shown in* Fig. 106.



Space diagram. Vector diagram.

FIG. 106.

Draw to scale the space diagram and letter this diagram according to Bow's notation. Then three forces act at the joint, or node ABC and produce equilibrium. These forces are :

(*a*) **external load** of 5 cwt., referred to as AB ;

(*b*) **axial force in the jib,** referred to as CB ;

(*c*) **axial force in the tie,** referred to as AC.

Of those three forces, AB is known in magnitude, sense and direction, while CB and AC are known in direction only. This is sufficient data to enable the triangle of forces for this node to be drawn. From this diagram the magnitudes and senses of the two forces CB and AC can be found. That is, in the *vector diagram* the length C_1A_1 will represent the force in the tie and the length B_1C_1 the force in the jib (Fig. 106).

Construction of vector diagram (Fig. 106).

Draw the load line A_1B_1 to represent the external load of 5 cwt. (AB in the space diagram), and from A_1 and B_1 draw lines parallel to the tie AC and the jib BC respectively. Then $A_1B_1C_1$ is the triangle of forces for the node ABC. Measure C_1A_1 to obtain the force in the tie AC and B_1C_1 to obtain the force in the jib BC. Tabulate the results and add the nature of the force in each member, using the method outlined later in the chapter.

Answers.

Member	Force	Nature
AC	4·3 cwt.	Tie
BC	6·6 cwt.	Strut

Nature of forces in a member. A tie is a member of a structure which is resisting tension, and is indicated according to the internal resisting forces by a straight line with a pair of arrows pointing *towards its centre* (Fig. 107).

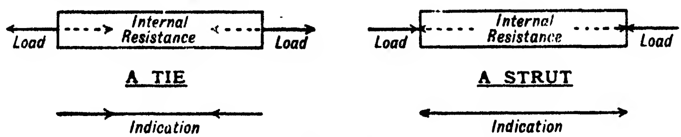


FIG. 107.

A strut is a member of a structure which is resisting compression, and is indicated according to the internal resisting forces by a straight line with a pair of arrows pointing *away from its centre* (Fig. 107).

NOTE.—*The distinction between ties and struts is made by placing arrows on the appropriate members in the space diagram.*

In the space diagram (Fig. 108), if the arrow points *towards* the node the member is a strut, for example, CB ; and if the arrow points *away* from the node the member is a tie, for example, AC.

Nature of forces from the vector diagram. If the direction, and sense, of one force in the vector diagram is known, all the other directions and senses can be found ; because the arrows follow around the diagram in a clockwise or anti-clockwise direction. For example (Fig. 109), A_1B_1 represents 5 cwt. acting downwards, and

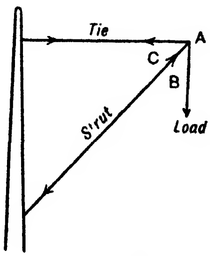


FIG. 108.

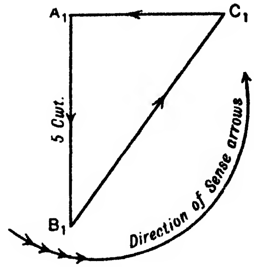


FIG. 109.

following this arrow around the triangle in an anti-clockwise direction from B_1 towards C_1 and from C_1 towards A_1, it is possible to indicate the sense of each of the forces represented. The senses of these forces may be transferred directly to the node ABC in the space diagram, thus :

For BC, in the vector diagram, the arrow points from B_1 toward C_1 ; therefore in the space diagram the arrow points along BC, in exactly the same sense and direction, *towards* the node, and indicates a strut.

For AC, in the vector diagram, the arrow points from C_1 towards A_1, and in the space diagram the arrow will point along AC *away* from the node and indicates a tie.

When the arrows are fixed for one end of a member the indication for the member may be completed by adding the complementary arrow at the other end of the member.

NOTE 1.—*If* A_1B_1 *acted in the upward direction it would indicate that the triangle of forces had to be followed around in a clockwise direction in order to fix the senses of the forces.*

NOTE 2.—*The direction of the forces in the tie and jib can be determined by imagining what would happen if either were to fracture. Their respective duties then become obvious.*

Example 2. A jib crane. *Find the forces in each of the members, assuming the forces are acting at a point* (Fig. 110).

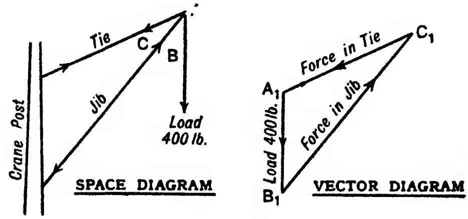


FIG. 110.

Answers.

Member	Force	Nature
AC	572 lb.	Tie
BC	820 lb.	Strut

Example 3. The toggle joint (Fig. 111). This is a mechanism for exerting a pressure in a direction at right angles to the effort applied.

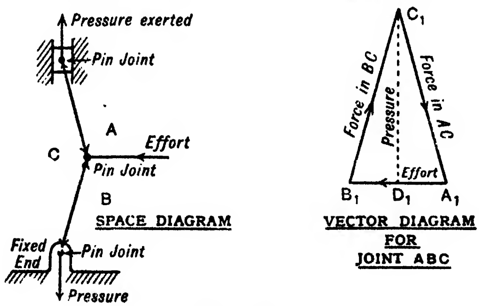


FIG. 111.

The pressure exerted can be made very large compared with the effort and, in consequence, the arrangement is frequently employed in machine design where this aim is desired.

The actual pressure exerted is represented by the line C_1D_1 in the vector diagram for the joint ABC.

Graphical treatment of the forces acting upon a body on an inclined plane.

Example 1. *A weight is supported on a frictionless inclined plane by a cord acting parallel to the plane. Find the tension in the cord and the reaction of the plane when the body is in equilibrium* (Fig. 112).

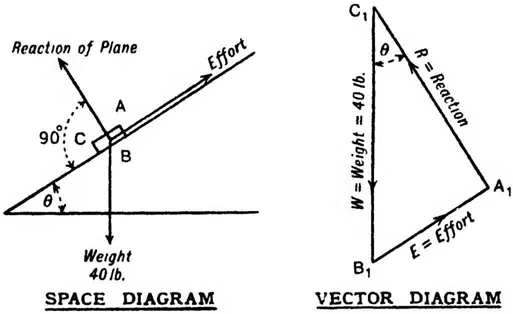


Fig. 112.

A body supported on an inclined plane is under the action of three forces, (*a*) the weight of the body, (*b*) the reaction of the plane, which is at right angles to the plane, and (*c*) the pull or tension in the cord. These three forces are in equilibrium, when the body is at rest, and since they are concurrent, and may be made co-planar, they can be found by the triangle of forces, if one force is completely known.

Reaction of plane = 34 lb., tension in cord = 21·2 lb.

Example 2. *Find the reaction of the plane and the tension in the cord when the supporting cord is not parallel to the plane* (Fig. 113).

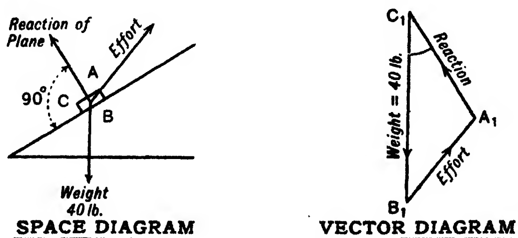


Fig. 113.

Reaction of plane = 26 lb., tension in cord = 22·8 lb.

Example 3. *Find the reaction of the plane and the tension in the cord when the supporting cord is acting along a horizontal line* (Fig. 114).

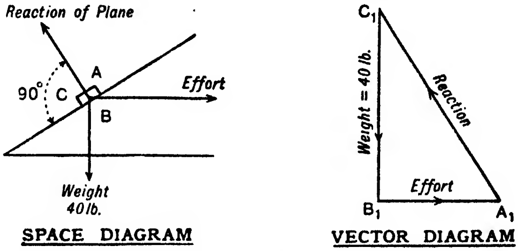


FIG. 114.

Reaction of plane = 48 lb.,
tension in cord = 27 lb.

EXPT. 7. OBJECTS.—(1) *To obtain the effort, and the reaction of the plane, when a roller is supported in equilibrium on an inclined plane with negligible friction.*

(2) *To verify the vector diagrams for the forces acting upon the roller and producing equilibrium.*

APPARATUS. An inclined plane which may be attached to a suitable stand, a roller with an extended axle, weights, stands, pulleys, and the necessary fine cords. See Fig. 115.

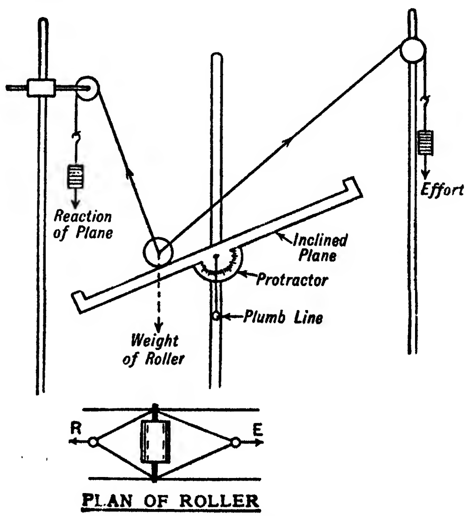


FIG. 115.

METHOD OF PROCEDURE. (1) Arrange the apparatus as shown in the diagram, taking particular care to see that the cord carrying the reaction is at right angles to the inclined plane.

(2) Carefully adjust the effort and reaction loads until the roller is just on the point of being lifted from the plane

(a thin piece of paper can be tried between the plane and the roller to ascertain whether this condition has been reached), and again make sure that the reaction cord is at right angles to the plane.

(3) Tabulate the observations obtained from the experiment and repeat the work, (*a*) with the effort parallel to the plane, (*b*) with the effort horizontal.

(4) Place a piece of paper behind the effort and reaction cords and transfer their directions to the paper. Complete the space diagram for the three forces acting upon the roller and draw the vector diagram, assuming a knowledge of the weight of the roller.

(5) Measure the lines in the vector diagram representing effort and reaction, and convert the measurements to forces by means of the scale employed. Tabulate these forces and compare them with the forces actually measured in the experiment.

OBSERVATIONS.

Direction of effort	Weight	Effort		Reaction	
	Weight of Roller	Expt.	Vector Diagram	Expt.	Vector Diagram
Inclined at 20° to plane	1·5 lb.	0·72 lb.	0·7 lb.	1·15 lb.	1·15 lb.
Parallel to plane - -	1·5 lb.	0·63 lb.	0·6 lb.	1·31 lb.	1·3 lb.
Horizontal - - -	1·5 lb.	0·94 lb.	0·9 lb.	1·72 lb.	1·7 lb.

CONCLUSIONS. (1) The observed effort and reaction of plane approximately agree with the values obtained by measurement of the vector diagram.

(2) This amounts to a further proof of the law of the triangle of forces for three co-planar, concurrent forces.

NOTE.—This experiment may now be extended, and a series of observations taken with the effort cord inclined at different angles to the plane. In this manner it may be proved that the effort required is least when the effort cord is parallel to the plane, providing the inclination of the plane is unaltered.

Treatment of framed structures with more than three members. When a structure contains more than three members, or more than one node, it can be treated by a form of composite diagram known as a **reciprocal figure.**

This reciprocal figure is composed of triangles and polygons of forces, one for each node in the structure.

METHOD OF PROCEDURE. (*a*) Calculate all necessary reactions due to external forces or loads.

(*b*) Select a node in which the magnitude and senses of not more than two forces are unknown. The directions of all the forces at the node are shown in the space diagram.

(*c*) Draw the vector diagram for this node.

(*d*) Select another node connected to the first with, as before, but two unknown magnitudes and senses to be determined. Then draw the vector diagram for this node, *using only forces which have been determined by the vector diagram for the previous node, as part of the diagram.*

(*e*) Continue with the remaining nodes until the vector diagram, or reciprocal figure for the whole structure is complete.

Example 1. A braced cantilever (Fig. 116). *Find the forces in the members of the loaded braced cantilever.*

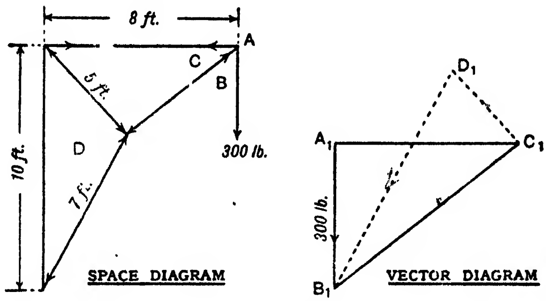


FIG. 116.

Method of constructing the vector diagram.

(*a*) Select the node ABC and draw, to scale, the load line A_1B_1. Complete the vector diagram for the node ABC, that is, the triangle $A_1B_1C_1$, shown in full in the diagram.

(*b*) Measure, and tabulate, the forces in the members AC and CB, and determine the nature of the forces in them from the triangle $A_1B_1C_1$.

(*c*) Take the node CBD, the force in CB is represented on the vector diagram by the line C_1B_1 and forms the base on which to construct the

vector diagram for the node CBD, that is, the triangle $B_1C_1D_1$, shown in chain lines in the diagram.

(*d*) Measure, and tabulate, the forces in the members CD and DB from the triangle $B_1C_1D_1$.

(*e*) Determine the nature of the forces at the node CBD, using the triangle $C_1B_1D_1$.

Results.

Member	Force	Nature
AC	380 lb.	Tie
CB	480 lb.	Strut
CD	195 lb.	Strut
DB	510 lb.	Strut

NOTE.—The *complementary* arrow on the member CB points towards the node CBD, so that the sense of the force represented by C_1B_1 in the vector diagram is from C_1 towards B_1. Following this arrow around the diagram $C_1B_1D_1$, the force represented by B_1D_1 becomes, in sense of direction, from B_1 towards D_1, thus the member BD *is in compression.* Similarly the sense of D_1C_1 is from D_1 towards C_1 and the member DC is *also in compression.*

Example 2. A roof truss. *To find the force in each member and the reaction of the supports for a symmetrically loaded roof truss* (Fig. 117).

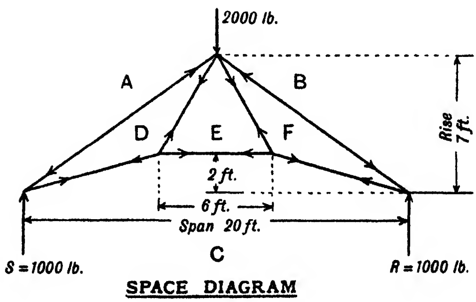


FIG. 117.

Reactions at supports. The truss carries a central load of 2000 lb. and the reactions *R* and *S* will therefore be equal, that is, they will each be 1000 lb.

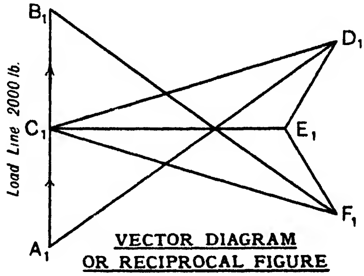


FIG. 118.

Construction of the vector diagram.

(1) Draw the load line B_1A_1, 2000 lb. to scale, and mark the point C_1, to show the reactions A_1C_1, C_1B_1, each of 1000 lb. (Fig. 118).

(2) Draw the vector diagram for the node ADC, that is, $A_1D_1C_1$, and continue the diagram for the node DEC, that is, add the section $D_1E_1C_1$.

(3) Continue the diagram for each node until the complete reciprocal figure is obtained, and measure and tabulate the forces in the members and their natures.

Results.

Member	Force	Nature
AD	2930 lb.	Strut
DC	2480 lb.	Tie
DE	800 lb.	Tie
EC	1980 lb.	Tie
EF	800 lb.	Tie
FC	2480 lb.	Tie
FB	2930 lb.	Strut

Notice that the external forces balance among themselves.

Resultant of two forces at right angles. Two forces acting at a point and at right angles to each other can be replaced by a single force, the resultant (Fig. 119). This resultant, from the law of the triangle of forces, will be seen to be represented by the hypotenuse of a right-angled triangle, in which the other two sides represent the forces of which the resultant is required.

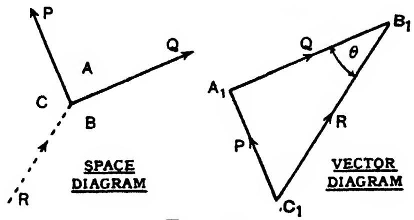


FIG. 119.

Let P and Q be the two forces and R the resultant.

Then the vector diagram is a right-angled triangle, and by the Theorem of Pythagoras,

$$\mathbf{R}^2 = \mathbf{P}^2 + \mathbf{Q}^2 \text{ or } \mathbf{R} = \sqrt{\mathbf{P}^2 + \mathbf{Q}^2}.$$

Example. *Find the resultant force on a ball which is subjected to a horizontal force of* 10 *lb. and a vertical force of* 8 *lb. simultaneously.*

Let R be the resultant,

then $$R = \sqrt{10^2 + 8^2} = \sqrt{100 + 64} = \sqrt{164}.$$

Resultant = 12·81 lb.

Direction of the resultant. The direction of the required resultant may be obtained by actually measuring the angle θ in the vector diagram. A student familiar with the trigonometrical properties of triangles will realise that the angle θ has a **tangent of P/Q**, that is, the ratio between the opposite and adjacent sides of the angle θ in the vector diagram.

Resolution of forces. A second consideration of the law of the parallelogram of forces shows that a force is often the resultant of two other forces. These two forces are said to be the **components** of the resultant.

In the diagram (Fig. 120), R is the resultant of P and Q, and P and Q are the components of R.

When the components are at right angles (Fig. 121) they are **called resolutes**, but are more generally referred to as the **horizontal and vertical components** of the resultant force.

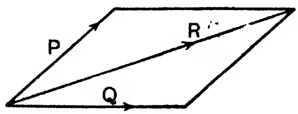


FIG. 120.

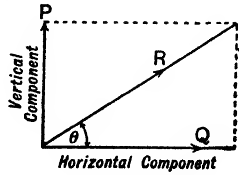


FIG. 121.

Trigonometrical proof.

$$\text{Horizontal component} = R\cos\theta = Q.$$
$$\text{Vertical component} \quad = R\sin\theta = P.$$

By the theorem of Pythagoras,

$$P^2 + Q^2 = R^2,$$

and $$R^2 = (R\sin\theta)^2 + (R\cos\theta)^2$$
$$= R^2(\sin^2\theta + \cos^2\theta),$$

but $$\sin^2\theta + \cos^2\theta = 1,$$

therefore $$R^2 = R^2 \times 1,$$

that is $\mathbf{R^2 = P^2 + Q^2}$ **where** $\mathbf{P = R\sin\theta}$ **and** $\mathbf{Q = R\cos\theta}$.

Example 1. *A steel column receives a force of* 300 *lb. directed at* 30° *to the horizontal and acting upon its top. Find the thrust, due to this force, on the foundation, and the force tending to overturn the column.*

NOTE.—(*a*) The vertical component presses upon the foundation.
(*b*) The horizontal component tends to overturn the column.

Vertical component $= 300\sin 30° = 300 \times 0{\cdot}5 = 150$ **lb.**
Horizontal component $= 300\cos 30° = 300 \times 0{\cdot}866 = 259{\cdot}8$ **lb.**

Example 2. *Find the reaction of the plane and the value of the effort for the inclined plane if* $W = 220$ *lb.,* $\theta = 30°$ (*see Fig.* 112).

In the vector diagram, which is a right-angled triangle, R, the reaction of the plane, is one rectangular component of W, and E is the other rectangular component of W.

Therefore, $R = W \cos \theta$ and $E = W \sin \theta$.

$$\mathbf{R} = 220 \times 0{\cdot}866 = 190{\cdot}5 \text{ lb.}$$
$$\mathbf{E} = 220 \times 0{\cdot}5 = 110 \text{ lb.}$$

NOTE.—Since W is vertical and R is perpendicular to the plane, then the angle θ between W and R must be the same as the angle of the plane.

Equilibrium of a body under the action of forces in one plane. In dealing with framed structures the structure itself may be considered as a body subjected to external forces. Since the structure is at rest, these external forces must be in equilibrium among themselves and subject to the above conditions.

Hence (1) the external forces are in equilibrium, and (2) the internal and external forces at each node, or point, are in equilibrium.

This assumption has been made in the examples in which the forces in members of a structure have been determined, and a special case has arisen, in which three forces maintain a body in equilibrium.

NOTE.—If three forces maintain a body in equilibrium the lines of action of the three forces must meet in *one point*. This is because the resultant of two of the forces must be equal and opposite and act in the same straight line as the third force.

General conditions of equilibrium for co-planar forces. If a body is in equilibrium under the action of a number of forces two conditions must be satisfied.

(1) **The vector polygon for the forces must close**; otherwise there would be motion, or tendency for motion, of the body in the direction of the resultant force acting upon it.

(2) **The algebraic sum of the moments of all the forces about any one point in the plane of the forces must be zero**; otherwise the body would rotate, or tend to rotate, in the direction of the resultant moment.

In the consideration of the conditions of equilibrium of a body acted upon by forces (*a*) tending to produce translation, (*b*) tending to produce rotation, it has been previously shown that the closure of the vector polygon is a proof of the equilibrium of the translational forces.

It remains to be shown that another type of polygon, the closure

of which is a proof of the equilibrium of the rotational forces, exists. This polygon is variously known as the **Link,** or **Funicular,** or **Moment Polygon** and is a vector representation, not of the forces, but of their moments as exercised upon the body with a tendency to produce rotation.

Complete equilibrium arises as a result of both the force, or vector polygon and the moment, or funicular polygon closing.

Experimental investigation of the general conditions of equilibrium. An experiment may now be performed with a view to verifying these conditions of equilibrium. In this experiment use will be made of the polygon known as the funicular polygon or link polygon. This polygon is obtained by a special treatment of the vector diagram and projection of certain lines back to the space diagram.

EXPT. 8. OBJECTS.—*To show that if* **any** *system of co-planar forces acts on a body which is in equilibrium then*

(1) *the vector polygon closes ;*

(2) *the algebraic sum of the moments of all the forces about any one point in their plane is zero ;*

(3) *the funicular or link polygon closes.*

APPARATUS. Place a number of large ball bearings or other uniform metal spheres on a horizontal table and let a smooth board rest on the bearings. To five different points on the edge of the board apply different co-planar, non-concurrent forces in arbitrary directions through spring balances as shown in Fig. 122. The smooth board constitutes the body upon which the forces act. The ball bearings practically eliminate friction and allow the board to take up freely a position of equilibrium under the action of the five forces. A short roller under the centre of gravity of each spring balance will also facilitate freedom of movement.

METHOD OF PROCEDURE. (*a*) Pin a sheet of drawing paper to the top of the board and transfer the directions of the five forces to the paper to obtain the space diagram. Letter the spaces A, B, C, D and E according to Bow's notation. Read the spring balances to obtain the magnitude of the forces.

(*b*) Draw the vector diagram *abcde* on the paper, the forces AB, BC, CD, DE and EA being represented in magnitude and direction respectively by the sides *ab*, *bc*, *cd*, *de* and *ea*. With care a closed vector polygon will be formed.

(*c*) Choose a pole *o*, which may be within or without the vector diagram, and join *o* to the vertices of this diagram.

(*d*) In the space A in the space diagram draw a line or link 1, 2, starting from any point 1 in the line of action of the force AE, parallel to *oa* on the vector diagram. In space B draw 2, 3 parallel to *ob* and so on. It will be found that the funicular polygon 1, 2, 3, 4, 5 connects or links together the five forces and closes on the line of action of the force *AE* at point 1.

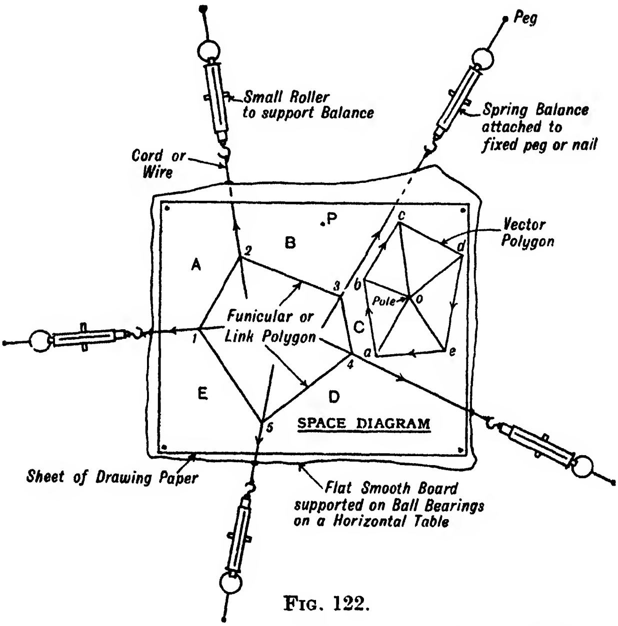


FIG. 122.

(*e*) Choose any point such as P in the plane of the forces and take moments of the five forces acting on the board about P. It will be found that the sum of the clockwise moments will be equal to the sum of the anti-clockwise moments.

CONCLUSIONS. The three objects of the experiment are shown to be both separately and collectively true, provided there is equilibrium. Hence they may be employed separately to solve problems involving co-planar forces acting on bodies in equilibrium.

Conversely, **to produce equilibrium, two conditions are necessary :**

(1) the vector polygon must close,

(2) the algebraic sum of the moments of all the forces about any one point in their plane is zero.

This should be readily understood, because if the vector polygon closes there will be no resultant force tending to move the body in any direction, while if the principle of moments is satisfied there will be no tendency to rotate the body about any point. Hence the body will be in equilibrium or at rest.

Alternatively, the necessary and complete conditions for producing equilibrium may be expressed as,

(1) the vector polygon to close and,
(2) the funicular or link polygon to close.

It will be noticed that condition (2) replaces the principle of moments mentioned above.

Examples of the use of the funicular polygon.

Example 1. *To find the reactions at the supports of a beam carrying several loads, and the position of the resultant load* (Fig. 123).

METHOD OF PROCEDURE. (1) Draw the space diagram to a suitable scale.

(2) Produce the force lines downward and draw the vector diagram $A_1B_1C_1D_1$, which is a straight line because the loads are parallel.

(3) Select a pole O and complete the polar diagram.

(4) Construct the funicular polygon by drawing in the space A a line parallel to OA_1, in the space B a line parallel to OB_1 and so on. Draw the closing line of the funicular polygon, that is *de*, and from O draw OE_1 in the vector diagram to meet the load line at

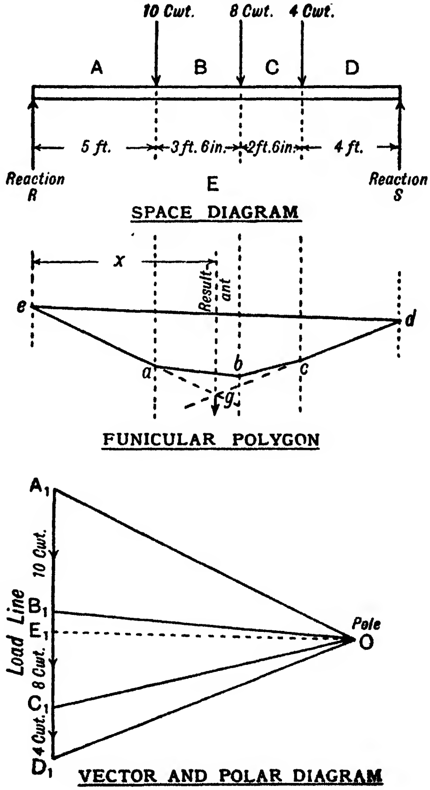


FIG. 123.

E_1. Then A_1E_1 *is the reaction* **R** to scale and E_1D_1 is **the reaction S** to scale.

(5) In the funicular polygon produce the lines *ea* and *dc* to meet at *g*. Then *g is the point through which the resultant load acts.*

NOTE.—The magnitude of the resultant is the sum of the loads, that is,

$$10+8+4=22 \text{ cwt.}$$

Answers. Resultant 22 cwt. acting along a vertical line 7 ft. 4 in. from R.
Reactions R = 11·2 cwt., S = 10·8 cwt.

Verification by the principle of moments.

Moments about R,

$$10\times5+8\times8\tfrac{1}{2}+4\times11=15S,$$
$$15S=162,$$
$$S=10\cdot8$$
$$R=22-10\cdot8=11\cdot2$$

Position of the resultant. Let the resultant be x ft. from R; then taking moments about R,

$$10\times5+8\times8\tfrac{1}{2}+4\times11=\text{resultant}\times x=22x.$$
$$22x=162 \text{ or } x=7\cdot4.$$
$$\therefore \text{ Resultant is } 7\cdot4 \text{ ft. from R.}$$

Example illustrating the conditions of equilibrium. The space diagram shown (Fig. 124) gives the forces acting on a shaft due to the pulls of belts on two pulleys. The latter are in parallel planes close together. In this question the forces are assumed to be in the same plane and bending action on the shaft and its weight are neglected.

The four forces and the equilibrant keep a portion of the shaft in equilibrium. Hence,

(*a*) the **vector polygon** $A_1B_1C_1D_1E_1$ **closes,** the line E_1A_1 giving the equilibrant in magnitude and direction;

(*b*) the **link polygon** 1, 2, 3, 4, 5 **closes,** the point 5 giving a point in the line of action of the equilibrant. (*a*) and (*b*) thus fix the line of action, sense and magnitude of E_1A_1 completely.

The **principle of moments** may also be used to find a point in the line of action of the equilibrant.

Clockwise moments of the given forces about shaft centre P in lb. ft.

$$=400\times1\tfrac{1}{2}+600\times1=1200$$

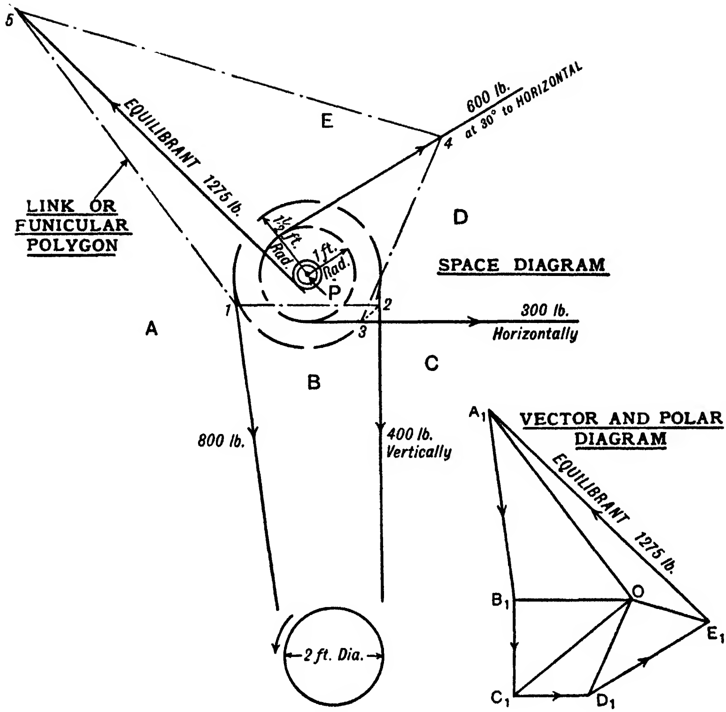


FIG. 124.

Similarly, anti-clockwise moments about P in lb. ft.

$$=800 \times 1\tfrac{1}{2} + 300 \times 1 = 1500$$

Therefore, for equilibrium, the equilibrant must have a clockwise moment of 300 lb. ft. about P.

Hence the perpendicular distance of the equilibrant from P in ft.

$$=\tfrac{300}{1275} = 0{\cdot}235 \text{ or } 2{\cdot}8 \text{ in.}$$

This result agrees with that obtained by the funicular polygon.

NOTE.—In practice this equilibrant would have to be supplied by the shaft bearings, which are not in the same plane as the pulleys, and in consequence the shaft itself would be subjected to bending.

EXERCISES ON CHAPTER V

1. A detail in a mechanism receives a thrust of 70 lb. directed at an angle of 30° to the horizontal. In addition, the piece is pressed on to its horizontal bed by a force of 50 lb. Find the magnitude and direction of the resultant force acting upon the bed.

2. Find the magnitude and direction of the equilibrant of the two forces in Fig. 125. What alteration is required to be made to this equilibrant in order that it may become the resultant of the two forces *A* and *B* ?

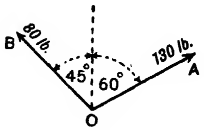


Fig. 125.

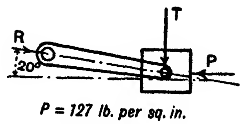


Fig. 126.

3. Find the force on the cylinder walls and the thrust on the connecting rod in an I.C. engine if the piston is 4 in. in diameter (Fig. 126). In the diagram *T* is shown as the resultant of *R* and *P*.

4. Fig. 127 shows the outline of a wall bracket for a horizontal shaft. Find the forces in the horizontal and inclined portions of the bracket and indicate the nature of these forces.

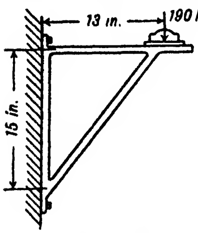


Fig. 127.

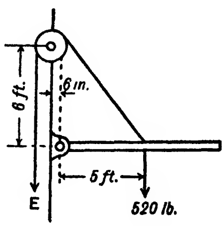


Fig. 128.

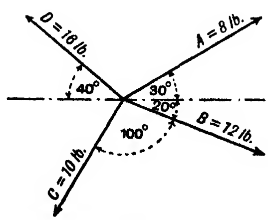


Fig. 129.

5. The diagram (Fig. 128) shows a gangway for use on a floating pontoon. Find the value of the force *E* when the gangway is just about to lift. What is the turning moment of *E* about the hinge at this period ?

6. Find the magnitude and direction of the single force which will replace the four forces shown in Fig. 129. What is this force called, and what name is given to an equal and opposite force ?

7. The rotor of a generator is slung from two points, on its axis, 4 ft. 3 in. apart. If the hook is 5 ft. 6 in. above the axis of the generator rotor and the rotor weighs 630 lb., find the tension in the sling.

8. The space diagram (Fig. 130) shows the plan of the forces acting on a telegraph pole. AB and BC are the effective horizontal forces due to the tension in the wires which are changing direction. Find the magnitude of the forces in the horizontal stays AD and CD.

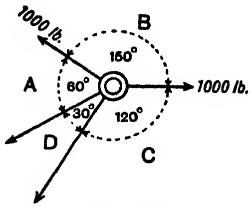


Fig. 130.

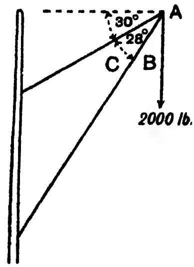


Fig. 131.

9. Find the forces in the members of the simple jib crane shown in Fig. 131. Indicate the nature of the forces and mark the jib.

10. A wheel has five spokes spaced with equal angles between them. If the forces in three adjacent spokes are 1200, 1400 and 1000 lb. respectively, find the forces required, for equilibrium, in the remaining two spokes.

11. An apparatus for the manufacture of briquettes is shown in Fig. 132. The toggle is operated through a hydraulic ram 4 in. in diameter. Find the maximum force, for the position shown, on the plunger A if the hydraulic pressure on the ram is 700 lb. per sq. in.

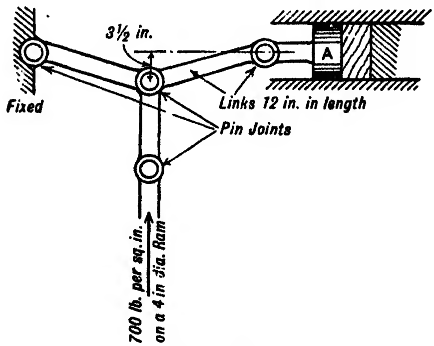


Fig. 132.

12. Find the forces, and the nature of the forces, in the members of the framed structures shown in Fig. 133.

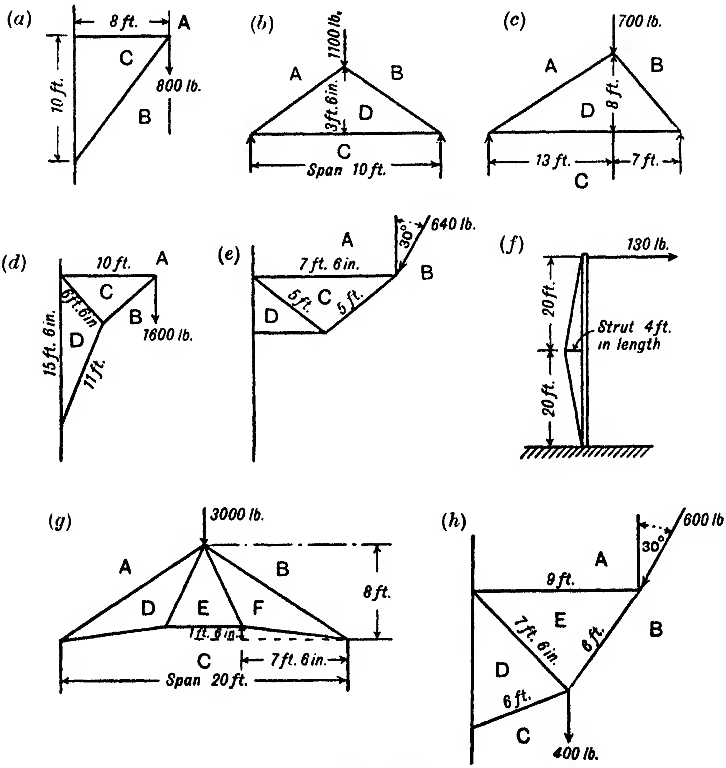


FIG. 133.

13. Find the forces and the nature of the forces in the members of the framed structures shown in Fig. 134.

14. A mooring mast is in the form of a right-angled triangle of height 20 ft. and base 8 ft. It receives a pull of 1100 lb. directed at its top and at 20° to the horizontal on the sloping side of the mast. Find the forces, and the nature of the forces in the members, and the moment tending to overturn the mast about the base of its sloping side.

15. Draw the space diagram of the bicycle shown in Fig. 135, and find the forces in the bars DA, AB and in each of the arms of the fork AC and the fork BC. State the nature of each of the forces.

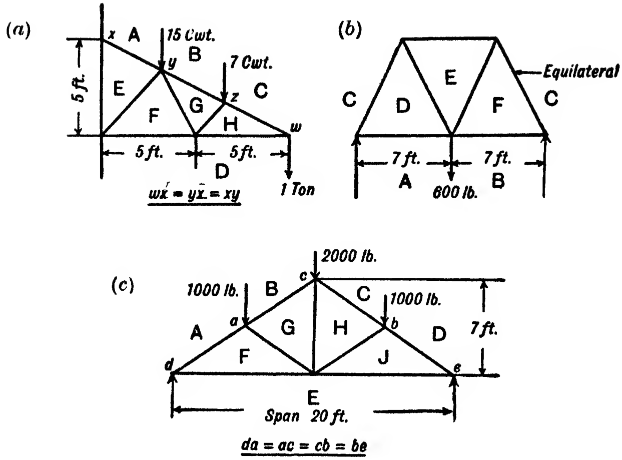


FIG. 134.

16. Find the horizontal and vertical components of a force of 70 lb. acting at 50° to the horizontal.

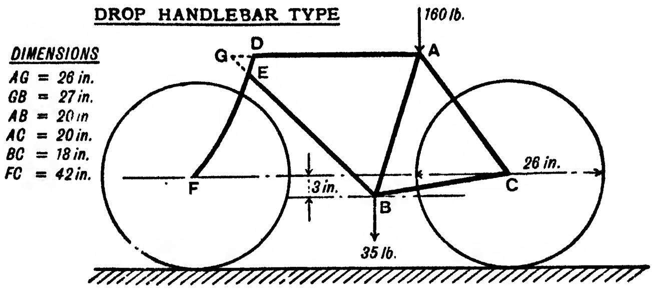


FIG. 135.

17. Use the funicular polygon to find the reactions at the supports, and the resultant load on the following beams :

(a) of 9 ft. span with loads of 7 and 10 cwt. situated 3 ft. and 6 ft. respectively from the left-hand support.

(b) of 14 ft. span with loads of 4, 7 and 9 cwts. respectively, 5, 7 and 12 ft. from the left-hand support.

18. A ladder weighing 70 lb. rests against a wall and is inclined at 60° to the vertical. If the ladder is 25 ft. in length and its weight may

be assumed to act at a point 10 ft. from the lower end, find the resultant force on the ground. Assume the reaction of the wall to be at 15° to the horizontal.

19. Forces of 2 lb., 4 lb. and 5 lb. act, in order, along the sides of an equilateral triangle of 1 ft. side. Determine the equilibrant in magnitude, line of action and sense in relation to the triangle. Use a graphical construction and verify by measurement and calculation.

20. The ends of a piece of rope, 10 ft. in length, are connected to two points A and B. B is 6 ft. from A, measured horizontally, and 2 ft. below the horizontal through A measured vertically. If a weight of 2 cwt. is slung on the rope so that it is free to move along it, determine its position of equilibrium and the tension in the rope.

21. A machine is subjected to three forces as follows: A 1000 lb., B 800 lb. at right angles to A, and C 400 lb. parallel and in the same direction as A, but with its line of action 2 ft. removed from A. Find the direction in which the machine will tend to move and the resultant force acting upon it.

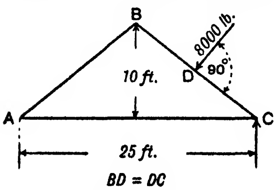


FIG. 136.

22. Find the magnitude and direction of the force at A, and the magnitude of the force at C to maintain the equilibrium of the simple roof frame, loaded as shown at the mid point of BC (Fig. 136).

23. The skeleton or space diagram of a simple jib crane is shown (Fig. 137). The weight of the jib is 500 lb. and it is 30 ft. in length. If the action of gravity on the jib is equivalent to a force of 500 lb. acting at a point in it, 14 ft. from the hinge, and the jib is in equilibrium when C is 15 ft. vertically above A, find the magnitude of the force in the horizontal tie T and the thrust of the jib on the hinge.

24. Find the force required in hoisting, and in the guy rope, for the configuration shown in the space diagram of a lifting device (Fig. 138). Also determine the magnitude and direction of the resultant force on the pulley.

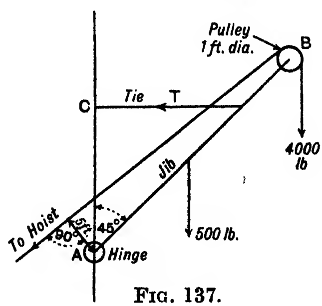


FIG. 137.

25. A capstan is operated by four deck hands exerting, on a radius of 5 ft., in a clockwise direction, forces of 100 lb., 120 lb., 80 lb. and 90 lb. spaced symmetrically around the capstan. Find the resultant force on the capstan, using the methods of the funicular polygon, the principle of moments and resolution of forces.

26. Determine by calculation the effort and reaction of the plane when a smooth plane of inclination 40° to the horizontal supports in equilibrium a body of weight 220 lb.,

(a) with the effort parallel to the plane,
(b) with the effort at 20° to the angle of the plane,
(c) with the effort horizontal.

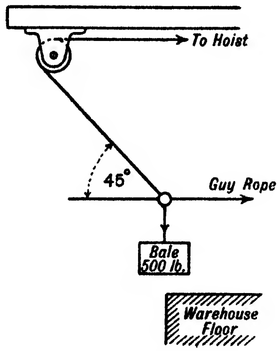


FIG. 138.

27. A ship is being towed at constant speed by two tugs. The angles made by the tow-ropes with the direction of motion of the ship are 60° and 30° respectively. The force opposing the motion of the ship is 6 tons acting along the line of its motion. What is the pull in each of the tow-ropes? (U.L.C.I.)

28. Two forces OA and OB of 5 lb. and 7 lb. respectively, pull on a body at a point O, the angle AOB being 70°. Find the magnitude and direction of a third force which will balance these two. (U.L.C.I.)

29. The base of a machine exerts a thrust of 5 tons, inclined at an angle of 10° to the vertical, upon its horizontal bed plate. Determine the vertical and horizontal components of the thrust and state what the component forces tend to do. (U.L.C.I.)

30. A carriage mounted on wheels which may be assumed to be frictionless rests on a plane inclined at 30° to the horizontal. If the carriage weighs 8 lb., find by graphical construction the force required to keep it in equilibrium—

(a) when the force is applied horizontally,
(b) when the force is applied in a direction parallel to the plane. (U.L.C.I.)

31. A crank is 1 ft. long and its connecting rod 4 ft. long, and the force along the axis of the latter is 10,000 lb. Resolve this force along the axis of the crank and perpendicular to it. The crank has turned 30° from its inner dead centre. Explain the effect each component will have on the crank. (U.L.C.I.)

CHAPTER VI

FRICTION AND THE EFFECTS OF FRICTION, BEARINGS AND LUBRICATION

Nature of friction between dry surfaces. Surfaces which appear, upon casual examination, to be perfectly smooth, when microscopically examined prove to have a large number of irregularities and undulations. When two such surfaces slide, one over the other, the irregularities provide a resistance to the motion. The resistance to the motion is termed **friction** and the force necessary to overcome it, **the force of friction** or **effect of friction.**

It is found that the force of friction to start a body moving is greater than that necessary to keep it moving. In other words, the **static friction is greater than the friction of motion.**

Forces acting upon two sliding surfaces. Two surfaces (Fig. 139), in sliding contact, are in the simplest condition acted upon by three forces, which are :

(1) The resultant force acting at right angles, and hereafter called the pressure between the surfaces.

(2) The frictional resistance, which is *tangential* to the surfaces.

(3) The force of friction, which is equal and opposite to the frictional resistance.

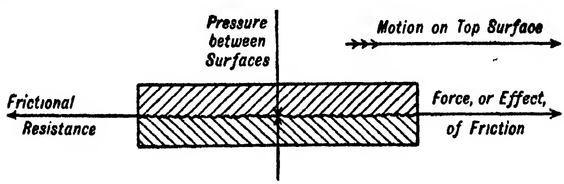


FIG. 139.

The action of these three forces produces equilibrium, and any increase of the force in the direction of the force of friction will produce motion.

Thus, **friction always opposes motion,** and may set up sufficient resistance to prevent it ; but, as the forces tending to cause motion

increase in magnitude, the frictional resistance reaches an upper limiting value when motion is just about to take place.

The **coefficient of friction** is the name given to the ratio

$$\frac{\text{force of limiting friction}}{\text{pressure between surfaces}},$$

a ratio which is referred to by the Greek letter "μ", pronounced *mu*.

Experiments to verify the laws of friction for dry surfaces.

EXPT. 9. OBJECTS. (*a*) *To measure the force of friction.*

(*b*) *To show that the force of friction depends upon the materials in contact.*

(*c*) *To obtain the coefficient of friction for each pair of materials.*

APPARATUS. The apparatus (Fig. 140) consists of a smooth wooden plane, set horizontally, about 2 ft. 6 in. in length and 6 in. in width.

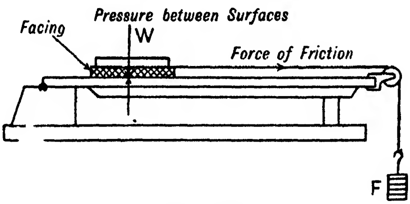


FIG. 140.

At one end is fitted a pulley with a good bearing, as free from friction as possible. A series of sleighs, of equal weight and surface dimensions, are obtained, each made of, or faced with, a different material. The sleigh in use is connected by a light cord parallel with the plane and passing over the pulley to a hanger upon which weights can be placed.

METHOD OF PROCEDURE. To measure the force of friction take one of the sleighs and thoroughly dust both its contact surface and that of the plane. Then carefully add weights to the hanger, just sufficient to cause the sleigh to slide along the plane without appreciable alteration of speed. The total weight of the hanger, and added weights, is equal to the force of friction, and the weight of the sleigh is the pressure between the surfaces. The coefficient of friction is the ratio between the force of friction and the pressure between the surfaces in contact, and is a quantity which gives a ready indication of the amount of friction between two surfaces.

OBSERVATIONS.

	Materials	Mahogany on Oak	Deal on Oak	Leather on Oak	Oak on Oak	Gunmetal on Oak
W	Pressure between surfaces	2·0 lb.	2·0 lb.	2·0 lb.	2·0 lb.	2·0 lb.
F	Force of friction	0·7 lb.	0·8 lb.	1·3 lb.	0·9 lb.	0·6 lb.
$\frac{F}{W}$	Coefficient of friction	0·35	0·4	0·65	0·45	0·3

CONCLUSIONS. (*a*) **The force of friction for dry, that is, non-lubricated surfaces, depends upon the nature of the surfaces in contact and the materials from which they are made.**

(*b*) The coefficient of friction is a ratio which indicates the relative amount of friction existing between pairs of surfaces when compared with other materials in contact. Note that a high coefficient exists when leather is employed, and this property of leather makes it a suitable material for belt drives and the linings of clutches and brake shoes.

EXPT. 10. OBJECT. To compare the force of friction with the pressure between two dry surfaces in sliding contact.

APPARATUS. The apparatus employed is the same as for the previous experiment, but only one of the sleighs is used; to which is added weights to increase the pressure between the surfaces.

METHOD OF PROCEDURE. Measure the force of friction for the sleigh (Fig. 141) in the manner described in the previous experiment. Then increase the pressure between the surfaces, by adding weights to the sleigh, and find the force of friction for this increased pressure. Continue the process for steadily increasing pressures between surfaces, and calculate the coefficient of friction for each case. Plot a graph of force of friction against pressure between the surfaces, and attempt to find a relationship between them. The pressure between the surfaces will be the sum of the weights of the sleigh and the added weights. As before the limiting force of friction is the force which will just produce motion without appreciable alteration of speed.

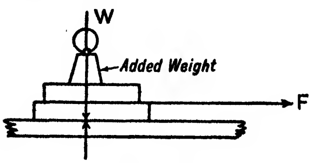


FIG. 141.

Observations.

Force of friction, F lb. - -	0·7	1·05	1·40	1·75	2·10	F
Pressure between surfaces, W lb.	2	3	4	5	6	W
Coefficient of friction $\mu = \frac{F}{W}$ -	0·35	0·35	0·35	0·35	0·35	$\frac{F}{W}$

Graph.

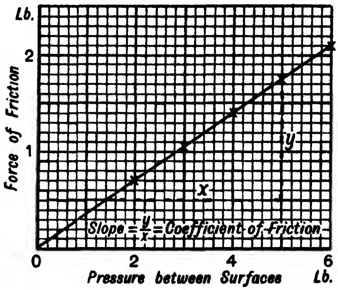


Fig. 142. Graph of force of friction *v.* pressure between the surfaces.

Conclusions. (*a*) Because the graph of force of friction against pressure between surfaces is a straight line passing through the origin, the **force of friction is directly proportional to the pressure between the surfaces.**

(*b*) The *gradient* of this graph, that is, the ratio

$$\frac{\text{force of limiting friction}}{\text{pressure between surfaces}},$$

is constant for all inter-surface pressures and is equal to the **coefficient of friction.**

Note.—The equation to a straight line graph passing through the origin is of the form $\mathbf{y=mx}$, where y is the vertical axis reading, or ordinate, and x the corresponding horizontal axis reading. The quantity "**m**" is called the gradient of the graph and is the **ratio y/x.** It follows, for this graph where y is the force of friction F, and x the pressure between surfaces W, that

$$F = mW \text{ and } m = \text{coefficient of friction,}$$

that is,

force of friction = coefficient of friction × pressure between surfaces,

and $\quad\quad$ **F = μW is the law of the graph.**

Expt. 11. Object. To show that the force of friction for dry surfaces, in sliding contact, is independent of the areas in contact.

Apparatus. The same plane is employed as is used in the previous experiments and a block of wood is prepared with two surfaces at right angles, these surfaces A and B of widely different dimensions.

Method of Procedure. Obtain the force of friction, with the larger surface A in contact with the plane, and then measure the

force with the smaller surface B in contact, maintaining the effort cord parallel to the plane. The same force should be required in each case.

CONCLUSION. The force of friction is independent of the areas in sliding contact, providing the materials and quality of the surfaces remain the same.

Laws of friction. The foregoing experiments illustrate the laws governing the friction of dry, non-lubricated surfaces, and these laws may be regarded as *approximately* true for such surfaces in practice.

Law 1. Friction always opposes motion.

Law 2. The force of friction depends upon the materials and the quality of the surfaces in contact.

Law 3. When **motion is about to take place** the magnitude of the limiting friction is directly proportional to the normal pressure between the surfaces in contact, that is :

force of friction = coefficient of friction × pressure between surfaces.

Law 4. The force of friction is independent of the areas in contact providing the quality of surfaces and materials in contact remain the same.

These laws apply to bodies at rest and are thus known as the **laws of static friction.** The force of friction can be shown to vary with the speed at which a body is moving, and this forms a subject for later study, that is, the friction between surfaces in motion, or **kinetic friction.**

If two dry surfaces are left under pressure for some time they appear, as time passes, to embed themselves, and the static friction slowly increases up to a point. In fact adhesion may occur as more and more air is slowly excluded from between the surfaces.

Example 1. *The resistance due to friction when a lathe saddle slides over its bed is* 68 *lb., and the weight of the saddle is* 340 *lb. Find the coefficient of friction assuming the surfaces to be dry.*

Force of friction = 68 lb.
Pressure between surfaces = 340 lb.
Coefficient of friction $= \frac{68}{340} = 0{\cdot}2.$

Example 2. *If the effect of lubrication is to reduce the coefficient of friction between the saddle and bed, of Example* 1, *to* 0·06, *find the force necessary to work the saddle.*

$$\mu = 0{\cdot}06$$
$$= \frac{\text{force of friction}}{\text{pressure between surfaces}} = \frac{F}{340}.$$
$$\frac{F}{340} = 0{\cdot}06, \quad F = 20{\cdot}4. \quad \textbf{Ans.} \quad \textbf{Required force} = \textbf{20·4 lb.}$$

Example 3. *In a recoil mechanism, a block is caused to return along a slide by a spring which exerts a force of* 17 *lb. Find the maximum weight which can be given to the block if the slide is horizontal and the coefficient of friction* 0·07.

Force of friction = 17 lb.

$$\mu = \frac{\text{force of friction}}{\text{pressure between surfaces}}.$$
$$0{\cdot}07 = \frac{17}{W}, \quad W = 242{\cdot}8. \quad \textbf{Ans.} \quad \textbf{242·8 lb.}$$

Friction of liquids, and lubrication. The laws of fluid friction cover the movement of particles of liquids in contact with each other and with other bodies.

The friction of liquids is believed,

(1) to be independent of the load,
(2) dependent upon the extent of the surface, and its roughness,
(3) to increase with the square of the speed, except at low speeds,
(4) to be proportional to the density of the liquid.

These results are illustrated by the resistance offered to the motion of ships, frequently referred to as **skin friction.**

Lubricated surfaces. The friction between well lubricated surfaces appears to approach the results obtained for friction in liquids, and the more efficient the lubrication, the more does the frictional resistance depart from that for dry surfaces. The modern trend, aiming at increasing the efficiency of bearings and sliding details, is met by,

(*a*) decreasing the contact surface area ;
(*b*) maintaining a wedge-shaped film of lubricant between the surfaces at such a pressure that it will not be forced away from them under load ;

(*c*) selecting a lubricant which will maintain its lubricating properties under load, and at the temperature involved ;

(*d*) employing materials and surface qualities which present the least frictional resistance.

Two classes of lubricating systems. Systems of lubrication may be divided into two classes,

(1) lubrication at, or near, atmospheric pressure,

(2) lubrication at high pressure, or **forced lubrication** ;

and it is the latter which is extensively employed in high-speed machinery carrying heavy bearing pressure.

Lubrication at atmospheric pressure. This is the basis for lubrication of the drip, oil feed, siphon, pad, and oil bath methods with which the student is probably familiar, either by workshop contact or in the study of machine drawing. The oil reaches the surfaces in contact, at approximately atmospheric pressure, and it can readily be seen that if the pressure between the surfaces is high, the lubricant will be forced away from them, and the conditions will revert to something approaching those operating for dry surfaces. In this type of lubrication it is necessary to keep the surfaces constantly coated with oil, a task which becomes difficult, in the case of moving parts working at high speed, or under heavy load. When lubricated surfaces have been at rest for some time the lubricant tends to be squeezed out, so that the starting or static friction may be much the same as with dry surfaces.

Forced, or pressure lubrication. In this system of lubrication the lubricant is pumped between the surfaces at such a pressure as to ensure a film of oil being maintained between them. The pressure of oil must be sufficient to prevent the oil being forced away from the surfaces by the pressure between them. This method of lubrication is extensively employed for high-speed engines and machines, and its effect is to lower considerably the frictional resistance between the bearing surfaces.

Lubrication of ball and roller bearings. Ball and roller bearings should be lubricated with a special ball bearing grease. The lubricating properties of this grease is a secondary consideration, its main function being the protection of the bearing against dust,

moisture and other injurious matter. This grease should be periodically renewed and should be selected according to the advice published by the bearing manufacturers.

Messrs. Skefco recommend, for general purposes, the following lubricants :

For temperatures of running up to 45° C. lime-soap grease, between 45° C. and 95° C. soda-soap grease, and for running temperatures above 95° C. mineral oil. Ball and roller bearings, efficiently installed and lubricated, give the following general advantages over plain bearings :

(*a*) **Reduction in the starting effort required,**
(*b*) **reduction of belt costs up to 30 per cent.,**
(*c*) **reduction of lubricating costs,**
(*d*) **an increase of line shafting speed with consequent smaller pulleys,**
(*e*) **reduction of horse power lost in friction up to 20 per cent.**

Effect of temperature. High temperature has two major influences on a lubricant :

(1) to increase its mobility, and thus its tendency to leave the bearing surfaces ;
(2) to bring about a tendency to chemical decomposition of the oil due to burning or oxidation.

For these reasons. oils selected for lubrication at high temperatures have to be carefully chosen ; those employed in cylinders of internal combustion engines have to possess a very high decomposition temperature, in addition to the necessary lubricating properties.

Coefficients of friction. The following table gives some general values for the coefficient of friction with different materials and conditions.

Average pressures and low speeds.

Materials	Dry	Lubricated
Wood on Wood -	0·25 – 0·5	0·02 – 0·1
Metal on Wood -	0·2 – 0·6	0·02 – 0·08
Metal on Metal - -	0·2 – 0·3	0·04 – 0·08
Leather on Metal -	0·4 – 0·6	0·1 – 0·25

The lower values are for continuous lubrication and the higher values for intermittent lubrication.

Ball and roller bearings. The Skefco Ball Bearing Co. Ltd. give the following general values for ball and roller bearings of their manufacture :

Bearing	Coefficient of friction
Ball and Cylindrical Roller Bearings -	0·001 – 0·0012
Taper Roller Bearings - - -	0·002 – 0·004

It is found that the friction due to one body rolling over another is considerably less than the corresponding sliding friction. This has led to the general use of ball and roller bearings, particularly for fast running machinery.

EXPT. 12. OBJECTS. (1) To study the effect of friction at a bearing. (2) To compare the effects of friction for plain and ball bearings. (3) To obtain the efficiency of a bearing at various loads.

APPARATUS. A pulley about 8 in. in diameter (Fig. 143) supported by a wall bracket, and fitted with interchangeable plain and ball bearings. A length of rope with eye-spliced ends, suitable weights and hangers.

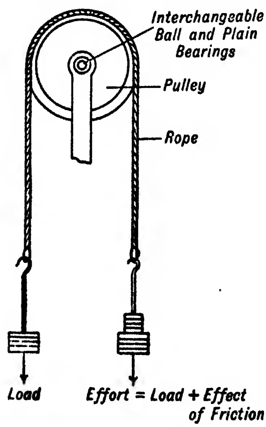


FIG. 143.

METHOD OF PROCEDURE. Attach, to the hangers, two equal weights and arrange their heights so that the two weights are level. Consider one of these weights as a load to be raised and the other the effort required to raise the load. Add weights to the effort side sufficient just to raise the load, then this additional weight will be equal to the effect of friction. Repeat the experiment for a series of increasing loads, and again carry out the experiment for the same range of loads, but after replacing the plain bearings by ball. Tabulate the results, and plot a graph of load against effect of friction for each set of bearings. To obtain the derived results, which have for their object the comparison of efficiencies, consider the load and effort to move

1 foot; then the work input, or work done by the effort in raising the load, and overcoming friction, will be (effort multiplied by 1) ft. lb.; whereas the work output, or useful work done on the load, will be (load multiplied by 1) ft. lb. The efficiency is always the ratio $\frac{\text{work output}}{\text{work input}}$, and this can be calculated for each observation. Plot a graph (Figs. 144, 145) of efficiency against load for each set of bearings and compare the graphs.

OBSERVATIONS.

	Load lb.	2	4	6	8	10	12	14
	Plain bearings, effort lb.	2·80	5·05	7·3	9·53	11·75	14·0	16·25
	Ball bearings, effort lb.	2·23	4·30	6·37	8·44	10·51	12·58	14·65
A	Effect of friction, plain bearings	0·8	1·05	1·30	1·53	1·75	2·0	2·25
B	Effect of friction, ball bearings	0·23	0·30	0·37	0·44	0·51	0·58	0·65
$\frac{A}{B}$	Ratio $\frac{\text{plain}}{\text{ball}}$	3·48	3·5	3·51	3·48	3·44	3·45	3·47

DERIVED RESULTS. Efficiencies calculated on a load movement of 1 ft.

Output	Work done on load, ft. lb.	2	4	6	8	10	12	14
Input, plain	Work done by effort, ft. lb.	2·8	5·05	7·3	9·53	11·75	14·0	16·25
Input, ball	Work done by effort, ft. lb.	2·23	4·30	6·37	8·44	10·51	12·58	14·65
Plain	Efficiency %	71·5	79·1	82·1	84·0	85·1	85·6	86·1
Ball	Efficiency %	89·9	93·0	94·2	94·9	95·1	95·4	95·6
Ratio	$\frac{\text{Efficiency, ball}}{\text{Efficiency, plain}}$	1·255	1·174	1·145	1·127	1·117	1·112	1·11

GRAPHS.

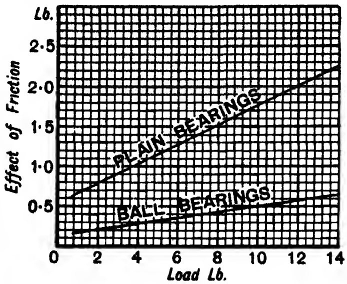


FIG. 144. Graph of load v. effect of friction for ball and plain bearings.

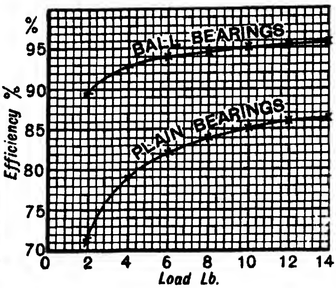


FIG. 145. Graph of load v. efficiency for plain and ball bearings.

CONCLUSIONS. (1) Because the graph of load v. effect of friction is a straight line which does not pass through the origin, the *increase* of effect of friction is directly proportional to the *increase* of load.

(2) The effect of friction with the use of ball bearings is much less than that with the use of plain bearings.

(3) The efficiency of each type of bearing increases with the load but *not proportionally*.

(4) The efficiency of ball bearings is much higher than that of plain bearings carrying the same load.

Types of ball and roller bearings. The types of ball and roller bearings in general use are illustrated in Figs. 146-151; many are made

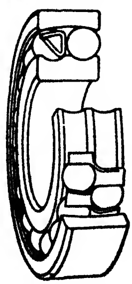


FIG. 146.
Double row self-aligning ball bearings.

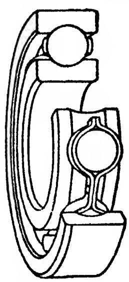


FIG. 147.
Single row ball bearings.

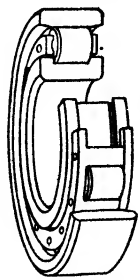


FIG. 148.
Cylindrical roller bearings.

(*The Skefco Ball Bearing Co., Ltd.*)

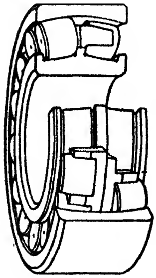


FIG. 149.
Self-aligning roller bearings.

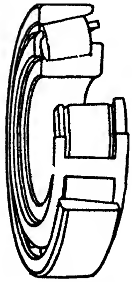


FIG. 150.
Taper roller bearings.

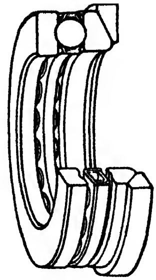


FIG. 151.
Single thrust bearings.

(*The Skefco Ball Bearing Co., Ltd.*)

with a self-aligning feature which allows for machine frame distortion, and cases of slight misalignment of shafting which occur in practice.

Employment of ball and roller bearings in machine design. The diagram (Fig. 152) shows a hypothetical transmission gear which lends itself to the use of many of the varieties of ball and roller bearings which are carried as standard stock by the makers. It consists of a heavy duty worm and worm-wheel transmission ; to which is geared, by a bevel wheel pair, a light transmission suitable for driving a lubricating pump.

The worm shaft demands suitable bearings for heavy duty rotation, in addition to a heavy duty thrust bearing to receive the end thrust, which is always present in worm and screw transmission. The worm wheel should be backed with a thrust bearing and supported on heavy duty bearings of either ball or roller construction. In view of the fact that a certain amount of shaft bending and frame distortion is possible, self-aligning bearings should be employed for rotation.

If any possibility exists for the worm to have a temporary reverse rotation, however small, a further thrust bearing should be fitted to the bevel wheel end of the worm shaft.

The light transmission from the bevel wheel pair demands self-aligning bearings for the rotating shaft, and a suitable thrust bearing in the footstep, to receive the weight of the rotating parts. These bearings may be of a light duty type and should be selected according to the size and weight of the transmission required.

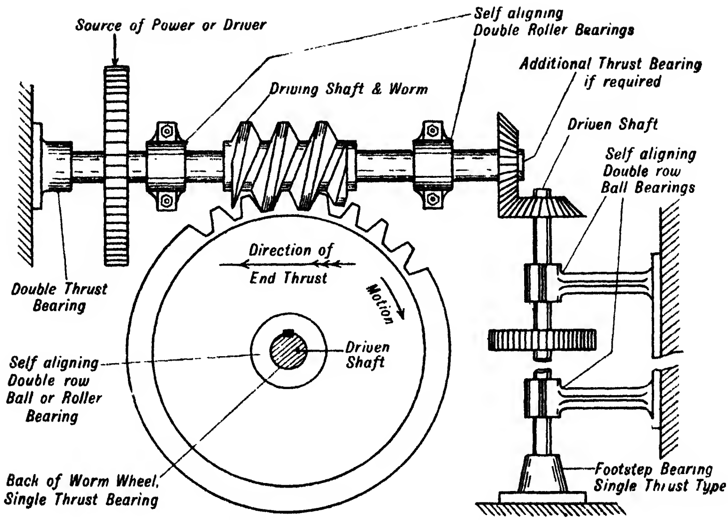


FIG. 152. Diagram to show the location of the various types of ball and roller bearings in a transmission set.

Double thrust bearing.—To receive the end thrust of the worm on the heavy duty side.

Double roller bearings.—To provide the rotation bearings on the heavy duty side.

Double row ball bearings.—To provide the rotation bearings on the light duty side, where a wide bearing is required.

Single thrust bearing.—To receive axial thrust in the footstep of the light duty transmission, and act as a back bearing for the worm wheel.

Self-aligning bearings are used where the shaft might have to withstand bending, or where machine frame distortion is possible.

NOTE.—Taper roller bearings can be employed where a shaft is subjected to a combination of forces producing end thrust as well as rotation.

EXPT. 13. OBJECTS. To find, for continuously oiled shaft bearings :

(*a*) the coefficient of friction,
(*b*) the work done against friction per revolution,
(*c*) the horse power used to overcome friction, at a speed of 400 revolutions per minute.

APPARATUS. This experiment may be carried out upon a length of shafting in a workshop; a countershaft, or the mandril of a lathe or a similar machine part. In the particular experiment described, a shaft 10 ft. in length, supported in two continuously oiled bearings, is employed. A light carrier, shown in Fig. 153, is made, and carefully balanced about the position of the shaft centre by means of attached counterpoise weights.

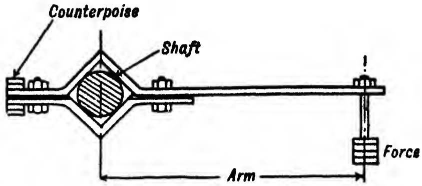


FIG. 153.

METHOD OF PROCEDURE. Adjust the counterpoise to produce a balance of the carrier and hanger about the position of the shaft bearings. Attach the carrier clip to the shaft and run the length of shaft, if possible, with power, to produce normal conditions of lubrication in the bearings. Disconnect the length of shaft so that the load on the bearings is simply its own weight and add weights to the hanger until the shaft slowly revolves against the bearing friction. The force thus obtained is the force in the friction moment at the bearings. By taking moments about the shaft centre, the effect of friction may be obtained, and the derived results calculated in the manner shown.

If pulleys are attached to the shaft employed, their weight must be taken into consideration in the bearing pressure and all belts must be removed before the experiment can be performed.

OBSERVATIONS.

Diameter of shaft	$=2$ in.
Distance between bearings	$=10$ ft.
Weight of shaft	$=\pi \times 1^2 \times 10 \times 12 \times 0{\cdot}29$ lb.
	$=109$ lb.
Length of arm of apparatus	$=12$ in.
F = force at end of the arm	$=0{\cdot}52$ lb.

DERIVED RESULTS. Let P lb. be the effect of friction.

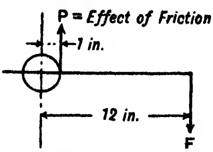


FIG. 154.

Then taking moments about the shaft centre (Fig. 154).

$$P \times 1 = F \times 12.$$

$$P = 0{\cdot}52 \times 12$$

$$P = 6{\cdot}24.$$

Coefficient of friction $= \dfrac{\text{effect of friction}}{\text{pressure between surfaces}}$

$= \dfrac{6{\cdot}24}{109} = \mathbf{0{\cdot}0572.}$

Work lost per revolution = effect of friction × shaft circumference

$= 6{\cdot}24 \times \pi \times 2 \div 12$ ft. lb. $=$ **3·26 ft. lb.**

H.P. lost at 400 r.p.m. $= \dfrac{3{\cdot}26 \times 400}{33000} =$ **0·0394 H.P.**

This calculated loss of H.P. is on a consideration of static friction and will vary from this figure under running conditions.

Friction on an inclined plane.

EXPT. 14. OBJECT. To find the angle of friction for various materials in contact, and its relation to the coefficient of friction between these surfaces.

APPARATUS. The plane employed in the earlier experiments can be used, providing it is hinged at one end and arrangements are provided for varying the slope (Fig. 155).

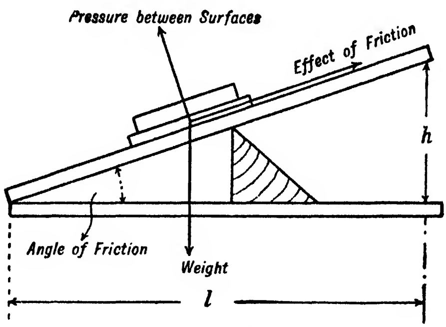


FIG. 155.

METHOD OF PROCEDURE. Place each of the sleighs provided, in turn, upon the plane. Tilt the plane until the sleigh is just induced to slide. Measure the height h and the corresponding length l and calculate the ratio h/l. Compare this ratio with the coefficient of friction obtained in Expt. 9 for this pair of surfaces. Measure the angle of inclination of the plane, in each case; this angle is known as the **angle of friction.**

Conclusion. This experiment will show that the tangent of the angle of friction, h/l, is equal to the coefficient of friction between the surfaces.

Example. *An automatic gauging machine has a feed shute supplying small parts to the machine for gauging. Find the least angle of this shute if the coefficient of friction between the parts and the shute is* 0·12.

$$\mu = 0{\cdot}12 = \text{tangent of the angle of friction.}$$
$$0{\cdot}12 = \tan\theta, \quad \theta = \tan^{-1}0{\cdot}12 = 6^\circ\ 51'.$$

Forces acting upon a body supported upon an inclined plane with friction. A body on an inclined plane is acted upon by the following forces :

(*a*) its weight,
(*b*) the pressure between the surfaces or the reaction of the plane,
(*c*) the frictional resistance which opposes motion.

In order to support this body under the action of these forces, the **equilibrant** of the forces must be applied, and this is referred to as the **effort** (Fig. 156 (*a*)). If no effort is required the body is in equilibrium under the action of forces (*a*), (*b*) and (*c*) and, when the limiting position is reached just prior to motion taking place, the **plane is inclined at the angle of friction,** or **angle of repose.**

Treatment. These four forces may be represented in magnitude and direction by the sides of a closed polygon (polygon of forces), and in this manner the magnitude of any unknown forces may be obtained. It is often better to combine the two forces, frictional resistance and pressure between surfaces, and find their resultant, afterwards considering the body to be under the action of three forces, that is, *this resultant, the effort, and the weight.* The solution then comes within the scope of the triangle of forces, or three forces acting upon a body and maintaining equilibrium.

This resultant is then regarded as the reaction of the plane, that is, the force the plane exerts on the body.

It is important, at this stage, to realise the conclusion drawn from Expt. 10, namely, when motion is just about to take place,

force of friction = coefficient of friction × pressure between surfaces.

Example 1. *A roller of weight* 40 *lb. is supported on a rough inclined plane by a rope parallel to the plane. Find the effort required to* support *the roller if the inclination of the plane is* 30° *to the horizontal and the coefficient of friction between roller and plane is* 0·3.

SOLUTION (*a*). **Graphical method.** The frictional resistance will oppose motion and be equal to μR where R is the normal reaction of the plane (Fig. 156). To simplify the problem the forces are taken as being concurrent; the frictional resistance really acts where contact takes place with the plane. Draw the parallelogram of forces for R and μR and thus find the direction of R_1, their resultant. (NOTE.—$\mu R = 0{\cdot}3R$.) Then draw the triangle of forces for R_1, E and W (which is a force of 40 lb. weight). Measure E to the scale employed for W, and this will be the required effort.

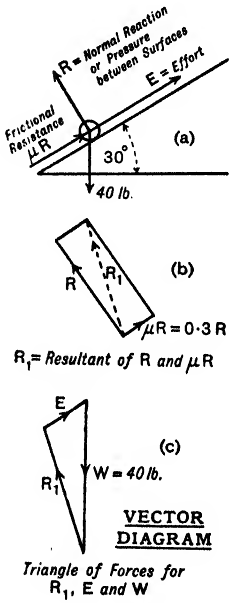


FIG. 156.

SOLUTION (*b*). **Method of resolution of forces.** Resolve all the forces along two directions, (*a*) parallel to the plane, (*b*) at right angles to the plane.

Parallel to the plane.

$$E + \mu R = W \sin 30°.$$
$$E + \mu R = 40 \times 0{\cdot}5 = 20.$$

At right angles to the plane.

$$R = W \cos 30° = 40 \times {\cdot}866.$$
$$\mathbf{R = 34{\cdot}64.}$$

Substituting $R = 34{\cdot}64$ in $E + \mu R = 20$,

$$E + 0{\cdot}3 \times 34{\cdot}64 = 20.$$
$$E = 20 - 10{\cdot}392 = 9{\cdot}61. \quad \textbf{Ans.} \quad \mathbf{9{\cdot}61\ lb.}$$

SOLUTION (*c*). **Alternative graphical method.** Since the reaction of a rough plane makes an angle equal to the angle of friction with the normal to the plane, the reaction can be drawn on the space diagram and then the triangle of forces can be drawn and the problem solved.

Coefficient of friction $= 0{\cdot}3$.

Angle of friction $=$ the angle whose tangent is 0·3.

$= 16° \ 42'$.

Now draw the reaction of the plane R_1 at an angle of $16° 42'$ to the normal to the plane, *so as to provide a component resisting motion* down the plane. Then draw the triangle of forces for R_1, E and W and measure E to the scale adopted for W, as in Fig. 156 (*c*).

NOTE.—If the roller were being *pulled* up the plane, R_1 would be drawn on the other side of the normal, making the same angle with it as before.

Example 2. *A machine weighing* 5000 *lb. is to be lowered into a sunk foundation by sliding it down rolled steel joists as shown. Find the effort required at E to prevent the machine taking charge, if the coefficient of friction is* 0·2.

Resolve all forces at right angles to and parallel to the plane.

At right angles to the plane (Fig. 157).

$$W \cos 35° - R = E \sin 15°.$$
$$5000 \times 0{\cdot}8192 = R + 0{\cdot}2588E.$$
$$(a) \qquad 4096 = R + 0{\cdot}2588E.$$

Parallel to the plane.

$$W \sin 35° = \mu R + E \cos 15°.$$
$$5000 \times 0{\cdot}5736 = 0{\cdot}2R + 0{\cdot}9659E.$$
$$(b) \qquad 2868 = 0{\cdot}2R + 0{\cdot}9659E.$$

Multiply (*b*) by 5 $\quad 14340 = R + 4{\cdot}8295E.$

Subtracting (*a*) $\quad 4096 = R + 0{\cdot}2588E.$

5 (*b*) - (*a*) $\quad 10244 = 4{\cdot}5707E.$

$$E = \frac{10244}{4{\cdot}5707} = 2241.$$

Ans. Effort = 2241 lb.

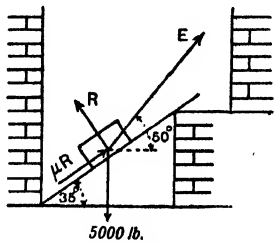


FIG. 157.

Friction braking. In many machines, particularly automobiles, it becomes necessary to reduce the speed, or stop, at will. The problem of braking demands the fullest possible use of the frictional resistance between two surfaces, and the selection of contact materials and conditions of surfaces which produce the greatest effect of friction. Brakes may be used with metal to metal contact, in which case the frictional resistance is comparatively low ; or the brake may be lined with a material which has a high coefficient of friction. Such materials as wood, leather, compressed canvas and fibre are used for this purpose, but probably modern practice leads

towards the extensive use of such a material as "Ferodo", a product of Messrs. Ferodo Ltd.

This material, in one form, consists of long strands of asbestos yarn, spun on to fine brass wire and woven to form a material which has the appearance and character of belting, but is more efficient as a brake lining. The fabric, so formed, is treated by trade processes to produce an excellently bonded material yielding an exceptionally high coefficient of friction, which under favourable conditions is as high as 0·7.

EXERCISES ON CHAPTER VI

1. What is meant by coefficient of friction? It requires a force of 23 lb. to cause a lathe saddle to slide over its bed. If the coefficient of friction is 0·12, find the probable weight of the saddle.

2. A machine is to be made to slide along level ground to its foundation. Draw a diagram to show the forces acting upon the machine during the process of sliding.

3. State the laws of static friction for dry surfaces and describe the experimental verification of one of them.

4. What effect does lubrication have upon the friction of dry surfaces? Outline the principles of efficient lubrication, and explain the advantages of pressure lubrication for bearings.

5. Sketch ball or roller bearings suitable for (*a*) shaft bearing, (*b*) thrust bearing, (*c*) heavy duty shaft bearing.

6. It is found that, during the work of unloading heavy cases from trucks, the cases just slide down a skidway from the truck when the incline of the skidway is 1 in 5. Find the coefficient of friction and the value of the angle of friction.

7. Work, in process of manufacture, has to pass from one operation to another by sliding down a shute. If the coefficient of friction between the work and the shute is 0·25, find the least angle of the shute to the horizontal.

8. A simple lever 6 ft. in length is pivoted at a point 1 ft. from one end. An effort of 4·5 lb. is just sufficient to raise a load of 20 lb. a vertical distance of 2 in. Find the effect of friction and efficiency of the lever.

9. An experiment with a simple pulley gave the following results: load 16 lb., effort 17·1 lb. Find the effect of friction and efficiency of the pulley at this load.

10. The efficiency of a simple pulley hoist is 88 per cent., find the load which could be raised by the hoist with an effort of 27 lb.

11. Find the least force required to start an electric iron, weighing 5 lb., sliding over a surface where the coefficient of friction is 0·3. To what angle may the surface be tilted before the iron commences to slide of its own accord ?

12. In a mechanism, a return spring causes a detail weighing 2·3 lb. to move up an incline of 1 in 40. Find the force which the spring must exert if the coefficient of friction is 0·28 and the spring acts in the direction of the slope.

13. A line of shafting $2\frac{1}{2}$ in. diameter, with its pulleys, weighs 2700 lb. and revolves at 350 revs. per min. If the bearing coefficient of friction is 0·05, find the horse power absorbed in overcoming friction.

14. The truck of a rack conveyer weighs, when loaded, 7400 lb. The combined coefficient of friction for slides and rack is 0·13 ; find (*a*) the force necessary to drive the truck, (*b*) the horse power absorbed in overcoming friction at a speed of 4 feet per second.

15. The total weight of a travelling crane, when loaded, is 19·3 tons. Find the force required to move it along its track, and the horse power absorbed in overcoming friction, if the speed is 70 feet per minute and the coefficient of friction 0·2.

16. A truck weighing 2390 lb. is drawn up a track by a cable parallel to the track. Find the tension in the cable if the coefficient of friction is 0·018 and the incline 1 in 7.

17. A turbine rotor weighing 3 tons revolves in bearings 4 in. in diameter. Determine the horse power lost in friction if the speed is 1440 revs. per min. and the coefficient of friction 0·02.

18. A masonry retaining wall is a trapezium in section, with one face vertical 20 ft. high. It is 4 ft. wide at the top and 8 ft. at the base. The density of the material is 130 lb. per cubic ft. Find the horizontal force per foot length of wall necessary to cause the wall to slide, if the coefficient of friction between the wall and the soil is 0·7.

19. The forces acting upon a 3 in. diameter journal bearing due to the weight of the shaft and the pulls on the pulleys due to the belts are 250 lb., 350 lb., 300 lb. and 200 lb., and the angles between these forces are respectively 45°, 30° and 90°. Find the resultant force on the bearing and the horse power lost in friction at a speed of 120 revs. per min. Coefficient of friction $= 0{\cdot}05$.

20. A revolving crane weighs, with its load and counterpoise, $6\frac{1}{4}$ tons. It revolves on a track 6 ft. 6 in. in diameter. What force is required to rotate the crane if $\mu = 0{\cdot}09$? Calculate the horse power required for this purpose if the track speed of rotation is 1·5 ft. per second.

21. The area of the working face of a D slide valve is 140 sq. in. and the steam pressure on the back of the valve is 100 lb. per sq. in. The stroke is $4\frac{1}{2}$ in. Calculate the H.P. lost in friction at 90 r.p.m. if $\mu = 0{\cdot}09$.

22. In a vertical steam engine the average pressure between the cross head and guides is 3000 lb. If the crank is 18 in. long and the coefficient of friction 0·07, find the H.P. lost in friction at 120 r.p.m.

23. A 4-pole enclosed motor has eight carbon brushes, the cross section of each being 1 in. × $\frac{3}{4}$ in. The diameter of the commutator is 9 in. and the speed 1000 r.p.m. The pressure of the brushes on the commutator is 2 lb. per sq. in. Calculate the horse power lost in friction, the coefficient of friction being 0·3. (U.L.C.I.)

24. The following observations were recorded during an experiment which was made to obtain the coefficient of friction of a journal bearing. Dimensions of journal bearing: 4 in. diameter, 6 in. long. Total load on bearing, 6500 lb. Revolutions per minute, 140. The moment of frictional resistance was equivalent to a weight of 4·9 lb. acting at a leverage of 24 in. from the centre of the shaft. Determine: (*a*) the load per square inch of projected area of the bearing; (*b*) the coefficient of friction of the bearings; (*c*) the horse power absorbed by friction. (U.L.C.I.)

25. What is the "coefficient of friction"?

A variously loaded slider was moved slowly but uniformly over a horizontal surface and the different total weights (W) of the cradle and the corresponding horizontal forces (F) necessary to move it were as follows:

W lb.	10	15	20	25	30	35
F lb.	1·9	3·1	4·1	4·9	6	7·1

Plot a graph connecting F and W and determine the coefficient of friction for the surfaces. What force would be necessary to move the cradle when loaded 27 lb.? (U.L.C.I.)

26. What do you understand by "coefficient of friction"?

A body weighing 120 lb. rests on a horizontal surface. If the coefficient of friction is 0·2, find the least horizontal force which would move the body.

How much work would be done in moving the body over the surface through a distance of 1 ft.? (U.L.C.I.)

27. Friction may be useful or objectionable in a machine. Mention one case of each.

Why does lubrication reduce friction?

A block of iron weighing 56 lb. lies on a horizontal table. A flexible string is tied to the block and after passing horizontally to the edge of the table, passes over a frictionless pulley, and has its other end fastened to a scale pan, which weighs $\frac{1}{2}$ lb. If the coefficient of limiting friction between the iron and table surface is 0·25, what weight must be placed in the pan to cause the iron just to move? (U.L.C.I.)

28. What force would be necessary just to maintain motion in a body weighing 18 cwt., (*a*) along a horizontal plane, (*b*) up an incline of 1 in 10? The coefficient of friction in each case is 0·08. (U.L.C.I.)

CHAPTER VII

INTRODUCTION TO MACHINES AND EFFICIENCY TESTS ON MACHINES

A machine may be said to be a contrivance by means of which a load may be raised, or a resistance overcome, to the advantage of the operator. It may overcome a large resistance with a small effort, or apply a force to a body at a position remote from the point of application of the effort, or overcome a resistance more conveniently than would be possible by direct application of the effort.

Examples of machines. Levers, screws, wedges, pulleys and pulley tackles, cranes, gearing and machine tools.

Principle of transmissibility of force. A force applied to one portion of a machine, or structure, at any point in its line of action, which is rigidly connected to the machine, is transmitted through the material of each rigidly connected member or unit, and can thus be made to act upon the resistance to be overcome.

General treatment of a machine. **The load, or resistance,** is the force which has to be usefully overcome by the machine.

The **effort** is the force used to operate the machine and overcome the load, or resistance, and the frictional losses in the machine.

The **velocity ratio** is the ratio

$$\frac{\text{distance moved by the effort}}{\text{distance moved by the load}} \text{ in the same time,}$$

or

$$\frac{\text{speed of effort}}{\text{speed of load}}.$$

These three observations provide the basis for what are known as the derived results.

Derived results. The general object of any machine test is to find

(*a*) the advantage gained by the use of the machine,

(*b*) its efficiency at various loads;

and this **efficiency** is always measured by the ratio

$$\frac{\text{work output}}{\text{work input}} \text{ or } \frac{\textbf{useful work done by the machine}}{\textbf{actual work put into the machine}}.$$

This is a ratio which, owing to the existence of friction, must always be **less than unity or less than 100%**

The **mechanical advantage** is the name given to the ratio

$$\frac{\textbf{useful load or resistance overcome}}{\textbf{effort employed}}$$

The **work output** is the work performed upon the load, or resistance

$$= \textbf{load} \times \textbf{distance moved by load.}$$

The **work input** is the work done by the effort during the process of raising the load, or overcoming the resistance

$$= \textbf{effort} \times \textbf{distance moved by the effort.}$$

The **work effect of friction** is the difference between the work input and the work output and is the work absorbed in overcoming friction.

NOTE.—*If the work output is considered as based on a load movement of* 1 *foot, the effort will move a distance equal to* 1 *foot* × *the velocity ratio, and the effect of friction becomes equivalent to an additional load or resistance to be overcome by the effort* or a **load effect of friction.**

Alternative method of obtaining efficiency. The efficiency is sometimes obtained by taking the **ratio of the actual resistance overcome to that of the ideal resistance, or load,** where the ideal load is the product of the effort and the velocity ratio. This, although yielding the same result, does not give a true impression of efficiency, which is essentially a ratio of the work output against the corresponding work input.

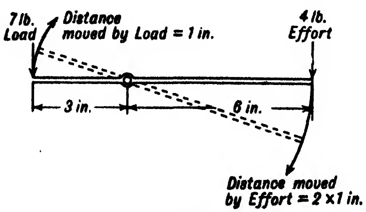


FIG. 158.

A simple lever. A lever is one of the simplest of machines and may be taken as a concrete example of this treatment.

Suppose, in the case of the particular lever shown (Fig. 158) that an effort of 4 lb. raises a load of 7 lb.

If the load moves 1 in. the effort will move 2×1 or 2 in. because the radius of the effort circle is twice that of the load circle.

$$\textbf{Velocity ratio} = \frac{\text{distance moved by the effort}}{\text{distance moved by the load}} = \frac{2}{1} = 2.$$

$$\textbf{Mechanical advantage} = \frac{\text{load}}{\text{effort}} = \frac{7}{4} = 1{\cdot}75.$$

Work input = work done by the effort $= 2 \times 4$ or 8 in. lb.

Work output = work done on the load $= 1 \times 7$ or 7 in. lb.

Effect of friction = (work input − work output) $\div$ 1 ft. $= (8 - 7)$ lb., is *equivalent to an additional load of* 1 lb. Thus, the useful load lifted is reduced by 1 lb. due to the friction at the fulcrum.

$$\textbf{Efficiency} = \frac{\text{work output}}{\text{work input}} = \frac{7}{8} \text{ or } 87{\cdot}5\%.$$

By the alternative method,

$$\textbf{Ideal load} = \text{effort} \times \text{velocity ratio} = 4 \times 2 \text{ or } 8 \text{ lb.}$$

$$\textbf{Efficiency} = \frac{\text{actual load}}{\text{ideal load}} = \frac{7}{8} \text{ or } 87{\cdot}5\%.$$

The efficiency may also be expressed by the ratio

$$\frac{\text{theoretical effort}}{\text{actual effort}} \times 100.$$

Relation between mechanical advantage, velocity ratio and efficiency.

$$\text{Mechanical advantage (M.A.)} = \frac{\text{load}}{\text{effort}}.$$

$$\text{Velocity ratio (V.R.)} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}.$$

$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}} = \frac{\text{load} \times \text{distance moved by load}}{\text{effort} \times \text{distance moved by effort}},$$

which is $$\text{M.A.} \div \text{V.R.}$$

or $$\textbf{efficiency} = \frac{\textbf{M.A.}}{\textbf{V.R.}}.$$

From this it can be seen that, in the absence of frictional losses, the mechanical advantage is equal to the velocity ratio and the efficiency is 100 *per cent. This is the ideal machine, a condition which has never been reached in practice.*

Experimental tests of machines. An experimental test of a machine is conducted along lines which do not materially differ with the machines under test.

METHOD OF PROCEDURE. (*a*) Measure the distances moved by the effort and by the load in the same time, and calculate the velocity ratio.

(*b*) Check this calculated velocity ratio by measurement of pulley diameters, numbers of teeth, or pitch of screws as the case may be, and compare, if possible, the observed velocity ratio with that obtained by a theoretical consideration of the machine.

(*c*) Select a range of loads the maximum of which is reasonably near to that for which the machine was designed and find the efforts required to raise each of these loads without gain of speed.

NOTE.—*No useful purpose is served by testing a machine of lifting capacity say* 500 *lb. with a load of* 10 *lb. In such a test the frictional losses would be out of all reasonable proportions when compared with the load, and the efficiency obtained would be very low. Loads for such a machine might be* 100 *lb.*, 150 *lb.*, 200 *lb.*, 250 *lb. up to* 500 *lb.*

(*d*) Prepare a suitable table in which both observations and derived results may be entered; calculate the derived results and complete the entries in the table.

Summary of the observations and results for a machine test.

OBSERVATIONS. (*a*) the velocity ratio,

(*b*) the effort required at different loadings.

DERIVED RESULTS. (*a*) the mechanical advantage,

(*b*) the work input (on the basis of 1 foot load movement),

(*c*) the work output (on the basis of 1 foot load movement),

(*d*) the load effect of friction,

(*e*) the efficiency,

all calculated for each of a range of loadings.

The graphs. A graph, or series of graphs, of the results of an experiment often form a very valuable leader towards the conclusions to be drawn. The quantities to be graphed should, for this reason, be carefully selected in order that they may give a visual impression of the performance of the machine. *The full objects of a machine test are,* (*a*) to find the effort required to raise certain loads, (*b*) to find the effect of friction at each of these loads, (*c*) to find the

efficiency at each of the loads, (*d*) to study the variation of effort, effect of friction and efficiency with increase of load. With these objects in view the best graphs appear to be :

(*a*) **Load against effort,**
(*b*) **Load against effect of friction,**
(*c*) **Load against efficiency,**

and the multiple axis graph shown later in this chapter lends itself readily to this representation.

The conclusions. Conclusions may be drawn from the graphs obtained, and verified by a study of the results set out in the table of results. An equation, or the **law of the machine,** can then be obtained from an algebraic treatment of the load-effort graph.

This treatment of a machine test is spoken of as the **log of the experiment** and is the accepted method of recording experimental results.

Typical machine experiments. A series of experimental tests of machines follows, which, although yielding widely different efficiencies, serves to illustrate the general method of test and lead to conclusions which are generally applicable to lifting machines.

OBJECTS OF THE EXPERIMENTS.

(*a*) *To find the effort required to raise each of a range of loads.*
(*b*) *To find the effect of friction at each load.*
(*c*) *To find the efficiency at each load.*
(*d*) *To study the variation of effort, effect of friction and efficiency with the load.*

EXPT. 15*a*. **A simple pulley tackle, 2–1 type.** Rope tackles may be arranged in many ways : the one selected for this experiment (Fig. 159) is of what is known as the 2–1 type ; that is, there are two pulleys at the top, and one at the bottom. The rope is brought from an eye in the block of the bottom pulley, around a pulley wheel of the top block, thence it is reeved through the bottom pulley and its free end passes around the second pulley, at the top, to form the effort rope. The load is attached to the bottom block.

The weight of the lower block and the ropes should on no account be included in the load, as they form part of the machine itself and they must of necessity have mass in order to play their part in the functioning of the machine. The strength, and consequently the

mass of the lower blocks and ropes, will depend largely on the maximum load for which the tackle is designed. Thus, even without any load on the machine, an effort will be necessary in order to sustain, or raise with uniform velocity, the lower movable block and ropes. Later it will be seen that this effort is indicated on the Load v. Effort graph by the intercept on the effort axis; or, in other words, the value of the effort with no load. As the load increases, so the masses of the parts mentioned become proportionately of less importance, and the efficiency increases.

Similar remarks to the above will apply to all the subsequent tests on machines.

OBSERVATIONS AND DERIVED RESULTS.

$$\text{Velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = 3.$$

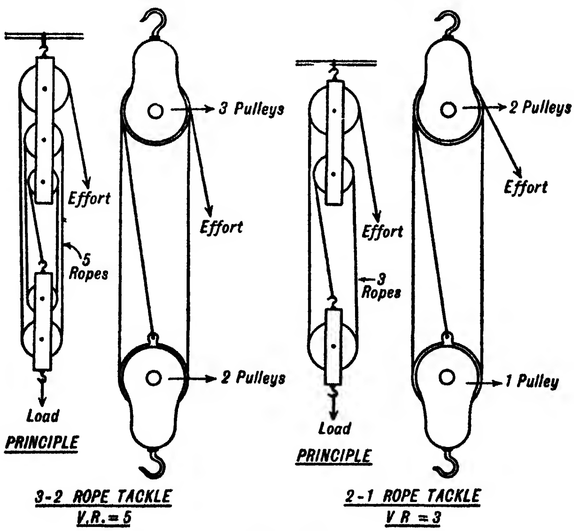


FIG. 159.

NOTE.—*In all rope tackles the velocity ratio is equal to the number of ropes supporting the load, for if the load is lifted 1 foot, each of these supporting ropes will shorten 1 foot; that is, the effort will have to move n feet where n is the number of supporting ropes. Thus, V.R. of 2–1 tackle = 3, of 3–2 tackle = 5.*

Load, L lb.	Effort, E lb.	M.A., $\frac{L}{E}$	Work output, W.O., $L \times 1$ ft. lb.	Work input, W.I., $E \times$V.R.$\times 1$ ft. ft. lb.	Effect of friction, $\frac{\text{W.I.} - \text{W.O.}}{1 \text{ ft.}}$	Efficiency, $\frac{\text{W.O.}}{\text{W.I.}} \times 100$
10	7·0	1·43	10·0	21·0	11 lb.	47·7%
20	11·1	1·80	20·0	33·3	13·3 lb.	60·0%
30	15·2	1·98	30·0	45·6	15·6 lb.	65·7%
40	19·3	2·07	40·0	57·9	17·9 lb.	69%
50	23·2	2·16	50·0	69·6	19·6 lb.	71·7%
60	27·3	2·20	60·0	81·9	21·9 lb.	73·3%

Load increases.	Effort increases with the load.	M.A. increases with the load.	Work input is always greater than the work output.	Effect of friction increases with the load.	Efficiency increases with the load.

POINTERS TO CONCLUSIONS

NOTE.—The effect of friction is equivalent to an increase of load, since the work output is calculated on a movement of the load equal to one foot.

POINTERS TO CONCLUSIONS. The student should carefully study these indications with a view to framing the conclusions to the experiment. When the graphs are drawn they should be compared with the pointers and if, for example, the graph of load v. effort is a straight line, not passing through the origin, the following conclusion may be drawn:

The pointer shows an increase of effort with load and the graph gives a straight line, therefore the increase of effort is proportional to the increase of load. See p. 138.

GRAPHS.

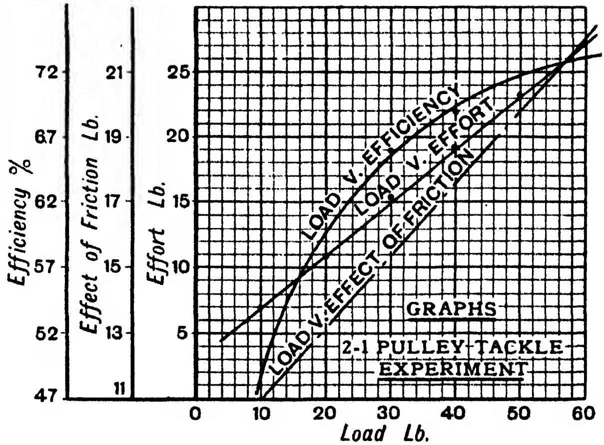


FIG. 160.

EXPT. 15*b*. **A pulley tackle of the 3–2 type.**

OBSERVATIONS AND DERIVED RESULTS.

Velocity ratio = 5 = No. of supporting ropes.

Load, lb.	Effort, lb.	M.A.	Work output, ft. lb.	Work input, ft. lb.	Effect of friction, lb.	Efficiency, %
20	8·2	2·44	20·0	41·0	21·0	48·8
40	13·8	2·90	40·0	69·0	29·0	58·0
70	22·5	3·11	70·0	112·5	42·5	62·2
90	27·75	3·24	90·0	138·75	48·75	64·8
110	33·25	3·31	110·0	166·25	56·25	66·2
160	47	3·40	160·0	235·0	75·0	68·0

In a comparison of the derived results of Experiments 15*a* and 15*b*, it is noticeable that the efficiency of the 3–2 tackle at the same load is lower. This is due to the additional friction when the number of pulleys is increased.

GRAPHS.

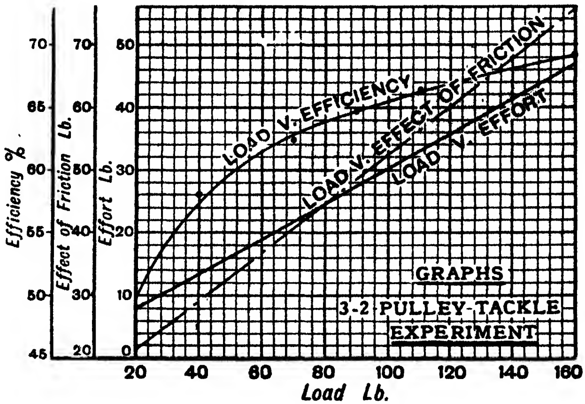


FIG. 161.

EXPT. 16. **A wheel and differential axle** (Fig. 162).

NOTE.—This apparatus is representative of that type of winch in which an effort applied to a large wheel winds the load rope on to a smaller wheel having a common axle with the larger. When the

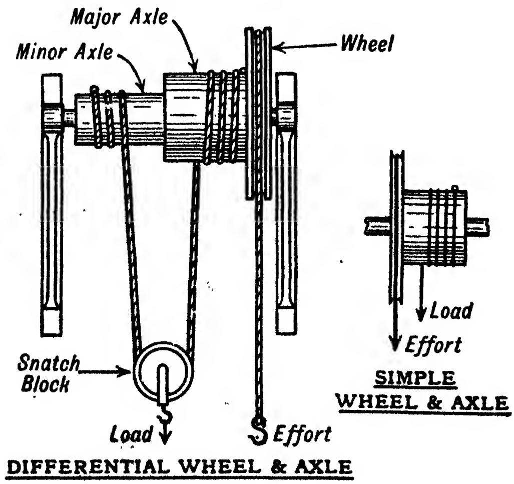


FIG. 162.

machine is of the differential type, there are two axles of varying diameters and the load rope is wound on to the larger axle at the same time as it is wound off the smaller. The result is that for every revolution of the effort wheel the load is raised a distance equal to one-half the difference between the circumferences of the axles. This is because the loop, or bight, supporting the load pulley is shortened an amount equal to the differences in the circumferences of the two axles, and half the shortening will occur on each half of the bight.

OBSERVATIONS AND DERIVED RESULTS.

Measured velocity ratio $= \dfrac{\text{distance moved by effort}}{\text{distance moved by load}} = \dfrac{35 \text{ in.}}{5 \text{ in.}} = 7.$

Calculated velocity ratio. Diameter of effort wheel $= 10\frac{1}{8}$ in.
Diameter of major axle $= 5\frac{3}{16}$ in.
Diameter of minor axle $= 2\frac{3}{16}$ in.

NOTE.—These diameters are to the centre of the rope.

In one revolution, distance moved by effort $= 10\frac{1}{8}\pi$ in.
distance moved by load $= \frac{1}{2}\pi(5\frac{3}{16} - 2\frac{3}{16})$ in.
$= 1\frac{1}{2}\pi$ in.

$$\text{Velocity ratio} = \frac{10\frac{1}{8}\pi}{1\frac{1}{2}\pi} = 6{\cdot}75.$$

This provides a check on the measured V.R., which is the value taken in the following derived results.

Load, lb.	Effort, lb.	M.A.	Work output, ft. lb.	Work input, ft. lb.	Effect of friction, lb.	Efficiency, %
6	1·14	5·26	6·0	7·98	1·98	75·1
10	1·81	5·53	10·0	12·67	2·67	79·0
14	2·44	5·74	14·0	17·08	3·08	82·0
18	3·08	5·85	18·0	21·56	3·56	83·6
22	3·70	5·95	22·0	25·90	3·90	85·0
26	4·33	6·0	26·0	30·31	4·31	85·7

GRAPHS.

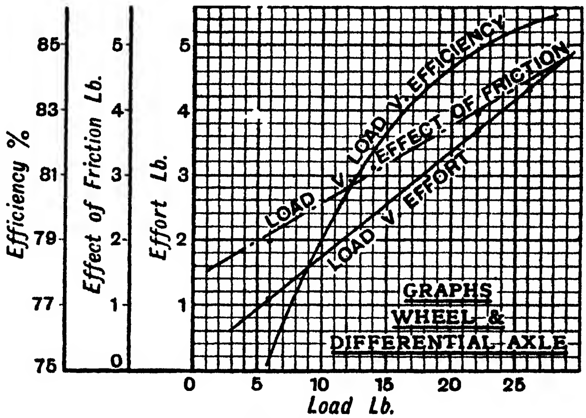


FIG. 163.

NOTE.—*It will be noticed that the load* v. *efficiency graph, in this experiment, does not flatten out at the higher loads as in previous experiments. This is an indication that the machine has not been loaded to its full capacity and detracts from the value of the test.*

EXPT. 17. **Worm and worm-wheel** (Fig. 164). This machine is representative of not only screw or worm-driven lifting machines, but of traversing gears for machine tools, steering gears, mechanisms controlling brakes, and many other mechanisms where the worm and worm-wheel are employed. It must be remembered that its efficiency as a lifting device is equally true when it is used in one of its other capacities. Modern worm gear reduction units can be operated at worm speeds as high as 4000 r.p.m., making them suitable for connecting direct to steam turbines. Double reduction worm gear units can be used for reduction ratios up to 10,000 to 1.

METHOD OF PROCEDURE. The effort is applied by hanging weights on the small hook (Fig. 164) while the load is raised by the large hook. The thrust along the worm axis is taken up by a thrust block.

OBSERVATIONS AND DERIVED RESULTS.

$$\textit{Measured}\ \text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{67{\cdot}5\ \text{in.}}{1{\cdot}5\ \text{in.}} = 45.$$

Calculated velocity ratio.

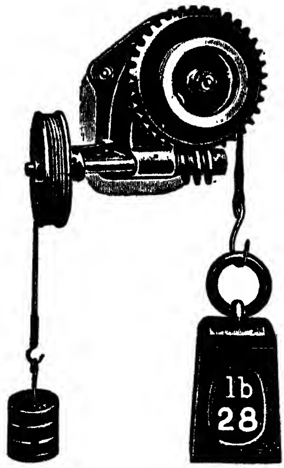


FIG. 164. Experimental worm and worm-wheel.
(*Messrs. G. Cussons, Ltd., Manchester.*)

Diameter of effort wheel = 5 in.
Diameter of load drum = 4 in.
No. of teeth on worm-wheel = 36.

Since the worm is single start (see p. 145), one revolution of the effort wheel will cause the worm-wheel, to which is attached the load drum, to move 1 tooth, that is, $\frac{1}{36}$ of a revolution.

So that, in one revolution of the effort wheel,

distance moved by effort
$= \pi \times 5$ in.

distance moved by load
$= \pi \times 4 \times \frac{1}{36}$ in.

$$\text{Velocity ratio} = \frac{5\pi}{\frac{4}{36}\pi} = \frac{36 \times 5}{4} = 45.$$

Load, lb.	Effort, lb.	M.A.	Work output, ft. lb.	Work input, ft. lb.	Effect of friction, lb.	Efficiency, %
10	0·85	11·76	10·0	38·25	28·25	26·14
20	1·60	12·50	20·0	72·0	52·00	27·78
30	2·30	13·05	30·0	103·5	73·50	29·0
40	2·95	13·56	40·0	132·8	92·80	30·14
50	3·65	13·70	50·0	164·3	114·3	30·44
60	4·35	13·79	60·0	195·8	135·8	30·64

NOTE.—The efficiency is comparatively very low, and this is characteristic of all screw and worm apparatus. This is due to the fact that the effect of friction is very high; in every case it is considerably in excess of the load and there is an end thrust due to the slope of the thread.

GRAPHS.

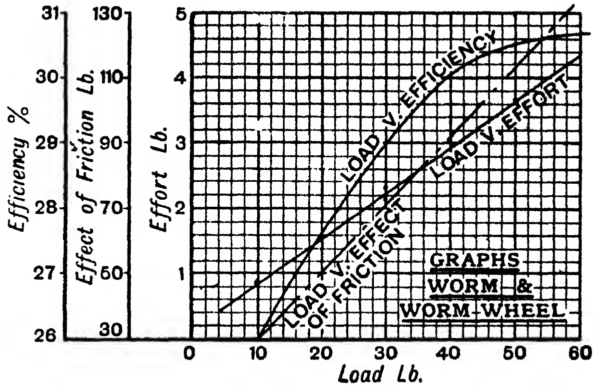


FIG. 165

Example. *The motor of an electric wire hoist drives a single start worm directly through a flexible coupling. Geared to the worm is a worm-wheel of 45 teeth, and this wheel rotates the wire drum used for hoisting the load. If the diameter of the drum to the centre of the wire is* 10 *in., calculate :*

(1) *the speed at which the load is being raised, if the motor speed is* 1000 *r.p.m.*

(2) *the necessary H.P. of the motor at this speed, if the overall efficiency of the hoist is* 30% *and the load being raised is* 2 *cwt.*

(1) Velocity of load = Revs. of drum × Dia. of drum × π

$$= \frac{1000}{45} \times \frac{10}{12} \times \pi \qquad = 58{\cdot}19 \text{ ft. per min.}$$

(2) Work output $= 58{\cdot}19 \times 224$ ft. lb. $= 13030$ ft. lb.

Work input $= 13030 \times \dfrac{100}{30}$ ft. lb.

$$\therefore \text{ Horse Power required} = \frac{13030 \times 100}{33000 \times 30} \qquad = 1{\cdot}32.$$

EXPT. 18. **The screw jack.** This machine is representative, not only of lifting machines, but also of the many mechanisms where screw and nut operation is employed. The efficiency obtained by experiment is not only the efficiency of a lifting device, but can apply to a similar screw and nut mechanism wherever it is employed.

Method of Procedure. The turning moment on the screw is applied as a pure couple, by dividing the effort to pull equally on each side of the effort drum. A large hook (Fig. 166) is provided to carry the load which is supported by a nut that can be raised or lowered by rotating the screw.

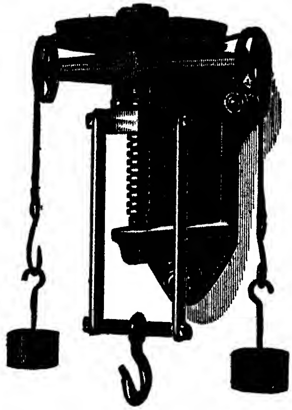


Fig. 166. Experimental screw jack.
(*Messrs. G. Cussons, Ltd., Manchester.*)

Observations and derived results.

Measured velocity ratio

$$=\frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$=\frac{100\text{ in.}}{2\text{ in.}}=50.$$

The effort, in this experiment, must be provided with a long drop, otherwise the test is carried out on a very small portion of the screw.

Calculated velocity ratio :

Diameter of load drum + rope = 8·10 in.

Pitch of screw = 0·5 in.

Since the screw is single started, 1 revolution of the load drum causes the load to lift 0·5 in. and the effort to move $\pi \times 8{\cdot}10$ in.,

$$\text{therefore the velocity ratio}=\frac{\pi\times 8{\cdot}10}{0{\cdot}5}=50{\cdot}87.$$

Load, lb.	Effort, lb.	M.A.	Work output, ft. lb.	Work input, ft. lb.	Effect of friction, lb.	Efficiency, %
56	10	5·6	56·0	500	444	11·2
112	13·8	8·12	112·0	690	578	16·24
168	17·7	9·49	168·0	885	717	18·98
224	22·0	10·18	224·0	1100	876	20·36
280	26·0	10·77	280·0	1300	1020	21·54
336	30·1	11·16	336·0	1505	1169	22·32

GRAPHS.

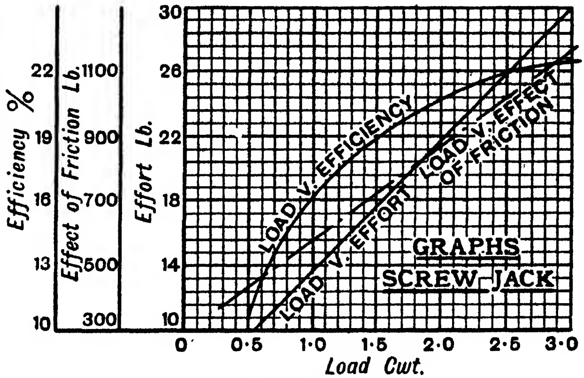


FIG. 167.

NOTE.—The screw jack employed for this experiment is adapted from the screw and nut of an elevating mechanism. It can be seen that the efficiency is comparatively low, a characteristic of screw mechanisms.

The graphical interpretation of experimental results. It is very seldom found that a series of experimental results, when plotted, fall perfectly on a straight line, or curve. The general placing of the points indicate whether a straight line or curve may be expected, and it is customary to draw what is known as **a fair straight line** or **curve** through the points if the indication is in favour of a straight line or a curve.

General conclusions from machine experiments. The conclusions to be drawn from a series of machine experiments are, in their general character, the same, although the efficiencies may vary considerably.

The conclusions are drawn from the combined evidence of the tabulated results and the graphs. After the table of results for Experiment No. 15*a* (p. 127) is a series of pointers which serve to indicate the conclusions which may be deduced from the tabulated results.

Interpretation of the graphs. Mathematically, if a graph of two variable quantities is drawn, and this graph is a straight

line, the following properties of the variable quantities are known :

(*a*) **Straight line through the origin.** The quantities are directly proportional to each other.

(*b*) **Straight line but *not* through the origin.** The *increase* of one quantity is directly proportional to the *increase* of the other quantity.

The graphs of load v. *effort and load* v. *effect of friction are straight lines which do not pass through the origin.*

A curve, connecting the two quantities, may indicate a mathematical relationship, but not a direct proportion between them.

The graphs of load v. *efficiency are curves, and thus the efficiency is not directly proportional to the load itself.*

Conclusions. (1) The effort increases with the load, and the increase of effort is proportional to the increase of load.

Reason.—Straight line graph but not through the origin.

(2) The work put into the machine, that is, the work input, is always greater than the work done in raising the load, that is, the work output.

(3) The effect of friction increases with the load and the increase of effect of friction is proportional to the increase of load.

Reason.—Straight line graph but not through the origin.

(4) The efficiency increases with the load, but the increases are not in proportion.

Reason.—The load v. *efficiency graph is a curve.*

The efficiency is a proper fraction or less than 100 per cent.

Reason.—The work output is always less than the work input.

NOTE.—The efficiency curve tends to flatten, and assume a maximum value as the load increases. It may be suggested that this maximum value is at the best load for the machine, and any increase of load, beyond this, will lead to a fall in efficiency, that is, overloading.

Overhauling. A machine is said to be capable of overhauling when the load is able to raise the effort, that is, the operation of the machine can be reversed. Such a condition is only possible when

the effect of friction is less than the load or when the efficiency is over 50 per cent.

Machines likely to overhaul are provided with a preventative such as a pawl and ratchet to prevent the reversal or overhauling of the machine when the effort rope is not secured.

In the previous experiments rope tackles and the wheel and axle are likely to overhaul, whereas the worm and worm-wheel and screw jack, definitely, will not overhaul as the efficiency is less than 50 per cent.

The law of a machine. The law of a machine is the name given to the algebraic equation connecting load and effort, and which is valid at all loads within the range of the machine.

Determination of the law. The procedure of finding the law for a machine can be illustrated by determining the law for the screw jack (Expt. 18).

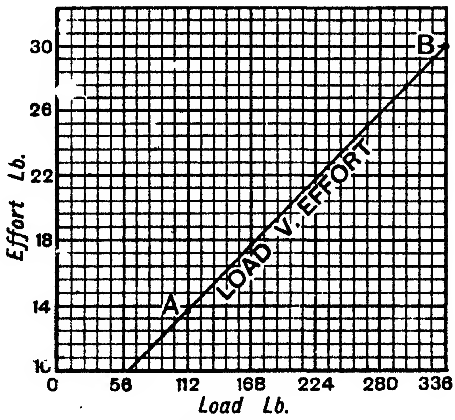


FIG. 168.

Consider the load *v.* effort graph in which the load scale has been converted for convenience to pounds (Fig. 168).

The mathematical form of the law of a straight line is

$$y = mx + c,$$

where y is the vertical reading, x the horizontal reading, m the gradient and c a constant for the graph.

In this case y is replaced by the effort E, and x by the load L. The law then becomes $\mathbf{E = mL + c}$.

Take two points A and B on the graph of load against effort and read off the values of E and L at each point (Fig. 168).

Substitute these values in $E = mL + c$.

Thus at A where $L = 112$, $E = 13{\cdot}6$, $\rightarrow 13{\cdot}6 = m \times 112 + c$.

At B where $L = 336$, $E = 30$, $\rightarrow \underline{30{\cdot}0 = m \times 336 + c.}$

Eliminating c by subtraction, $\rightarrow -16{\cdot}4 = -224m$,

or
$$m = 16{\cdot}4/224 = 0{\cdot}0732.$$

Substitute this value of m in the equation at A or B.

At B,
$$30{\cdot}0 = 0{\cdot}0732 \times 336 + c,$$
$$c = 30 - 24{\cdot}60 = 5{\cdot}4.$$

Then, **law is $\mathbf{E = 0{\cdot}0732L + 5{\cdot}4}$**

LIMITING EFFICIENCY OF A MACHINE.

The efficiency of a machine may be written :

$$\frac{\text{Actual load } L}{\text{Theoretical load}} = \frac{\text{Actual load } (L)}{\text{Effort } (E) \times \text{velocity ratio } (v)} = \frac{L}{(mL + c)v}.$$

Dividing numerator and denominator by L,

$$\text{Efficiency} = \frac{L \div L}{(mLv + cv) \div L} = \frac{1}{mv + cv/L}.$$

As L increases the efficiency tends towards the limiting value :

$$\frac{1}{\mathbf{mv}}$$ (See Ex. 62, p. 361.)

Example 1. *Find the effort required to raise a load of* 300 *lb. with a screw jack known to have a law* $E = 0{\cdot}0732L + 5{\cdot}4$.

$$E = 0{\cdot}0732 \times 300 + 5{\cdot}4$$
$$= 21{\cdot}96 + 5{\cdot}4 = 27{\cdot}36.$$ **Ans. 27·36 lb.**

Example 2. *Find the load which can be raised by the jack if an effort of* 15 *lb. is employed.*

$$E = 0{\cdot}0732L + 5{\cdot}4,$$
$$15 = 0{\cdot}0732L + 5{\cdot}4.$$
$$L = \frac{9{\cdot}6}{0{\cdot}0732} = 131{\cdot}1.$$ **Ans. 131·1 lb.**

NOTE.—*The law of a machine is often expressed in the complementary form $L = mE + c$, but the treatment remains the same.*

Example 3. *A crane raises a load of 5000 lb. a distance of 7 ft. The efficiency is 46% and the effort 200 lb. Find the velocity ratio, distance moved by the effort, and effect of friction.*

Let the distance moved by the effort be x ft.

Then, the work output $= 7 \times 5000$ or 35000 ft. lb.,
and the work input $= 200 \times x$ ft. lb.

$$\text{Efficiency} = \frac{46}{100} = \frac{35000}{200x}.$$

$$\therefore \quad 46 \times 200x = 35000 \times 100,$$

$$x = \frac{35000 \times 100}{46 \times 200} = \mathbf{380{\cdot}5}.$$

$$\text{Velocity ratio} = \frac{380{\cdot}5}{7} = \mathbf{54{\cdot}36}.$$

$$\text{Effect of friction} = 54{\cdot}36 \times 200 - 5000 = \mathbf{5872\ lb.}$$

Velocity ratio = 54·36, distance moved by effort = 380·5 ft., effect of friction = 5872 lb.

Example 4. *The leading screw of a lathe has a pitch of 0·5 in. and has to exert a force of 534 . in order to operate the saddle. Find the effort required on a driving wheel of effective diameter 15 in. if the efficiency is 25%.*

Distance moved by effort in 1 revolution $= 15\pi$.
Distance moved by the saddle $= 0{\cdot}5$ in.

Work output $= 0{\cdot}5 \times 534$ in. lb.
Work input $=$ effort $\times 15\pi$ in. lb.

$$\text{Efficiency} = \frac{25}{100} = \frac{0{\cdot}5 \times 534}{E \times 15\pi},$$

$$\therefore \quad 15\pi \times 25 \times E = 100 \times 0{\cdot}5 \times 534,$$

$$E = \frac{100 \times 0{\cdot}5 \times 534}{15\pi \times 25} = \mathbf{22{\cdot}66\ (lb.)}.$$

The inclined plane and wedge as machines. When a body of weight W lb. is moved up an inclined plane by the application of an effort E lb., there is a mechanical advantage W/E and this advantage depends upon the inclination of the plane. The theory of the inclined plane is important and has an application in mechanical handling by gravity roller conveyors, shutes, lifting devices and screw threads.

Inclined plane without friction. Consider an inclined plane (Fig. 169):

(*a*) with the effort acting horizontally,

$$\text{mechanical advantage} = \frac{W}{E} = \frac{1}{\tan\theta};$$

(*b*) with the effort acting along the plane (Fig. 112),

$$\text{mechanical advantage} = \frac{W}{E} = \frac{1}{\sin\theta}.$$

If there is no friction the efficiency $= 100\%$ and the

velocity ratio = mechanical advantage.

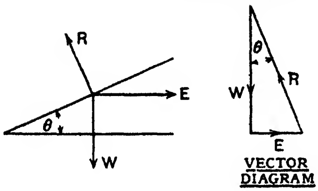


FIG. 169.

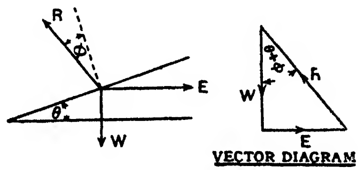


FIG. 170.

This may also be viewed from the work aspect:

Suppose the load is raised 1 foot, then the effort will move, in case (*a*) $\frac{1}{\tan\theta}$, and in case (*b*) $\frac{1}{\sin\theta}$, so that the velocity ratios are $\frac{1}{\tan\theta}$ or $\cot\theta$ and $\frac{1}{\sin\theta}$ or $\operatorname{cosec}\theta$ respectively.

Inclined plane with friction (effort horizontal). Fig. 170 shows a diagram of the forces acting upon a body when it is being raised by an effort on an inclined plane against friction. The reaction of the plane R makes an angle ϕ, equal to the angle of friction, with the normal to the plane and from the vector diagram,

$$\frac{E}{W} = \tan(\theta + \phi),$$

and the mechanical advantage is

$$\frac{W}{E} = \text{tangent of the complement of } (\theta + \phi)$$
$$= \tan\{90 - (\theta + \phi)\} = \cot(\theta + \phi).$$

The velocity ratio is independent of friction and is $\dfrac{1}{\tan\theta}$.

$$\text{Therefore the efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \cot(\theta+\phi)\tan\theta.$$

The wedge. It is probable that the wedge was one of the first machines employed by primitive man. To-day the engineer makes

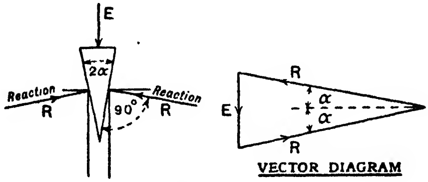


Fig. 171.

extensive use of wedges for such purposes as levelling machines on their foundations, breaking flanged pipe joints, and in the form of taper keys for fastening purposes. Cutting tools are also shaped to a wedge form.

Neglecting friction, the forces acting upon a wedge are shown in Fig. 171, where the driving force E is balanced by the two reactions R. These reactions, determined from the triangle of forces, represent the bursting or splitting forces exerted by the wedge, but are *opposite in sense.*

Mechanical advantage = velocity ratio (neglecting friction)

$$= \frac{\text{splitting or bursting force}}{\text{driving force or effort}} = \frac{R}{E} = \frac{R}{2R\sin\alpha} = \frac{1}{2\sin\alpha}.$$

The effect of friction is to alter the direction of the reactions R by an amount equal to ϕ, the angle of friction (Fig. 172). The value

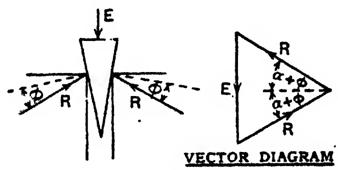


Fig. 172.

of R can then be found by the method of the triangle of forces as shown in the vector diagram.

Example 1. *The angle of a wedge is 6°. Find the magnitude of the force which must be supplied by a hammer blow in order to open a flanged joint which is held together by a normal force estimated at 5 tons. Neglect friction.*

Let T tons be the bursting force,

then since $$T = R\cos 3°,$$

and $$E = 2R\sin 3°,$$

then $$E = 2T\tan 3°,$$

or $$E = 5\tan 3° \times 2 = 0{\cdot}0524 \times 10 = 0{\cdot}524.$$ **Ans. 0·524 ton.**

The mechanics of the screw. When a screw revolves once within its nut the screw advances, or retracts, a distance equal to the lead or pitch of the screw. If a piece of paper is cut to the form of a right-angled triangle and wound around a cylinder, as shown (Fig. 173), it can be seen that the hypotenuse of the triangle forms, on the cylinder, the curve of a screw. Thus a screw thread may be reduced to an **inclined plane.**

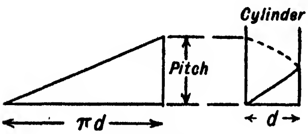
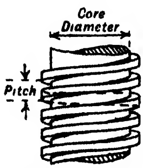


FIG. 173.

Single start thread. In this, the commonest, type of thread the groove is cut so that, at any section of the screw at right angles to the axis, the groove intersects the section once only, and the lead equals the pitch and contains one space and one thread. See Fig. 174.

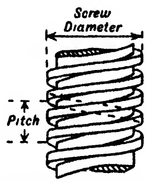
NOTE.—There is, even in British practice, some inconsistency in the definition of pitch. The British Engineering Standards Association defines the pitch of a screw in terms which give it as the distance, measured parallel to the axis, between two corresponding and consecutive points on the same thread. This amounts to the pitch being the distance advanced or retracted by the nut during one complete revolution of the screw. In American practice, and sometimes in this country, the distance measured axially between one thread and the next is termed the pitch, while the distance moved by the nut during a complete revolution is called the lead.

Multiple start threads. These may be two, three or more starts, and the section at right angles to the axis is intersected by the groove, at equiangular points, in number equal to the number of starts. The pitch is increased so that a number of spaces and threads equal to the number of starts may be accommodated in the pitch. Thus a two start thread (Fig. 175) has two threads and two spaces to each pitch length. The axial distance from one thread to the next is called the divided pitch or sometimes just the pitch, although it is better to regard the pitch as the movement of the nut in one revolution of the screw.



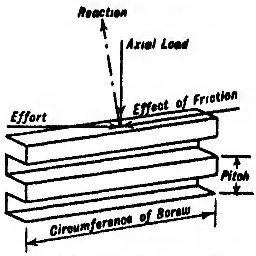
SINGLE START SCREW

FIG. 174.



MULTIPLE START SCREW

FIG. 175.



EQUIVALENT INCLINED PLANE

FIG. 176.

Use of multiple threads. Screws are threaded to multiple threads in order to decrease their velocity ratio. For example, in a three start thread, one revolution of the screw in its nut will advance the screw a distance equal to the pitch, which is *three threads* and not one thread, as is the case for a single start thread.

NOTE.—This increase of pitch also increases the slope of the inclined plane, to which, for mechanical consideration, the screw may be reduced (Figs. 176, 177), and the corresponding effort needed to operate the screw is increased.

Forces acting upon a screw thread. When a screw thread is reduced to its equivalent inclined plane the forces acting upon it are:

(*a*) the load acting upon the surface of the thread,

(*b*) the reaction of the plane, or thread, in a direction at right angles to the thread,

(*c*) the effect of friction, opposing motion and acting along the thread,

(*d*) the effort acting at right angles to the axial load.

The problem, to assess the amounts of these forces, then resolves itself into one in which four concurrent forces are in equilibrium and may be treated, either graphically, or by the method of resolution of forces.

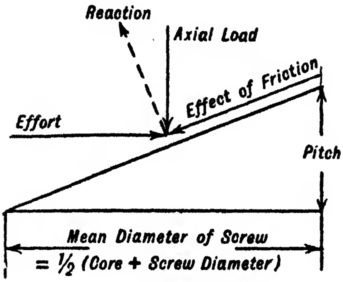


FIG. 177.

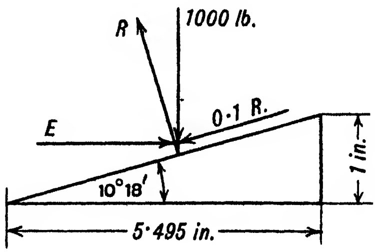


FIG. 178.

Example. *A two start, square thread, screw has a screw diameter of* 2 *in., pitch* 1 *in., and core diameter* $1\frac{1}{2}$ *in. Find the effort required to operate the screw by means of a spanner* 18 *in. in length, and the efficiency, if the coefficient of friction is* 0·1, *and load* = 1000 *lb.*

Mean diameter of screw = 1·75 in.

Mean circumference = $1{\cdot}75\pi = 5{\cdot}495$ in.

Lead or pitch = 1 in. Divided pitch = ½ in.

$$\text{Angle of thread, } \theta = \tan^{-1}\frac{1}{5{\cdot}495} = 10^\circ\ 18'.$$

Resolving horizontally, Fig. 178.

$$R\sin 10^\circ\ 18' + 0{\cdot}1R\cos 10^\circ\ 18' = E,$$

$$0{\cdot}1788R + 0{\cdot}1 \times 0{\cdot}9839R = E,$$

$$(1) \qquad 0{\cdot}2772R = E.$$

Resolving vertically, $R\cos 10^\circ\ 18' = 1000 + 0{\cdot}1R\sin 10^\circ\ 18'$,

$$0{\cdot}9839R - 0{\cdot}1 \times 0{\cdot}1788R = 1000,$$

$$0{\cdot}966R = 1000,$$

$$(2) \qquad R = 1035 \text{ lb}.$$

Substituting in (1), $E = 0{\cdot}2772R = 0{\cdot}2772 \times 1035$

$$= 286{\cdot}9 \text{ lb}.$$

This is the effort required at a radius equal to the mean radius of the screw, that is 0·875 in.

To find the effort on an 18 in. spanner.

Moments about the axis (Fig. 179).

$$286{\cdot}9 \times 0{\cdot}875 = 18E.$$

$$\textbf{Effort} = \mathbf{13{\cdot}95\ lb.}$$

To find the efficiency of this screw.

Consider one revolution of the screw.

$$\text{Work input} = E \times \pi \times 18 \times 2 \text{ in. lb.}$$
$$= 13{\cdot}95 \times \pi \times 36 = \mathbf{1578\ in.\ lb.}$$
$$\text{Work output} = 1000 \times \text{pitch} = \mathbf{1000\ in.\ lb.}$$
$$\textbf{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{1000}{1578} \text{ or } 63{\cdot}4\%.$$

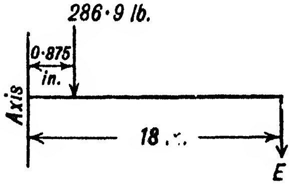


FIG. 179.

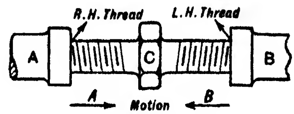


FIG. 180.

This is a theoretical efficiency and it takes no account of the inefficiency of the screw bearings, which invariably produce losses in any screw machine. For example, if the screw bearings had an efficiency of 70% the overall efficiency would be $0{\cdot}7 \times 63{\cdot}4$ or 44·4%.

The use of right- and left-hand threads. When it is desired, by the rotation of a screw, to draw two components together, a screw cut with right- and left-hand threads is employed. One rotation of C (Fig. 180) will move each of the components A and B together, or apart, according to the direction of rotation of C, a distance equal to the screw pitch.

Weston's differential pulley block. This block is in very general use, as a portable lifting device, in workshops, aboard ship or on buildings. It consists of two chain pulleys working on a common axle in the top block. These pulleys are of unequal radii

R and R_1, and a chain register and groove is cut in their rim to accommodate an endless chain, which passes over the larger pulley, around the rim of the snatch block pulley and finally around the smaller pulley in the block (Fig. 181). The effort is applied to the free portion of the endless chain, and the load is carried upon the snatch block hook.

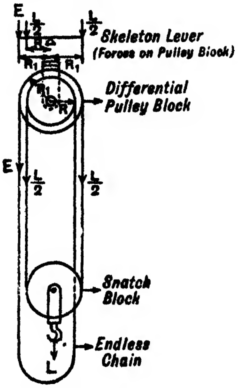


FIG. 181. Weston's pulley block.

The principle of working may be identified with that of the wheel and differential axle, and if the top block is considered as a lever with the axle as the fulcrum, the principle of moments may be applied to find the effort for a given load, if friction is ignored.

Consider the skeleton lever for the top block: the tensions in each of the chains supporting the snatch block will be equal to one half of the load, and if the senses of these tensions are taken into consideration, it will be noticed that one of these tensions is assisting the effort.

Taking moments about the fulcrum, E = effort,

$$R_1 \times \frac{L}{2} = R \times \frac{L}{2} + R_1 \times E,$$

$$\therefore \quad R_1 E = \frac{L}{2}(R_1 - R),$$

$$\text{or} \quad E = \frac{L(R_1 - R)}{2R_1}.$$

NOTE.—The effort, in actual practice, is much greater than this theoretical figure, due to the presence of considerable friction and lack of flexibility in the chain.

Types of lifting machines. Modern workshop and building practice demands lifting machines of the maximum possible efficiency coupled with convenience of manipulation and mobility.

The stationary types of machine, such as pulley tackles, winches and cranes, have been to some extent superseded by machines employing the same lifting device, but free to vary their sphere of

FIG. 182. Morris differential block.

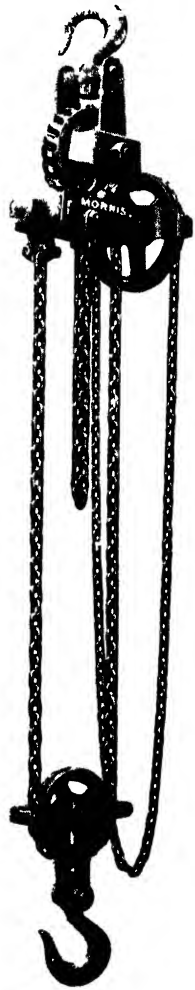


FIG. 183. Worm gear pulley block.

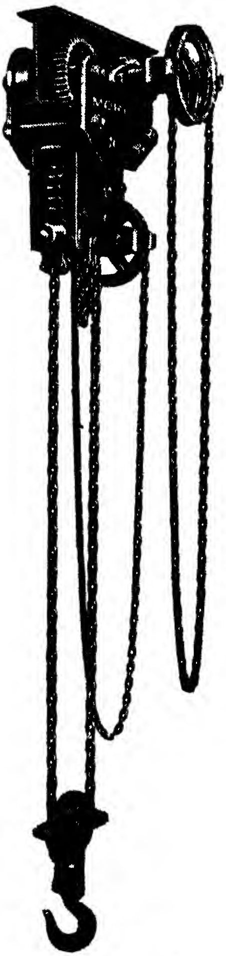


FIG. 184. Morris worm gear travelling pulley block.

operations on runways and overhead rails. The modern factory is equipped to meet this need, and the tackles can be attached to the runway, and thus operate over the range of its service.

Hand power is sometimes replaced by motor power, and controls are provided to vary the speed of load ascent and descent.

The range of machines shown in Figs. 182-188 is supplied by courtesy of Messrs. Herbert Morris Ltd. of Loughborough, and show a small number of the applications in which pulley, screw and worm and worm-wheel principles are employed.

EXERCISES ON CHAPTER VII

1. What observations would you take during an efficiency test upon a lifting machine? Distinguish between the results actually obtained from measurement and those derived from the measured results.

2. A machine raises a load of 180 lb. a distance of 10 in. The effort, a force of 24 lb., moves 90 in. during the process. Calculate the velocity ratio, mechanical advantage, effect of friction and efficiency at this load.

3. A lever, in which the fulcrum is placed at a point one-third of the length of the lever from the load, has an efficiency of 92%. If the available effort is 32 lb., find the load which the lever could raise.

4. A simple wheel and axle has a wheel 14 in. in diameter, and an axle 6 in. diameter. Find its velocity ratio. If it is converted to a differential wheel and axle by the addition of a further axle 4 in. in diameter, what velocity ratio would result?

5. A worm is driven by a handle having an effective radius, measured from the centre of the worm shaft, of 15 in. Calculate the velocity ratio of a worm and worm-wheel machine, operated by this worm, if the worm-wheel has 60 teeth and the hauling drum is 9 in. in diameter.

6. A screw return motion for a stamping machine has a screw of $\frac{3}{4}$ in. pitch. It is operated by a toothed wheel of 17 in. pitch diameter. Find the force required between the wheel teeth if the resistance is $2\frac{1}{4}$ cwt. and the efficiency 30%.

7. A rope tackle has an efficiency of 67% and a velocity ratio of 7. Calculate the effort required to lift a load of 650 lb.

8. The efficiency of a rope tackle is 62%, and the tackle is reeved on a 3-2 system of pulleys. Calculate the mechanical advantage and the load which an effort of 36 lb. would raise.

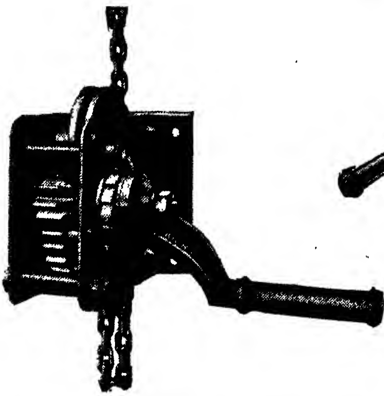


Fig. 185. Spur gear self-sustaining hoist with chain lift.

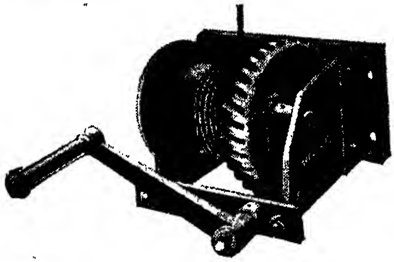


Fig. 186. Worm geared wire rope wall hoist.

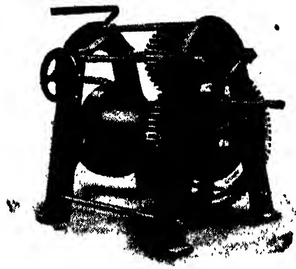


Fig. 187. Morris crab winch.

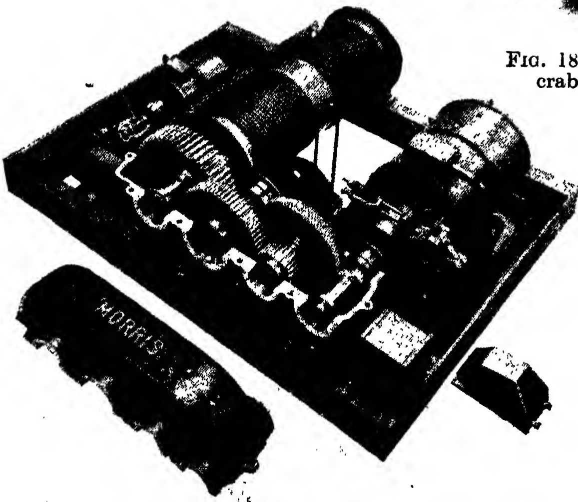


Fig. 188. Morris electric overhead crane with ball bearings and enclosed gears.

9. A worm-wheel hoist consists of a single threaded worm, driven at 200 revolutions per minute through a pulley 12 in. in diameter. If the worm-wheel has 40 teeth and the load rope drum is 8 in. diameter, calculate (*a*) the velocity ratio, (*b*) the H.P. of the driving motor when the load lifted is $\frac{1}{4}$ ton and the overall efficiency 28%.

10. What is meant by overhauling? State the conditions which must exist in order that overhauling may take place, and name the types of machines in which overhauling is unlikely.

11. The ram and crosshead of a hydraulic press together weigh 1930 lb. The arrangement for raising them consists of two screws of $1\frac{1}{2}$ in. pitch double started, operated simultaneously. Find the force required by each of two men in order to raise the crosshead if they employ a spanner 12 in. in length and the screw efficiency is known to be 32%.

12. The punch of a screw indenting machine is driven by a screw, of 2 in. pitch, double started, actuated by a pulley 3 ft. in diameter. Find the force on the punch if the force of 1000 lb. is applied tangential to the pulley rim. Overall efficiency 25%.

13. A machine for rolling metal cylinders is operated by a screw of 3 in. pitch, 4 in. mean diameter, treble started. Find the tangential force required to drive a worm-wheel 2 ft. 4 in. in pitch diameter, threaded on to the screw, if the axial resistance to rolling is 14 tons, and the overall efficiency is 22%.

14. A circular turn-table 4 ft. 6 in. in diameter, and cut on its circumference with 100 teeth, is driven by a worm, attached to which is a driving pulley 16 in. in diameter. If the effective pull on the belt is 82 lb., find the resistance at pitch radius offered by the table, allowing an efficiency of 24%.

15. The law of a machine, taken from the load *v.* effort graph, where L and E are in lb., is $L = 2{\cdot}91E - 16{\cdot}4$.

Find: (*a*) the effort which can raise a load of 2 cwt.,
(*b*) the load which an effort of 25 lb. will lift,
(*c*) the effort required to operate the machine at no load,
(*d*) the efficiency under a load of 1 cwt. if the velocity ratio is 3.

16. The following results were obtained during the test of a screw arranged with a velocity ratio of 50. Find the law of the machine and its efficiency under a load of 300 lb.

Load, lb. -	60	120	180	240	300	360
Effort, lb. -	7·8	12	16·8	21·6	25·8	30

17. Obtain the law connecting load and effort for a machine, which when under test, gave the following observations.

Load, lb. -	20	30	40	50	60	70	80
Effort, lb. -	16	22·5	28·5	34·5	41	47·5	54

If the velocity ratio of the machine is 2, find the effect of friction and efficiency under a load of 75 lb.

18. A portion of a heavy machine weighs 957 lb. It is to be moved along a slide against a coefficient of friction of 0·2, by a screw of $\frac{1}{2}$ in. pitch. What horse power will be required for this purpose if the pulley driving the screw revolves at 150 revolutions per minute ? Efficiency overall = 18%.

19. A worm drives a toothed sector, the teeth of which are $1\frac{1}{4}$ in. pitch. What force would be available at the end of the lever *B* (Fig. 189) if the worm is driven by a hand-wheel of effective radius 20 in. and a force of 30 lb. ? The efficiency of the worm mechanism is 27% and that of the lever 92%. Find the overall velocity ratio of this mechanism.

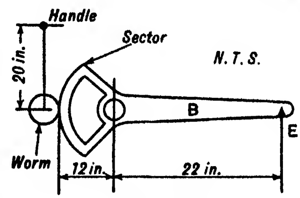


Fig. 189.

20. A winch for hauling cables through their conduits is made on the wheel and differential axle principle. If the axles are, respectively, 10 in. and 5 in. diameter, find the necessary length of handle to receive an effort of 64 lb., and to haul with a force of 700 lb. Efficiency = 45%.

21. Prepare a diagram to show the forces acting upon a screw thread when it is on the point of raising the load. Find the effort required on the mean screw diameter to raise a load of 620 lb. by means of a double start screw of mean diameter 2·1 in. and pitch 0·5 in. if the coefficient of friction is 0·15.

22. Calculate the effort and efficiency of a screw of mean diameter 1·7 in., pitch 0·2 in., which raises a load of 300 lb. and is driven by a hand-wheel 10 in. in diameter. Coefficient of friction 0·15.

23. What would you expect to be the effect of overloading a machine

(*a*) on the load *v*. effort graph,

(*b*) on the load *v*. effect of friction graph,

(*c*) on the load *v*. efficiency graph ?

How would you select the best range of loads for this machine, and how would you fix the maximum load to be employed ?

24. Fig. 190 shows a form of hand baling press in which a worm and worm-wheel transmit power through a crank and connecting rod to a piston. Find with the help of a scale drawing the velocity ratio between the handle A and the gudgeon pin for three positions of the crank, (*a*) as shown, (*b*) after the crank has moved upwards another 45°, (*c*) 10° before the end of the working stroke. What does the value of the velocity ratio approach towards the end of the working stroke? If 30 lb. be applied to the handle, find the theoretical force compressing the material in each of the above positions.

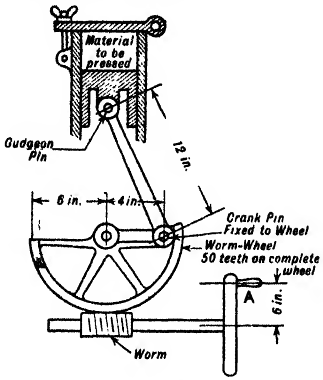


FIG. 190.

25. The dimensions of a toggle mechanism operating a small hand punch are shown in Fig. 191. By means of a scale drawing measure the distance moved by the punch and by the effort for each 5° movement of the effort handle, starting with the punch in its lowest position. Calculate the average velocity ratio during each of these 5° intervals and plot a graph of velocity ratio against angular movement of the handle. Hence show that the velocity ratio is greatest where it is most required. What is the theoretical force available at the punch if the effort is 30 lb. and the effort handle is (*a*) $7\frac{1}{2}°$, (*b*) $12\frac{1}{2}°$ from the bottom of its stroke?

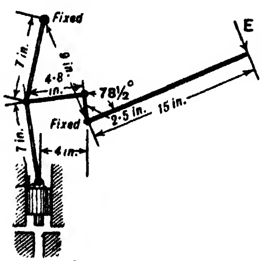


FIG. 191.

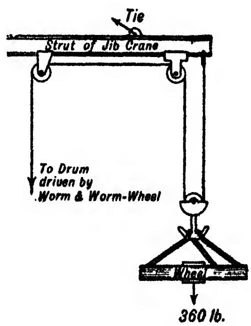


FIG. 192.

26. An arrangement for lifting wheels in the process of shrinking them on axles is shown in Fig. 192. The wheel of the worm geared wall hoist has 40 teeth and the handle turning the worm has an effective radius of 12 in. The winding drum is 8 in. in diameter. Calculate the

velocity ratio of the arrangement and the force at the handle to lift a wheel weighing 360 lb. Efficiency 30%.

27. In the brake mechanism shown in Fig. 193, the force E is 20 lb. Find the force between the brake shoe and the wheel, using the principle of moments. Use this result to find the velocity ratio between E and the brake shoe. If the efficiency of the system is 60%, what is the actual braking force ?

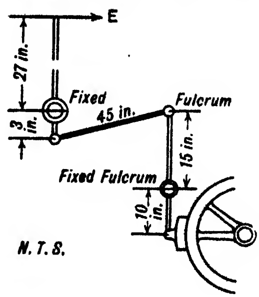


Fig. 193.

28. Fig. 194 shows a timber grab. Find for the position shown, by means of a scale drawing, how far the points A and B move for 1 in. upward travel of the lifting ring. Hence determine the velocity ratio of the mechanism and the force at A (or B), measured at right angles to the $17\frac{1}{2}$ in. dimension, if the baulk of timber weighs 15 cwt. To check your answer use the triangle of forces to find the tension in each chain and then, by considering the equilibrium of one bent lever, take moments about the fulcrum to find the balancing force at A or B.

29. The large pulley of a Weston's block is 7 in. in diameter, and the smaller 6 in. in diameter, measured in each case to the chain centre. Find the theoretical and actual efforts required to raise a load of 7 cwt. if the efficiency is 35%. What is the efficiency if an effort of 80 lb. raises a load of 3 cwt. ?

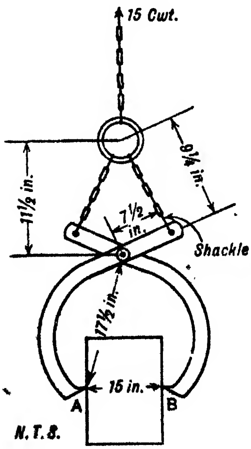


Fig. 194.

30. A worm-geared pulley block with a double threaded worm is employed to raise a load of $\frac{1}{2}$ ton. The effort chain wheel has an effective radius of 6 in. while for the load chain drum it is $2\frac{1}{2}$ in. From the load drum the chain is passed round a snatch block and then fixed to the machine. There are 45 teeth on the worm-wheel. Calculate the velocity ratio of the block and the effort required if the efficiency is 30%. What is the effect of friction ?

31. The angle of a wedge is 6°. Find the magnitude of the force which must be applied by means of a hammer blow to open up a flanged joint if the force pressing the flanges together is estimated at 7 tons. Neglect friction.

32. A load of $\frac{1}{2}$ ton is to be raised 2 ft. by means of a rough inclined plane making an angle of 15° with the horizontal. If the angle of friction

is 17°, find (*a*) the effort necessary parallel to the plane, (*b*) the mechanical advantage of the machine, (*c*) the velocity ratio, (*d*) the efficiency.

33. Assuming an inclined plane to be smooth and frictionless, what is the mechanical advantage and velocity ratio of a plane if an effort of 20 lb. applied horizontally maintains a load of $1\frac{1}{2}$ cwt. in equilibrium? What angle does the inclined plane make with the horizontal?

34. An effort of 300 lb. is applied to a wedge with an angle of 7°. Neglecting the effect of friction, find the splitting force normal to the wedge produced by this effort. What is the mechanical advantage of the wedge?

35. The angle of a taper key is 1°. Neglecting friction, find the force with which the key is pressed against the shaft and pulley if the driving force is 300 lb.

36. Define the *velocity ratio, mechanical advantage,* and efficiency of a lifting machine in terms of the load W, the effort P, the distance D moved by the load, and the distance d moved by the effort.

In a wheel and differential axle the wheel has a diameter of 24 in., and the drum has diameters of 7 in. and 6 in. respectively. Calculate the velocity ratio. (U.L.C.I.)

37. Explain how you would determine the actual velocity ratio of a Weston's differential pulley block if you were experimenting with it in the laboratory.

The efficiency of such a lifting appliance is 35% and the velocity ratio 24; find what pull should be exerted on the chain of the block in order to raise a load of $\frac{1}{2}$ ton. (U.L.C.I.)

38. The effective length of the handle of a screw jack is 24 in. and the pitch of the screw thread is $\frac{3}{8}$ in. What force applied at the end of the handle would be required to raise a load of 15 cwt., if the efficiency of the jack is 15%? (U.L.C.I.)

CHAPTER VIII

TRANSMISSION OF POWER BY BELTS AND PULLEYS AND TOOTHED WHEELS

TRANSMISSION of the power developed by a prime mover, such as an engine or motor, is usually performed through lines of shafting and belt pulleys. This is only possible where plenty of overhead room is available to accommodate the shafting and belt drives. Where it is necessary to transmit the power underground, or in a confined space, such as within a machine itself, toothed gearing is employed.

Power transmitted by a belt. When a pulley is driven by a belt (Fig. 195), two belt tensions operate; T_1, the tension due to the power transmitted, and T_2, an opposite tension due to the sag and inertia on the non-pulling side. It follows that the effective force driving the pulley is the difference between the tension on the tight or driving side, and the tension on the slack or non-pulling side of the belt, that is $T_1 - T_2$.

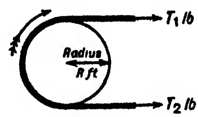


FIG. 195.

Consider a pulley R ft. radius driven at N revs. per min. with belt tensions of T_1 and T_2 lb. (Fig. 195).

Effective pull $= T_1 - T_2$ lb.

Work done per revolution $= (T_1 - T_2) \times 2\pi R$ ft. lb.

Work done per minute $= (T_1 - T_2) \times 2\pi R \times N$ ft. lb.

$$\text{H.P. transmitted} = \frac{(T_1 - T_2) \times 2\pi RN}{33000},$$

$$\text{or} \quad \text{H.P.} = \frac{2\pi \text{RN}(\text{T}_1 - \text{T}_2)}{33000},$$

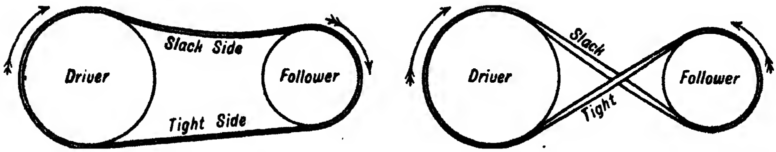
$$\text{or} \qquad \text{H.P.} = \frac{\text{speed of belt in ft. per min.} \times \text{effective pull in lb.}}{33000}.$$

Direction of transmission and velocity ratio. Belt drives may be arranged to transmit the power from one pulley to another, (*a*) in

the same directions of rotation, (*b*) in opposite directions of rotation, by the use of open belt drives or crossed belt drives respectively (Fig. 196).

The pulley receiving the power is referred to as the **driver** and the pulley to which power is transmitted the **follower.**

Velocity ratio is the name given to the **ratio of the speed of the driver to the speed of the follower**, measured in revolutions per minute. It is evident that the circumferential speed of a point on the driver must be equal to the corresponding speed of a point on the follower, otherwise the belt connection could not be maintained.



Open belt drive—same directions. Crossed belt drive—opposite directions.

FIG. 196.

Suppose in Fig. 196 the driver is D ft. in diameter and revolves at N revs. per min., while the follower is d ft. in diameter and makes n revs. per min.

$$\text{Velocity ratio} = \frac{\text{speed of driver}}{\text{speed of follower}} = \frac{N}{n}.$$

The speed of the belt $= \pi DN$ ft. per min., which is the circumferential speed of both driver and follower, so that

$$n = \frac{\pi DN}{\pi d} = \frac{DN}{d}, \text{ or } \mathbf{V.R.} = \frac{\mathbf{N}}{\mathbf{n}} = \frac{\mathbf{d}}{\mathbf{D}},$$

which, as a proportion, is $\mathbf{N : n = d : D}$.

In other words, the speeds are inversely proportional to the diameters of the pulleys.

Slip. In practice the speed of the follower does not realise the figure obtained from the preceding calculation, because of a certain small slip of the driving belt over the pulleys. This slip is generally expressed as a percentage of the calculated speed; its amount largely depends upon the condition of the belt and the pulley surfaces.

Example 1. *A belt is driven by a pulley* 16 *in. in diameter. Find the speed of a follower* 6 *in. in diameter if the driver speed is* 200 *revs. per min. and slip is equal to* 3%.

$$\text{V.R.} = \frac{\text{speed of driver}}{\text{speed of follower}} = \frac{\text{diameter of follower}}{\text{diameter of driver}}.$$

$$S_d : S_f = D_f : D_d.$$

$$200 : S_f = 6 : 16.$$

$$6S_f = 3200 \text{ or } S_f = 533\tfrac{1}{3}.$$

Allowing 3% slip. **Speed of follower = 517 r.p.m.**

Example 2. *A pulley* 10 *in. in diameter is to be driven at* 400 *r.p.m. by a shaft revolving at* 150 *r.p.m. Calculate the diameter of the shaft pulley if the slip is* 4%.

$$\text{Speed to be transmitted} = \tfrac{100}{96} \times 400 \text{ r.p.m.}$$
$$= 417 \text{ r.p.m.}$$

$$S_d : S_f = D_f : D_d.$$

$$150 : 417 = 10 : D_d.$$

$$D_d = \tfrac{4170}{150} = 27{\cdot}8.$$ **Ans. 27·8 in.**

NOTE.—*A* 28 *in. diameter pulley would be employed.*

Example 3. *A pulley* 3 *ft. in diameter is driven by a belt at* 200 *r.p.m. If the tensions are* 200 *lb. and* 70 *lb. on the tight and slack sides respectively, find the H.P. transmitted* (*a*) *without slip,* (*b*) *with* 3% *slip.*

(*a*) Effective driving force $= 200 - 70$ or 130 lb.

Speed of belt in ft. per min. $= \pi DN = \pi \times 3 \times 200$.

Work done per minute $= 130 \times \pi \times 3 \times 200$ ft. lb.

$$\text{H.P.} = \frac{130 \times \pi \times 3 \times 200}{33000} = 7{\cdot}43.$$

(*b*) With 3% slip the speed will be reduced by 3% with a consequent loss of H.P. $= 3\% \times 7{\cdot}43$.

$$\text{H.P. (with slip)} = \tfrac{97}{100} \times 7{\cdot}43 = 7{\cdot}2.$$

Texrope drives. A very popular drive, which has the advantage that it can be used on pulleys with their axes very close together, is the Texrope drive. The drive is made between (*a*) two Vee grooved pulleys, or (*b*) a Vee grooved small pulley and a flat large pulley and is, in the latter case, known as a Vee flat Texrope drive.

Example 4. *The H.P. transmitted by a belt drive is* 16·4 *and the follower speed is* 120 *revs. per min. If the slip is* 3% *calculate the effective pull in the belt before the slip occurs. Pulley* 4 *ft. diameter.*

$$\text{H.P.} = 16{\cdot}4.$$

$$\text{H.P. allowing } 3\% \text{ slip} = \tfrac{100}{97} \times 16{\cdot}4.$$

$$\text{Work equivalent} = \frac{100 \times 16{\cdot}4 \times 33000}{97} \text{ ft. lb.}$$

$$= \text{speed of belt} \times \text{effective pull}$$

$$= \pi \times 4 \times 120 \times F \text{ ft. lb.}$$

$$F = \frac{100 \times 16{\cdot}4 \times 33000}{97 \times 4 \times 120 \times \pi} = 369{\cdot}8. \quad \textbf{Ans. 369·8 lb.}$$

Example 5. *A pulley* 3 *ft.* 6 *in. diameter revolving at* 240 *r.p.m. is to drive a line of shafting at* 600 *r.p.m. and transmit* 10 *H.P. Find the diameter of the follower pulley and the effective belt tensions if the slip is* 4%. *If the tight side tension is* $2\frac{1}{2}$ *times that on the slack side, find the tensions.*

$$\text{Speed of follower, allowing } 4\% \text{ slip} = \tfrac{100}{96} \times \tfrac{600}{1} = 625 \text{ r.p.m.}$$

$$D_d : D_f = S_f : S_d.$$

$$42 : D_f = 625 : 240.$$

$$\text{Diameter of follower in in.} = \frac{240 \times 42}{625} = 16{\cdot}13 \text{ (say 16 in.).}$$

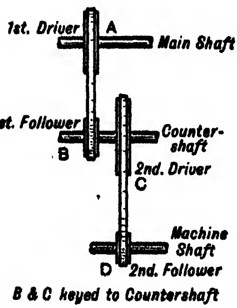


FIG. 197. Countershaft drive.

$$\text{Speed of belt} = 625 \times \pi \times \tfrac{16}{12} \text{ ft. per min.}$$

$$\text{H.P.} = \frac{625 \times \pi \times 16 \times P}{33000 \times 12} = 10.$$

$$P \text{ in lb.} = \frac{33000 \times 12 \times 10}{625 \times \pi \times 16} = 126.$$

Let T = slack side tension,

then $2\frac{1}{2}T$ = tight side tension.

$$2\tfrac{1}{2}T - T = 126, \quad T = \frac{2 \times 126}{3} = 84.$$

Tensions, 210 lb. and 84 lb.

Compound belt and countershaft drives. The power transmitted to a machine is frequently conveyed by a compound belt drive through a countershaft, so that the speed of the machine is obtained at the desired velocity ratio by compounding the velocity ratios of two driver and follower pairs (Fig. 197).

In the arrangement shown A is the mainshaft pulley, and B and C are keyed to the countershaft: *thus the speeds of* B *and* C *are alike*, and C is the driving pulley for the machine pulley D.

Consider the pulleys A *and* B,

$$\frac{\text{speed A}}{\text{speed B}} = \frac{\text{diameter B}}{\text{diameter A}}$$

$$\text{or speed B} = \frac{\text{speed A} \times \text{diameter A}}{\text{diameter B}}.$$

Consider the pulleys C *and* D,

$$\frac{\text{speed C}}{\text{speed D}} = \frac{\text{diameter D}}{\text{diameter C}}$$

$$\text{or speed D} = \frac{\text{speed C} \times \text{diameter C}}{\text{diameter D}};$$

but the speed C = speed B, therefore

$$\frac{\text{speed A} \times \text{diameter A}}{\text{diameter B}} \times \frac{\text{diameter C}}{\text{diameter D}} = \text{speed D}.$$

$$\text{V.R.} = \frac{\text{speed A}}{\text{speed D}} = \frac{\text{diameter B} \times \text{diameter D}}{\text{diameter A} \times \text{diameter C}},$$

$$\text{that is,} \quad \frac{\text{speed A}}{\text{speed D}} = \frac{\text{product of follower diameters}}{\text{product of driver diameters}}.$$

Multiple countershaft drives. The use of countershafts may be extended in order to obtain a desired velocity ratio. In the double countershaft drive shown (Fig. 198) the velocity ratio between the pulleys A and F can be shown to be

$$\frac{\text{speed A}}{\text{speed F}} = \frac{\text{product of follower diameters}}{\text{product of driver diameters}}.$$

The wheels A, B, C and D form the first countershaft drive and C, D, E and F the second countershaft drive.

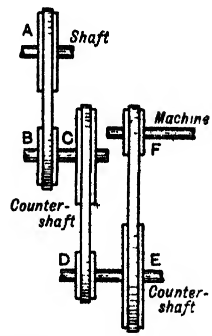


FIG. 198. Double countershaft drive.

Example 1. *A lathe is to be driven at* 600 *revs. per min. The main shaft pulley is* 20 *in. in diameter and runs at* 230 *revs. per min. while the countershaft pulleys are* 12 *in. and* 16 *in. respectively. Calculate the diameter of the lathe pulley if a slip of* 5% *is allowable.*

Diameters of drivers are 20 in. and 16 in.
Diameters of followers, 12 in. and x in.

Speed of lathe pulley allowing 5% slip $=\frac{100}{95}\times\frac{600}{1}$ or **632 r.p.m.**

$$\text{V.R.}=\frac{\text{speed of shaft}}{\text{speed of lathe}}=\frac{\text{product of follower diameters}}{\text{product of driver diameters}},$$

or
$$\frac{230}{632}=\frac{12x}{20\times16},\quad x=\frac{230\times20\times16}{632\times12}=9{\cdot}7.\quad \textbf{Ans. 9·7 in.}$$

Example 2. *In a double countershaft drive* (Fig. 198), *the main shaft rotates at* 90 *revs. per min. The pulley diameters* A, B, C, D, E *and* F *are respectively* 28, 12, 20, 10, 16, 6 *inches. Calculate the speed of* F *if the slip is* 3%.

$$\text{Velocity ratio}=\frac{\text{speed of A}}{\text{speed of F}}=\frac{\text{product of follower diameters}}{\text{product of driver diameters}}.$$

$$\frac{90}{S_f}=\frac{12\times10\times6}{28\times20\times16}.$$

$$S_f=\frac{90\times28\times20\times16}{12\times10\times6}=1120.$$

Allowing for 3% slip,

Speed of F $=\frac{97}{100}\times\frac{1120}{1}$ or **1086 r.p.m.**

Example 3. *A motor pulley* 12 *in. in diameter runs at* 1200 *revs. per min. It is required to drive a machine at* 300 *revs. per min. through a countershaft drive. If the machine pulley is* 10 *in. in diameter, calculate the ratio between the diameters of the countershaft pulleys and suggest suitable diameters for them. Neglect slip.*

Let the pulley diameters be D and d where D is the driver, then

$$\text{V.R.}=\frac{1200}{300}=\frac{d\times10}{D\times12}.$$

$$\frac{d}{D}=\frac{1200\times12}{300\times10}=4{\cdot}8.\quad \textbf{Ans. (1).}$$

Suggested diameters, **driver, 5 in.; follower, 24 in. Ans. (2).**

Transmission of power by toothed wheels. In the records of old machinery it is found that power is often transmitted from one shaft to another through two cylinders, pressed together at their circumference, and fastened rigidly to the shafts at their centre. Later these cylinders are found covered with some material having a high coefficient of friction, such as leather or raw hide ; the object being to prevent or reduce the slip between the cylinders. In very old clock mechanisms, and early winches, pins are inserted into the circumference of one cylinder, and either slots placed in the circumference of the other, or the second cylinder made up of two plates, connected by rods, which form the teeth and spaces. This probably illustrates the process of evolution of the toothed wheel drive in such general use to-day.

The toothed wheel, as now produced, is scientifically designed to give silent and smooth running, the minimum of slip, and the teeth are so shaped that they shall have the greatest possible strength.

Equivalent circles. A pair of toothed, or spur, wheels is equivalent to two cylinders running in contact, and these two equivalent contacting circles are called the **pitch circles.**

The velocity ratio. When two toothed wheels are transmitting power, the speed of a point on one pitch circle is equal to the speed of a point on the other pitch circle ; so that the velocity ratio is

$$\frac{\text{speed of driver}}{\text{speed of follower}} = \frac{\text{diameter of follower}}{\text{diameter of driver}},$$

where the speed is measured in revs. per min. and the diameters are those of the pitch circles.

Circular pitch is the name given to the distance, measured on the arc of the pitch circle, between corresponding points on two consecutive teeth, namely p (Fig. 199).

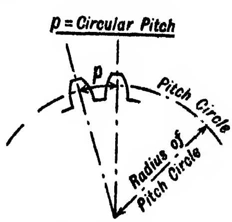


FIG. 199.

In order that two toothed wheels may correctly gear together, one of the conditions to be fulfilled is that the circular pitches shall be equal.

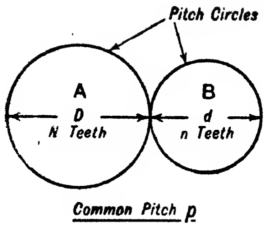


FIG. 200.

Relation between circular pitch and number of teeth.

Let d in. be the diameter of the pitch circle (Fig. 200) and n the number of teeth, then the

$$\text{circular pitch in in.} = \frac{\pi d}{n} = p = \frac{\pi D}{N}.$$

The velocity ratio may be expressed thus:

$$\text{V.R.} = \frac{\text{speed of driver}}{\text{speed of follower}} = \frac{\text{number of teeth on follower}}{\text{number of teeth on driver}},$$

or as a proportion:

speed of driver : speed of follower = number of teeth on driver : number of teeth on follower.

Example 1. *A toothed wheel has a circular pitch of $\frac{3}{4}$ in.; calculate the diameter of the pitch circle if the wheel is cut to 64 teeth.*

$$\text{Diameter of pitch circle in in.} = \frac{64 \times \frac{3}{4}}{\pi} = \frac{48}{\pi} = 15{\cdot}28.$$

Example 2. *Find the circular pitch of a spur wheel, pitch circle diameter 10 in. with 35 teeth.*

$$\text{Circular pitch in in.} = \frac{\pi D}{N} = \frac{\pi \times 10}{35} = 0{\cdot}898 \text{ in.}$$

The rack and pinion. When a series of teeth are cut upon a straight line instead of on a pitch circle, a **rack** is formed (Fig. 201) which, when in gear with a spur wheel, or pinion, is called a **rack and pinion.** The pitch circle of the rack may be regarded as a circle of infinite radius and is called the **pitch line.** The mechanism is equivalent to a circle, the pitch circle of the pinion, rolling along a straight line, the pitch line of the rack, which is a tangent to the pitch circle of the pinion. The faces and flanks of the rack teeth are straight and not of the curved profile described later for wheel teeth.

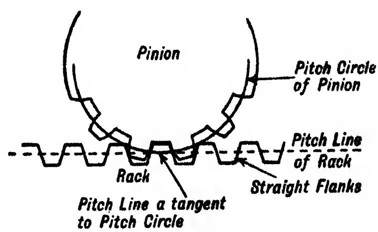


FIG. 201. Rack and pinion.

Example. *A pinion in a rack and pinion mechanism has* 30 *teeth of circular pitch* 1 *in. How far will the axis of the pinion move along the rack in* $2\frac{1}{2}$ *revolutions of the pinion ?*

Circumference of pinion pitch circle $= 30 \times 1$ or 30 in.

Distance travelled in $2\frac{1}{2}$ revolutions $= 2\frac{1}{2} \times 30$ or **75 in.**

Diametral pitch and module pitch. In modern practice, particularly in the mass production of toothed wheels, it is customary to specify the pitch as a **diametral pitch,** which is the number of teeth per inch of pitch circle diameter.

Thus, $$\textbf{diametral pitch} = \frac{\textbf{number of teeth}}{\textbf{diameter of pitch circle in inches}}.$$

The module pitch is the reciprocal of the diametral pitch, and is expressed thus :

$$\textbf{m} = \textbf{module pitch} = \frac{\textbf{diameter of pitch circle in inches}}{\textbf{number of teeth}}.$$

Continental practice. Where the metric system is in operation, it is now the general practice to specify wheel teeth by the **metric module pitch.**

$$\textbf{Metric module pitch} = \frac{\textbf{diameter of pitch circle in millimetres}}{\textbf{number of teeth}}.$$

Standard proportions of wheel teeth. British and American practice employs two standard forms for wheel teeth, namely, (*a*) the **Brown and Sharpe,** (*b*) the **Sellers.**

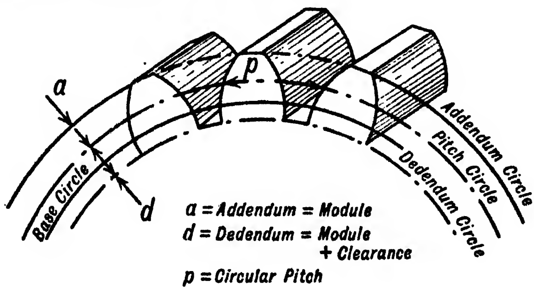


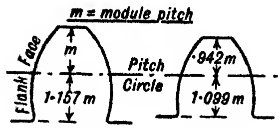
FIG. 202.

The **addendum circle** (Fig. 202) is the circle which passes through the crests of the teeth. The height of this addendum circle measured radially above the pitch circle is called the **addendum.**

The **dedendum circle** is the circle which passes through the roots of the teeth. The depth of this dedendum circle measured radially below the pitch circle is called the **dedendum.**

The **base circle** is a circle, concentric with the pitch circle, which has a bearing on the design of the tooth profile, and it is from this circle that the tooth profile is evolved.

Brown and Sharpe standard (Fig. 203). This proportion for teeth is that most generally used in British practice for machine cut teeth. The *width of tooth and space* are equal and together constitute the circular pitch. The addendum is made equal to the module pitch, and the dedendum the module pitch + a clearance of 0·157 of the module pitch. Thus the *addendum* $= m$ and the *dedendum* $= 1{\cdot}157m$ (Fig. 203), where m is the module pitch; or



Brown and Sharpe. Sellers.
FIG. 203.

$$\text{dedendum} = \text{m} + \frac{\text{thickness of tooth on pitch circle}}{10}.$$

Sellers standard (Fig. 203). In this standard *the addendum is shortened to* $0{\cdot}942m$ *and the dedendum to* $1{\cdot}099m$, where m is the module pitch. This gives a tooth which is shallower, with a consequent increase of root strength when compared with the Brown and Sharpe standard.

Profile of teeth. The curve employed for the profile of teeth has to satisfy certain conditions. Certain geometrical curves fully satisfy the required conditions, and among these the **involute of a circle** is the curve generally adopted for the profile of wheel teeth.

The involute of a circle *is the path of a point which starts from the circumference of a circle and moves so that the length of the tangent from the point to the circle is always equal to the arc of the circle between the starting point and the point of tangency.*

The circle from which this curve is evolved is known as the base circle, and in order that two wheels may correctly gear, the arc of contact of the teeth must have a tangent which is at right angles to the common tangent of the base circles.

The angle between the common tangent to the pitch circles and

the common tangent to the base circles, at the point of tooth contact, is called the **angle of obliquity.**

This angle varies between $14\frac{1}{2}°$ and $22\frac{1}{2}°$, according to the circumstances for which the tooth is designed.

Other tooth profiles. Before the general introduction of involute teeth it was common to make the portion of a tooth above the pitch circle an **epicycloidal curve,** and the portion below the pitch circle a **hypocycloidal curve.** These curves, in modern practice, are largely confined to internal gearing where the involute is impossible.

The epicycloid is the path of a point on the circumference of a circle which is allowed to roll without slipping around the circumference of another circle *externally.*

The hypocycloid is a similar curve formed by the rolling of one circle *inside* the other circle.

The construction of these curves is given in standard textbooks on Practical Geometry and in many books devoted to Machine Drawing.

Standardisation of wheel teeth. Toothed wheels are produced in very large quantities and are standardised for mass production. Apart from the profiles employed, it is general to manufacture wheels with diametral pitches which are either whole numbers or the common fractions; for example, a wheel may be cut to a diametral pitch of 2, 3, $2\frac{1}{2}$ or $1\frac{1}{2}$, in which case the module pitch would be $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$ or $\frac{2}{3}$, according to the diametral pitch selected.

Example 1. *A toothed wheel is cut to a diametral pitch of* 2 *and possesses* 40 *teeth. Calculate* (*a*) *the module pitch,* (*b*) *the diameter of the pitch circle,* (*c*) *the addendum and dedendum for both Brown and Sharpe and Sellers standard,* (*d*) *the circular pitch.*

(*a*) Module pitch $= \dfrac{1}{\text{diametral pitch}} = \frac{1}{2}$.

(*b*) Diameter of pitch circle $= \frac{40}{2} = $ **20 in.**

(*c*) Brown and Sharpe standard—**Addendum = 0·5 in.**

Dedendum = 1·157 × ½ = 0·579 in.

Sellers standard—**Addendum = 0·942 × ½ = 0·471 in.**

Dedendum = 1·099 × ½ = 0·549 in.

(*d*) Circular pitch $= \dfrac{\text{circumference of pitch circle}}{\text{number of teeth}} = \dfrac{20 \times \pi}{40} = \dfrac{\pi}{2} = $ **1·571 in.**

Example 2. *A toothed wheel has a circular pitch of* 0·393 *in. and* 100 *teeth. Calculate* (*a*) *the diameter of the pitch circle,* (*b*) *the diametral and module pitch.*

(*a*) Diameter of pitch circle $= \dfrac{100 \times 0{\cdot}393}{\pi} = 12{\cdot}5$ in.

(*b*) Diametral pitch $= \dfrac{100}{12{\cdot}5} = 8$.

Module pitch $= \frac{1}{8} = 0{\cdot}125$.

Example 3. *A toothed wheel has a metric module pitch of* 3, *and* 50 *teeth. Calculate the diameter of the pitch circle.*

$$\text{Metric module pitch} = 3 = \frac{\text{diameter of wheel in mm.}}{\text{number of teeth}}.$$

Diameter of pitch circle $= 3 \times 50 = 150$ mm.

Stepped, helical and double helical teeth. For smooth running, it is found that the smaller the tooth pitch the better, and a system of *stepping* the teeth as shown in Fig. 204 has been adopted. This system led to the use of an infinite number of steps, thus forming a curve across the wheel face and producing **helical teeth** (Fig. 205).

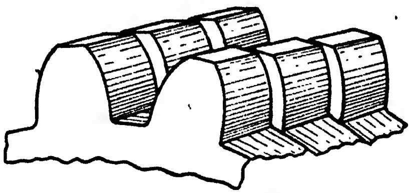


FIG. 204. Stepped teeth.

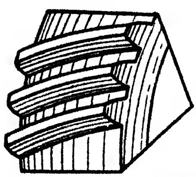


FIG. 205. Helical teeth.

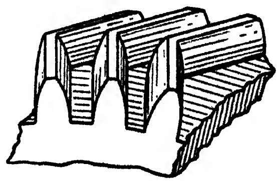


FIG. 206. Double helical teeth.

The disadvantage associated with the use of this type of tooth was the existence of an end thrust in one direction only. To balance this, by producing an end thrust in the opposite direction the **double helical** form of tooth was produced (Fig. 206). This form of wheel

is extensively used for turbine reduction gears in addition to the general purposes of power transmission; a certain amount of axial freedom must be allowed in the installation, unless effective thrust bearings are fitted to the ends of the transmission shaft. The single reduction type is manufactured to produce a reduction of 8 to 1 in shaft speeds. For higher ratios up to 60 to 1, two steps and a layshaft are required.

Bevel wheels. When it is required to change the direction of power transmission, **bevel or mitre wheels** are employed. These wheels are frustra of cones, and the essential condition for their operation is that the cones, of which they form part, shall have a *common*

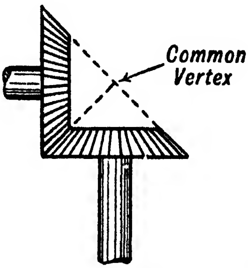


Fig. 207. Pair of mitre wheels.

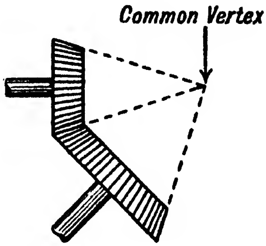


Fig. 208. Pair of bevel wheels.

vertex. When the shaft axes are at right angles and the angle at the base of the cone is 45°, the wheels are called **mitre wheels** (Fig. 207). If the shafts are at an angle other than 90°, the wheels are referred to as **bevel wheels** (Fig. 208).

Epicyclic gears. Up to the present, the axes of the gear wheels considered have been rigidly attached to the frame of the machine and the wheel has been confined to rotation on a fixed axis. It sometimes happens that the bar supporting the axis of a wheel is also capable of revolution about another axis, and this constitutes what is known as an **epicyclic gear**. A simple form of this type of gear is shown in Fig. 209 in which the wheel B can rotate upon the axis A and at the same time the bar D supporting the axis of the wheel C can rotate about A. Thus the rotation of C is a combined rotation made up of the rotation of D and that of B. The determination of the velocity ratio of this type of mechanism is outside the scope of

this book, but the student should realise that it does not conform to the theory for wheels having a fixed axis. These gears allow of

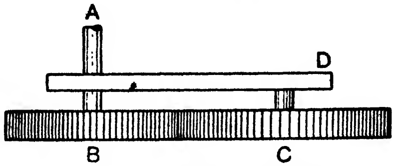


FIG. 209. Epicyclic gears.

speed reductions between shafts varying from 4 to 1 to as much as 500 to 1.

Advantages derived from the use of an involute profile. The adoption of an involute profile for wheel teeth possesses the following advantages over the use of epicycloidal and hypocycloidal curves:

(1) The velocity ratio between the wheels is unaltered as the depth of mesh is increased or decreased.

(2) The profile is one continuous curve from the base circle of the tooth to its crest.

(3) The width of the tooth steadily increases from the crest towards the base circle, thus providing additional strength to meet the increase of turning moment producing bending towards the root.

Toothed wheel trains. Direction of motion of the follower. In a pair of toothed wheels the follower rotates in the opposite direction to that of the driver (Fig. 210 (*a*)); when it is required for the follower to rotate in the same direction as the driver, an intermediate wheel is fitted to gear freely with driver and follower. This wheel, known as an **idle wheel** or **hunting cog,** has no effect on the velocity ratio but serves to reverse the normal direction of rotation of the follower (Fig. 210 (*b*)).

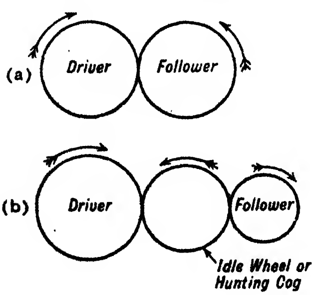


FIG. 210.

Simple and compound trains. When a wheel A drives a follower B, either directly or through an idle wheel, the arrangement is called a **simple train** (Fig. 211), and the

$$\textbf{velocity ratio} = \frac{\textbf{speed of driver}}{\textbf{speed of follower}} = \frac{\textbf{number of teeth on follower}}{\textbf{number of teeth on driver}}.$$

In a **compound train** (Fig. 212) the wheel A on the driving shaft drives a wheel B on a **lay shaft** to which is also keyed a wheel C, which acts as the second driver and drives the wheel D on the following shaft. The wheels B and C, keyed to the lay shaft, thus have the same speed of rotation.

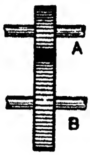


FIG. 211. Simple train or single purchase.

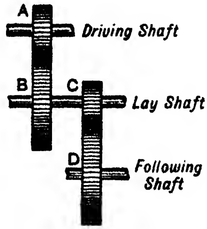


FIG. 212. Compound train or double purchase.

Consider the wheels A and B, which constitute a simple train.

$$\textbf{Speed of B} = \frac{\text{speed of A} \times \text{number of teeth on A}}{\text{number of teeth on B}},$$

but speed of B = speed of C, so that, for the train C and D,

$$\textbf{speed of D} = \frac{\text{speed of C} \times \text{number of teeth on C}}{\text{number of teeth on D}};$$

or, replacing the speed of C by the speed of B,

$$\text{speed of D} = \frac{\text{speed of A} \times \text{No. of teeth on A}}{\text{No. of teeth on B}} \times \frac{\text{No. of teeth on C}}{\text{No. of teeth on D}},$$

and

$$\textbf{velocity ratio} = \frac{\textbf{speed of A}}{\textbf{speed of D}} = \frac{\textbf{product of teeth on followers}}{\textbf{product of teeth on drivers}}.$$

This process of compounding may be extended to any extent in order to produce a given velocity ratio, a ratio which is always expressed by the above formula.

NOTE.—*The use of simple and compound trains in winches and hauling gears is very common, and they are generally referred to as* **single purchase** *and* **double purchase** *respectively.*

Example 1. *In a reduction gear a wheel of 50 teeth drives another of 240 teeth in simple train. Find the velocity ratio and the reduction ratio.*

$$\text{Velocity ratio} = \frac{\text{speed of driver}}{\text{speed of follower}} = \frac{240}{50} = 4{\cdot}8.$$

$$\text{Reduction ratio} = \frac{\text{speed of follower}}{\text{speed of driver}} = \frac{50}{240} = \frac{5}{24}.$$

Example 2. *Find the velocity ratio for a compound train if the driving shaft spur wheel has 50 teeth, the driver and follower on the lay shaft 40 and 120 teeth respectively and the follower shaft wheel 100 teeth.*

$$\text{Velocity ratio} = \frac{\text{product of teeth on followers}}{\text{product of teeth on drivers}} = \frac{120 \times 100}{50 \times 40} = 6.$$

Example 3. *A double purchase hauling winch* (Fig. 187) *has a wheel train as follows: driving shaft 30 teeth, lay shaft driver 40 teeth, follower 60 teeth, hauling shaft 120 teeth. If the driving shaft is driven by a hand lever 24 in. in length and the hauling shaft is keyed to a rope drum 8 in. in diameter, find*

(*a*) *the velocity ratio of the gears,*
(*b*) *the velocity ratio of the winch,*
(*c*) *the load which could be raised with an effort of 25 lb. if the efficiency is 60%.*

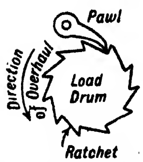


FIG. 213. Pawl and ratchet.

(*a*) $\text{V.R. of gears} = \frac{120 \times 60}{40 \times 30} = 6.$

(*b*) $\text{V.R. of winch} = 6 \times \frac{\pi \times 24 \times 2}{\pi \times 8} = 36.$

(*c*) $\text{Theoretical load} = 36 \times 25 \text{ lb.}$

$$\text{Actual load} = \frac{36 \times 25 \times 60}{100} = 540 \text{ lb.}$$

NOTE.—*This type of machine is fitted with a pawl and ratchet, or some similar device, to prevent overhauling* (Fig. 213).

The makers of the crab winch shown in Fig. 187, p. 151, Messrs. H. Morris, Ltd., Loughborough, also make a feature of a hand operated band brake to meet this necessity and to control the speed of load descent.

EXPT. 19. A jib crane fitted with single and double purchase winch.

OBJECTS. (1) *To find the velocity ratio of the winch in both single and double purchase gear.*

(2) *To determine the effort required, effect of friction, mechanical advantage and efficiency, in single and double purchase, for various loads.*

(3) *To find the laws of the machine for both single and double purchase operation.*

(4) *To determine the forces, and the nature of the forces, in the jib and each of the ties.*

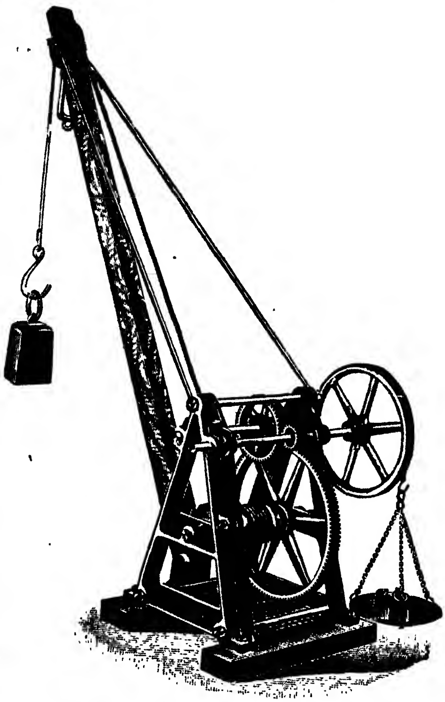


FIG. 214. Experimental jib crane. (*Messrs. G. Cussons, Ltd.*)

APPARATUS. The effort is applied by placing weights in the scale pan (Fig. 214), the effort wheel being 10 in. in effective diameter. With the single purchase the pinion on the effort wheel shaft turns the large wheel on the drum shaft, and the drum is $2\frac{1}{2}$ in. in diameter. By sliding the effort shaft axially it is possible to obtain a double purchase by driving the drum shaft through a lay shaft. The load rope is carried from the drum over the pulley at the top of the jib.

OBSERVATIONS AND DERIVED RESULTS. **Velocity ratio.**

(*a*) *Single purchase.* The V.R. is the product of two ratios :

$$(1)\quad \frac{\text{No. of teeth on following wheel}}{\text{No. of teeth on driving wheel}}=\frac{120}{20},$$

$$(2)\quad \frac{\text{circumference of effort wheel}}{\text{circumference of load drum}}=\frac{10\pi}{2\frac{1}{2}\pi},$$

$$\text{thus V.R.}=\frac{120}{20}\times\frac{10\pi}{2\frac{1}{2}\pi}=6\times4=24.$$

(*b*) *Double purchase.* The V.R. is again the product of two ratios.

(1) $\dfrac{\text{No. of teeth on 1st following wheel} \times \text{No. of teeth on 2nd following wheel}}{\text{No. of teeth on 1st driving wheel} \times \text{No. of teeth on 2nd driving wheel}} = \dfrac{50 \times 120}{20 \times 20}$,

(2) $\dfrac{\text{circumference of effort wheel}}{\text{circumference of load drum}} = \dfrac{10\pi}{2\frac{1}{2}\pi}$,

thus $\text{V.R.} = \dfrac{50 \times 120}{20 \times 20} \times \dfrac{10\pi}{2\frac{1}{2}\pi} = 15 \times 4 = 60.$

Single purchase. V.R. = 24.

Load, lb.	Effort, lb.	M.A., $\frac{L}{E}$	Work input, ft. lb.	Work output, ft. lb.	Effect of friction, lb.	Efficiency, %
20	1·24	16·11	29·76	20·0	9·76	67·1
40	2·31	17·32	55·64	40·0	15·64	71·9
60	3·40	17·65	81·6	60·0	21·60	73·5
80	4·50	17·78	108·0	80·0	28·0	74·1
100	5·60	17·86	134·4	100·0	34·4	74·4
120	6·70	17·91	160·8	120·0	40·8	74·7

Double purchase. V.R. = 60.

20	0·6	33·33	36·0	20·0	16·0	55·6
40	1·05	38·09	63·0	40·0	23·0	63·5
60	1·49	40·26	89·4	60·0	29·4	67·1
80	1·89	42·30	113·4	80·0	33·4	70·5
100	2·31	43·25	138·6	100·0	38·6	72·1
120	2·75	43·63	165·0	120·0	45·0	72·7

GRAPHS.

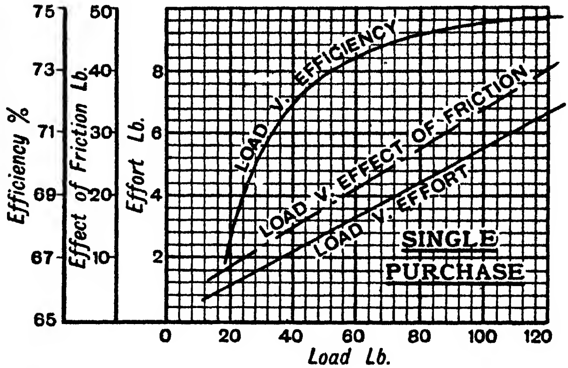


FIG. 215.

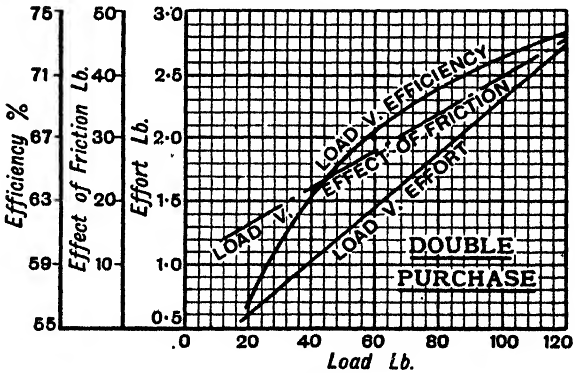


FIG. 216.

Laws of the machine: General law of a machine,

$$L = mE + F.$$

Single purchase at A, $L = 100$ lb., $E = 5{\cdot}58$ lb. } from the graph.
at B, $L = 40$ lb., $E = 2{\cdot}25$ lb. }

$$\begin{array}{ll} \text{(A)} & 100 = m \times 5{\cdot}58 + F \\ \text{(B)} & \underline{\ 40 = m \times 2{\cdot}25 + F\ } \end{array}$$

Eliminating F $\qquad 60 = m \times 3{\cdot}33 + 0$, $\mathbf{m = 18{\cdot}0}$.

Substituting in A, $\qquad 100 = 18 \times 5{\cdot}58 + F$, $\mathbf{F = -0{\cdot}44}$.

Law (1), L = 18E − 0·44.

Double purchase at C, $L=100$ lb., $E=2{\cdot}3$ lb.
at D, $L=40$ lb., $E=1{\cdot}0$ lb.

$$\text{(C)}\quad 100=m\times 2{\cdot}3+F$$
$$\text{(D)}\quad 40=m\times 1{\cdot}0+F$$

$$60=\ m\times 1{\cdot}3\ \ +0,\ \mathbf{m=46{\cdot}15.}$$

Substituting in (C) $\qquad 100=46{\cdot}15\times 2{\cdot}3+F,\ \mathbf{F=-6{\cdot}17.}$

Law (2) L = 46·15E − 6·17.

Determination of the forces in jib and ties. In this determination, it is possible to consider the two ties replaced by a single equivalent tie carrying a force which is the resultant of those in the two ties, and will be in the plane of the jib. The jib head axle is then the point of application of four forces, that is, (A) the pull in the rope, (B) the load, (C) the thrust in the jib, (D) the pull in the equivalent single tie, and these forces may be determined by graphical representation, since their directions are all known and the magnitudes of A and B are, each, 120 lb.

In vector diagram (A) (Fig. 217), the resultant thrust, A_1B_1, due to the rope and load pulls, on the jib head is found, and with this as

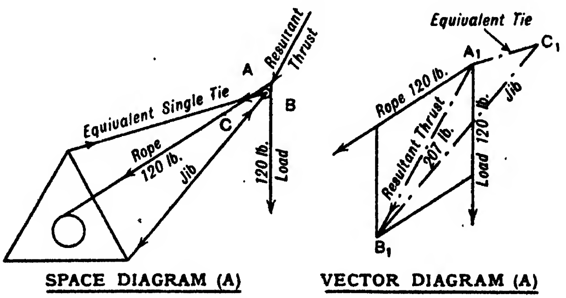


FIG. 217.

a known force the triangle of forces for the jib head $A_1B_1C_1$ is drawn. In this triangle B_1C_1 represents the force in the jib and A_1C_1 that in the equivalent single tie.

Vector diagram (B) is drawn to a larger scale with the object of resolving the force in the equivalent single tie into its two component forces, that is, the forces in the ties. D_1E_1 represents the force in

the equivalent single tie, and D_1F_1 parallel to DF, and E_1F_1 parallel to EF, represent the forces in the individual ties (Fig. 218).

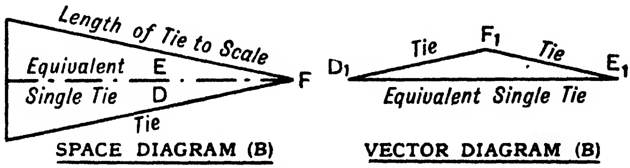


FIG. 218.

Member or Result	Force	Nature
Jib - - - - -	267 lb.	Strut
Tie - - - - -	36 lb.	Tie
Equivalent single tie - -	75 lb.	Tie
Resultant thrust on jib head -	210 lb.	Thrust

Screw cutting. Screw cutting, in a lathe, provides a good example of the use of gear wheels to produce a definite velocity ratio between two shafts. In the mechanism of a sliding surfacing and screw cutting lathe, the mandrel carries the rotating work. When screw cutting, the leading screw is operated through a train of wheels (Figs. 211 or 212), to produce a travel of the saddle along its bed. If the wheels A and B (Fig. 211) had equal numbers of teeth the saddle would move along the bed a distance equal to the pitch of the leading screw for every revolution of the mandrel, or work. Leading screws are cut to large pitches, often $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{1}{5}$ inch, and in order to produce a screw on the rotating work of a lesser or greater pitch than that of the leading screw a train of wheels must be employed to connect mandrel and leading screw and give the required velocity ratio between them. With a right-handed leading screw, the mandrel and the leading screw must rotate in like or opposite directions according as the thread or screw to be cut is right- or left-handed.

Suppose a lathe has a *leading screw of* 4 *threads per inch*, and it is required to cut a screw of 10 *threads per inch* on the rotating work. If the wheels A and B (Fig. 211) had an equal number of teeth the

saddle would move $\frac{1}{4}$ in., that is the pitch of the leading screw, per revolution of the work. In order to cut a screw of $\frac{1}{10}$ in. pitch, the saddle must only move $\frac{1}{10}$ in. per revolution of the work ; so that a *velocity ratio of* $\frac{1}{4} \div \frac{1}{10} = 2\frac{1}{2}$ must be established, by gearing, between the mandrel and leading screw.

The problem now resolves itself into finding two wheels, A and B, which will produce the required velocity ratio of $2\frac{1}{2}$.

$$\text{Velocity ratio} = \frac{\text{speed of driver, or mandrel}}{\text{speed of follower, or leading screw}}$$

$$= \frac{\text{number of threads per in. to be cut}}{\text{number of threads per in. on leading screw}}$$

$$= \frac{\text{number of teeth on follower}}{\text{number of teeth on driver}} = 2\tfrac{1}{2},$$

so that any pair of wheels in which the number of teeth on B is $2\frac{1}{2}$ times the number on A will cut the required screw. With this simple train and when cutting a right-handed screw it will be necessary to employ an idle wheel. Examples, 20 and 50 ; 40 and 100 ; 30 and 75.

Limitations to selection of wheels. A lathe is generally supplied by the manufacturers with a range of **change wheels** as they are called, and the selection of suitable wheels has to be made from this range. Further, the mandrel and leading screw are limited in the size of wheel they can accommodate, and in order to cut screws with a larger number of threads per inch than 10 or 12, it is necessary to employ a compound train to obtain the required velocity ratio (Fig. 212).

The velocity ratio is then $\dfrac{\text{product of teeth on followers}}{\text{product of teeth on drivers}}$,

that is,

$$\frac{\text{number of threads per in. to be cut}}{\text{number of threads per in. on leading screw}} = \frac{\text{product of teeth on followers}}{\text{product of teeth on drivers}}.$$

Example 1. *Select a train of wheels to cut screw threads of 7 and 36 threads per inch on a lathe with a leading screw of $\frac{1}{4}$ in. pitch.*

The range of change wheels will possibly be of 20, 25, 30, 35, 40, 45, 50, 55, 60, 70, 75, 80, 90, 100, 120 teeth with the 20 and 30 wheels in

duplicate. The mandrel probably will not accommodate a wheel with more than 40 teeth.

(*a*) 7 threads per inch:

$$\text{Velocity ratio} = \frac{7}{4} = \frac{\text{followers}}{\text{drivers}} = \frac{70}{40}.$$

A 40 teeth wheel driving a 70 teeth wheel with any intermediate wheel.

(*b*) 36 threads per inch:

$$\text{Velocity ratio} = \frac{36}{4} = 9 = \frac{\text{followers}}{\text{drivers}};$$

$$9 = \frac{9000}{1000} = \frac{90 \times 100}{20 \times 50}.$$

A compound train, drivers 20 and 50 teeth, followers 90 and 100 teeth.

This train can be varied to any extent providing the ratio between followers and drivers is maintained at 9.

Example 2. *Select a train of wheels to cut a worm of $1\frac{1}{2}$ in. pitch on a lathe of leading screw 2 threads per inch.*

$$\text{Velocity ratio} = \frac{2}{3} \div 2 = \frac{1}{3} = \frac{\text{pitch of leading screw}}{\text{pitch of worm}};$$

$$\frac{1}{3} = \frac{\text{followers}}{\text{drivers}} = \frac{1000}{3000} = \frac{20 \times 50}{60 \times 50} = \frac{20 \times 25}{30 \times 50}.$$

A compound train, drivers 30 and 50 teeth, followers 20 and 25 teeth.

NOTE.—It is often necessary to adapt a lathe specially when worms of long pitch have to be cut. This is because the numbers of the teeth on the followers must be small to give the required velocity ratio. The number of teeth on a wheel should not be less than 12 and preferably not less than 16. It is for this reason that worms are often cut in a milling or gear-cutting machine.

Power transmitted by toothed wheels. The power transmitted by a pair of toothed wheels is equivalent to that transmitted by a pulley of diameter equal to the pitch circle diameter and receiving a force tangential to the pitch circle equal to the inter tooth pressure (Fig. 219).

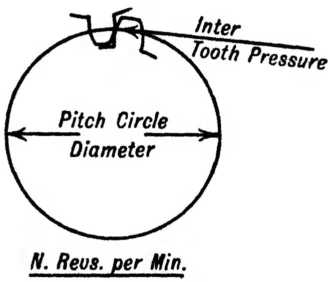


FIG. 219.

Let the diameter of pitch circle be D in. and the inter-tooth pressure P lb.

Then work done per revolution

$$=\pi \mathbf{D} \times \mathbf{P} \text{ in. lb.}$$

At N revs. per min.

$$\text{Work done per min.} = \pi DP \times N \text{ in. lb.} = \frac{\pi . D . P . N}{12} \text{ ft. lb.}$$

$$\text{H.P.} = \frac{\pi . \mathbf{D} . \mathbf{P} . \mathbf{N}}{33000 \times 12}.$$

Example 1. *A toothed wheel of diametral pitch* 2 *and* 45 *teeth makes* 80 *revs. per min. Calculate the H.P. it transmits if the inter-tooth pressure is* 410 *lb. and the transmission efficiency is* 95%.

Diameter of pitch circle $= \frac{45}{2} = 22{\cdot}5$ in.

Work done per revolution $= 22{\cdot}5 \times \pi \times 410$ in. lb.

Work done per minute $= 80 \times 22{\cdot}5 \times \pi \times 410$ in. lb.

$$\text{Theoretical H.P.} = \frac{80 \times 22{\cdot}5 \times \pi \times 410}{12 \times 33000} = 5{\cdot}86.$$

$$\textbf{Actual H.P.} = \tfrac{95}{100} \times 5{\cdot}86 = 5{\cdot}57.$$

Example 2. *A toothed wheel is to transmit* 10 *H.P. and have a diametral pitch of* $2\frac{1}{2}$. *Calculate the number of teeth if the pressure between teeth is not to exceed* 700 *lb. and the speed is to be* 100 *revs. per min.*

Let T be the number of teeth, then

$$\text{pitch circle diameter} = \frac{T}{2\frac{1}{2}} \text{ in. or } \frac{T}{30} \text{ ft.}$$

$$\text{H.P. or } \frac{\pi . D . P . N}{33000 \times 12} = \frac{\pi \times T}{30} \times \frac{700}{1} \times \frac{100}{33000}.$$

$$\text{H.P.} = \frac{2T}{9};$$

then $$\frac{2T}{9} = 10. \quad \mathbf{T = 45 \text{ teeth.}}$$

Example 3. *A compound train of wheels in which the drivers are of* 30 *and* 40 *teeth, and the followers* 80 *and* 100 *teeth, is to transmit* 7 *H.P.*

at 30 r.p.m. on the follower shaft. Find (a) the velocity ratio, (b) the inter-tooth pressure on the follower shaft wheel if the diametral pitch is 4.

$$(a)\ \text{Velocity ratio} = \frac{80 \times 100}{30 \times 40} = \frac{20}{3} = 6\tfrac{2}{3}.$$

$$(b)\ \text{Pitch circle diameter} = \tfrac{100}{4} = 25 \text{ in.}$$

$$\text{H.P.} = \frac{\pi \times 25}{12} \times \frac{30}{1} \times \frac{P}{33000} = 0{\cdot}00595P.$$

$$0{\cdot}00595P = 7.$$

$$P = \frac{7}{0{\cdot}00595} = 1177 \text{ lb.}$$

Belt drives between two pulleys not in the same plane. A belt can be made to drive a pulley directly when its axis is at an angle to that

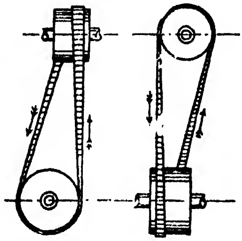


Fig. 220.

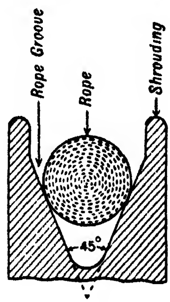


Fig. 221.

of the driving pulley. Fig. 220 shows two shafts at right angles and not in the same plane, connected and power transmitted by a belt. In order that this type of drive may be possible the point of contact of the belt as it leaves each pulley must be in the same plane as the other pulley on to which the belt is about to move.

Rope drives. For heavy transmission, ropes are frequently employed in place of belts. A rope is less flexible than a belt and on this account a small working load and a pulley diameter of not less than fourteen times the rope diameter are necessary to ensure a long working life, which may be as much as ten years. The ropes are made of hemp, cotton or a composition of rubber and fabric, and the pulleys are grooved, as shown in Fig. 221, to receive them. These grooves are designed to give better adhesion and the minimum of

slipping, and the angle of the grooves is generally 45°. A pulley may have many grooves to accommodate several ropes. Velocity ratios of 7 to 1 may be possible with rope drives, and a rope speed of 5000 ft. per min. gives the maximum H.P. Ropes made of a composition of rubber and fabric, for shaft centres up to 12 ft. apart, may be employed to transmit from $\frac{1}{2}$ to 2000 H.P. with practically no slip.

Chain drives. Chains are becoming increasingly popular as they have many advantages. Some of these are:

(*a*) high speeds up to 1500 r.p.m. are possible ;
(*b*) high efficiency of transmission, direct or reversed ;
(*c*) no slipping and therefore a fixed velocity ratio ;
(*d*) a wide range of power is possible whether on long or short centres ;
(*e*) heat, cold and moisture have no effect on the operation.

The tendency for noisy running has been overcome in the modern type of chain drive and velocity ratios may be as high as 15 to 1 for a single drive.

Use of guide and jockey pulleys. Small pulleys, known as guide pulleys, are often used to alter the direction of a belt, rope or chain in the transmission of power. In Fig. 222 a pair of guide pulleys

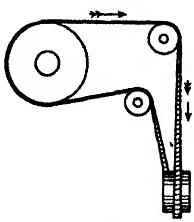


FIG. 222.

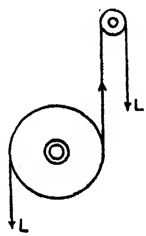


FIG. 223. Use of a guide pulley to produce a couple.

are used to direct a belt which is driving a follower not in the plane of the driver. Many similar arrangements can be devised to effect a power drive between pulleys which are so placed that a direct drive is impossible. Fig. 223 shows a method commonly employed to produce a couple on a torsion testing machine. Actually the

friction at the guide pulley defeats, to a small extent, the object of obtaining a perfect couple, but for all practical purposes the method suffices. Guide pulleys may also be used to support the slack in the case when the driver and driven pulleys are a long distance apart, as in teledynamic transmission. Jockey pulleys are mainly used to take up the slack of a belt or rope by being pressed continuously against the slack side. The pressure may be effected by means of a weighted arm as in Fig. 224 or by a spring. In every case a larger arc of contact is assured between belt and pulleys. New belts are very liable to stretch and the use of a jockey pulley may obviate the necessity of shortening the belt.

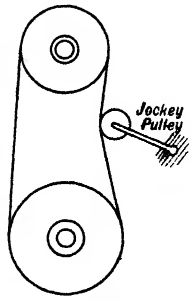


FIG. 224.

EXERCISES ON CHAPTER VIII

1. A belt pulley 20 in. in diameter is driven at 220 revs. per min. by an effective pull of 94 lb. Find the H.P. transmitted (*a*) without slip, (*b*) with 3% slip.

2. What must be the speed of rotation of a pulley 24 in. in diameter in order to transmit 4 H.P. when the effective belt pull is 110 lb. ?

3. The tensions on the tight and slack sides of a belt drive are 110 lb. and 32 lb. respectively. Find the H.P. transmitted if one pulley is 30 in. in diameter and revolves at 100 r.p.m. Slip is equal to 4%.

4. A belt drive is to transmit 20 H.P. at 110 revs. per min. Find the tensions on the tight and slack sides if the tight side tension is $2\frac{1}{2}$ times the slack side tension and the driving pulley is 4 ft. in diameter.

5. Calculate the width of belt required in order to transmit 10 H.P. through a pulley 32 in. in diameter making 170 revs. per min. The tight side tension is $2\frac{1}{2}$ times the slack side tension and the pull in the belt is not to exceed 80 lb. per inch of width.

6. A pulley 4 ft. 3 in. in diameter is driven by friction contact at 80 revs. per min. by another pulley 2 ft. 4 in. in diameter. Find the speed of the driver if the slip is equal to 10%.

7. A pulley A, 24 in. in diameter, drives another pulley B 15 in. in diameter at 400 revs. per min. with a slip of 3%. Find the velocity ratio between A and B, and the speed of A.

8. In an ordinary countershaft drive a wheel A of diameter 24 in., speed 220 revs. per min., drives a countershaft pulley B of 10 in. diameter. B is coupled to a wheel C, 20 in. in diameter, which drives the machine shaft through a wheel D, 8 in. in diameter. Find the speed of D and the velocity ratio between shaft and machine.

9. A velocity ratio of 10 is to be established between a main shaft pulley 24 in. in diameter and a machine shaft pulley 8 in. in diameter. Find suitable sizes for the countershaft pulleys in this drive.

10. A double countershaft drive is installed between a main shaft making 200 revs. per min. and a machine shaft. The pulley diameters are as follows: main shaft 20 in.; first countershaft, driver 16 in., follower 8 in.; second countershaft, driver 12 in., follower 7 in.; machine shaft pulley 6 in. diameter. Find the speed of the machine shaft, allowing for a total slip of 4%, and the theoretical velocity ratio of the drive.

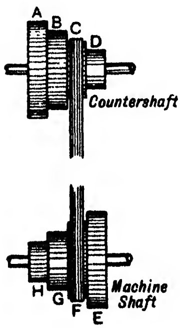


FIG. 225.

11. A pair of speed cones for variable speed driving are shown in Fig. 225. The belt may be transferred to each of the steps A, B, C, or D at will. Find the range of speeds available on the machine shaft if the countershaft runs at 420 revs. per min. Diameters of pulleys, A = E = 14 in.; B = F = $11\frac{1}{2}$ in.; C = G = 9 in.; D = H = $6\frac{1}{2}$ in.

12. State in general terms the conditions which must be satisfied in order that two toothed wheels may gear together. What types of curves are selected for wheel teeth profiles, and what are the standards of dimensions generally used?

13. Prepare a dimensioned sketch of two wheel teeth in gear clearly indicating the essential circles. Mark on the sketch (*a*) the circular pitch, (*b*) the addendum, (*c*) the dedendum, (*d*) the angle of obliquity.

14. A toothed wheel of 60 teeth has a diametral pitch of 3. Calculate the following: (*a*) module pitch, (*b*) circular pitch, (*c*) addendum, (*d*) dedendum, using the Brown and Sharpe standard.

15. A wheel A of 50 teeth, diametral pitch $2\frac{1}{2}$, drives a wheel B at 80 revs. per min. If the speed of A is 240 revs. per min., find the number of teeth on B and the pitch circle diameters of A and B.

16. A reduction gear is formed of two wheels: A of 54 teeth, speed 600 revs. per min., B of 270 teeth. Calculate the speed of B and the pitch circle diameters of A and B if the diametral pitch is $1\frac{1}{2}$.

17. A pinion has 20 teeth of circular pitch $1\frac{1}{4}$ in. Find the length of rack, and number of teeth per foot in a rack which will cater for 3 revolutions of the pinion.

18. A spring balance needle is operated by a rotating pinion of 12 teeth, the circular pitch of which is 0·175 in. If the springs extend

0·84 in. under a load of 400 lb., find the angle through which the needle will rotate per 100 lb. of load if the spring movement is transmitted direct to the rack operating the pinion.

19. In a compound reduction gear the driving wheel on the main shaft has 30 teeth. It drives a lay shaft fitted with coupled follower and driver of respectively 90 and 20 teeth, which drive the machine shaft wheel with 100 teeth. Calculate the velocity ratio of the train and the speed of the machine shaft wheel if the driving shaft makes 600 revs. per min.

20. A gear, for increasing the speed in a transmission, consists of the following spur wheels; A, a driving wheel making 10 revs. per min. and having 120 teeth, which drives a lay shaft with coupled follower and driver B and C of respectively 40 and 160 teeth. C drives a second lay shaft with coupled follower and driver D and E, of respectively 50 and 100 teeth; and E drives a machine shaft wheel F of 60 teeth. Find the overall velocity ratio and the speed of F in revs. per min.

21. Sketch a pair of mitre and a pair of bevel wheels. What is the essential condition that these may gear correctly?

22. What is meant by helical and double helical teeth? State the advantages and disadvantages attached to the employment of toothed wheels with these teeth.

23. How would you recognize an epicyclic gear? Sketch some example of such a gear with which you are familiar.

24. Obtain suitable trains of wheels to cut threads of the following pitches, $\frac{1}{10}$ in., $\frac{1}{30}$ in., and $\frac{3}{4}$ in., on a lathe of leading screw 4 threads per inch. Range of wheels: 2 of 20 teeth, 2 of 40 teeth, 30, 50, 60, 70, 75, 80, 90, 100, and 120.

25. A crab winch has an effort arm 15 in. in length and a hauling drum 5 in. in diameter. In single purchase the gears are 20 driving 120 teeth and in double purchase 20 driving 60 and 20 driving 120 teeth. Calculate the velocity ratio in single and double purchase and find the effort required to raise 400 lb. in each case if the respective efficiencies are 75 and 60%.

26. The diagram (Fig. 226) shows the back gear arrangement for a lathe. Calculate the range of speeds available, in direct and back gear, if the countershaft runs at 250 revs. per min. and there is a similar cone pulley on the countershaft.

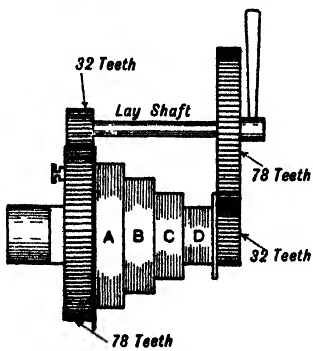


FIG. 226. Diameters, A = 10 in., B = 8 in., C = 6 in., D = 4 in.

27. The diagram (Fig. 227) shows the arrangement of a change gear box in which the driving shaft wheel A drives the lay shaft through the wheel B. The end of the tail shaft is splined to allow the wheels D and

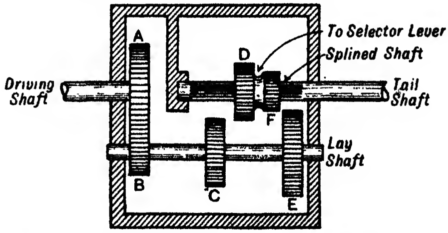


FIG. 227. A = 90 teeth, B = 50 teeth, C = 70 teeth, D = 60 teeth, E = 90 teeth, F = 40 teeth.

F, which are coupled, to engage with C, or E, wheels keyed to the lay shaft. If the driving shaft has a speed of 800 revs. per min., find the two speeds available for the tail shaft and the velocity ratio offered by the gear box.

28. A spur wheel of 54 teeth, diametral pitch $1\frac{1}{4}$, transmits power through an inter-tooth pressure of 480 lb. Find the H.P. transmitted if the speed is 170 revs. per min.

29. Find the pressure between the teeth of two spur wheels, one of which rotates at 150 revs. per min. and is cut with 70 teeth of circular pitch 0·75 in., if the H.P. transmitted is 3·8.

30. A spur wheel pinion of 15 teeth, circular pitch $1\frac{1}{2}$ in., is employed to drive a machine table of weight 730 lb., along a horizontal slide, by means of a rack attached to the table. Find the H.P. required if the coefficient of friction is estimated to be 0·1 and the table is to move at a speed of 120 ft. per min. Take the overall efficiency as 75%. What will be the speed of rotation of the pinion ?

31. A worm-wheel of 120 teeth is driven by a worm rotating at 80 revs. per min. The worm-wheel is coupled axially to a wheel of 70 teeth which drives a shaft through a pinion of 30 teeth. Find the pressure between the pinion teeth if the worm receives 1·2 H.P. and the overall efficiency is 35%. Diametral pitch of pinion teeth = 2.

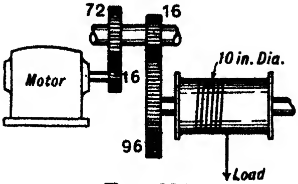


FIG. 228.

32. A hoist driven by a D.C. motor on a 240 volt supply is shown diagrammatically in Fig. 228. The numbers of teeth on the gear wheels of the double reduction gear are marked alongside the wheels. If the motor speed is 800 r.p.m., at what rate is the load being raised ? Determine in amperes the

current required to raise a load of 300 lb. at this speed if the efficiency of the hoist is 48% and that of the motor 85%. (See page 284.)

33. Show by means of sketches how belting may be used to transmit power between shafts in the following cases: (*a*) shafts parallel and motion in the same sense, (*b*) shafts parallel, motion in opposite senses, (*c*) shafts at right angles and not in the same plane. To what use are jockey pulleys put?

34. A double reduction gear transmitting 700 H.P. by means of double helical gearing for a ball mill crusher reduces the speed from 730 to about 20·5 r.p.m. If the numbers of teeth on the drivers are 26 and 21, find suitable numbers of teeth for the followers.

35. Two pulleys are connected by 45 in. wide camel hair belting and 1000 H.P. is being transmitted. The driver is 5 ft. in diameter and the follower 4 ft. diameter. Find the tension in the belt per inch width on the tight side for a belt speed of 2700 ft. per min. Take the tension on the tight side to be $2\frac{1}{2}$ times that on the slack side. Also find the speed of the follower, in each case allowing 1% for slip and creep.

36. Prove the rule for the speeds of shafts connected together by a belt and pulleys. A belt connects a stepped speed cone on a countershaft to a similar stepped speed cone of a lathe. The largest and smallest diameters of the speed cones are 14 in. and 8 in. Determine the highest and lowest speeds of the lathe mandrel when the countershaft is running at a speed of 150 r.p.m. (U.L.C.I.)

37. What do you understand by the "velocity ratio" of a lifting machine?

The length of the handle of a winch is 16 in. and the diameter of the barrel is 8 in. The pinion on the handle axis has 16 teeth and the spur wheel on the barrel axis has 90 teeth. Find the velocity ratio of the winch. If the winch were without friction, what force at the handle would lift 675 lb.? In an experiment a force of 40 lb. at the handle was necessary to lift 675 lb. Account for the difference between the actual force and that necessary in the perfect machine. (U.L.C.I.)

38. Show that if two toothed wheels are in gear their speeds are inversely proportional to their diameters or to their numbers of teeth.

Three spur wheels, A, B, and C, on parallel shafts, are in gear. A has 12 teeth, B 40 teeth, and C 48 teeth. Find the speed of C when A makes 80 r.p.m. What is the purpose of the wheel B? (U.L.C.I.)

CHAPTER IX

CENTRE OF AREA, CENTROID AND CENTRE OF GRAVITY

It is frequently necessary in the simplification of problems, to consider the mass, or weight, as concentrated at a certain point, either within or outside the body. Similarly the area of the section of a beam, or any area, may be considered to be concentrated at a point.

The point at which the whole of the mass, or weight, of a body may be considered concentrated is known as the **centre of gravity,** or **centre of mass,** or **centre of inertia** of the body, and the **centre of area** is the point at which the area may be considered concentrated when an area is considered. The term **centroid** is applied to homogeneous solids to indicate the mean centre, which in this case coincides with the centre of gravity. The centre of gravity is a fixed point in relation to the body and its position is unaltered when the body is rotated or moved. Similarly, however the body be placed, the centre of gravity is always the point through which the weight or resultant gravitational force may be considered as acting.

Centre of gravity, or centroid, of symmetrical plane laminae. A **lamina** is the name given to a thin plate or sheet of material of uniform thickness and density. Thus the centre of gravity of a lamina corresponds with the centre of area of the figure forming its shape.

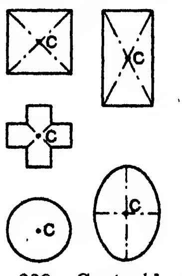


Fig. 229. Centroids of symmetrical figures.

Laminae of square, rectangular, circular or elliptical shape (Fig. 229), have centroids at the points of intersection of symmetrical axes, such as the intersection of the diagonals, or of the major and minor axes, in the case of an ellipse. Figures of irregular shape, without well defined axes of symmetry, have a centroid, the position of which has to be determined either experimentally or by the application of the conditions of equilibrium. For example, if

a lamina or solid be suspended by a single cord and allowed to hang freely, the equilibrant of all the parallel and downward vertical gravitational forces acting on the particles of the solid or lamina can only act upwards along the axis of the cord. The line of action of this equilibrant must pass through the point where the mass may be assumed to be concentrated—that is, the centre of gravity.

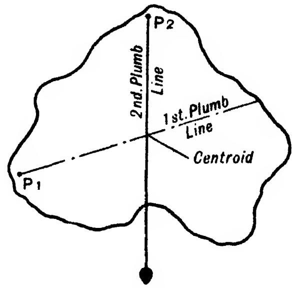


FIG. 230.

EXPT. 20. *To find the centroid of an irregular lamina* (Fig. 230).

METHOD OF PROCEDURE. Suspend the lamina so that it can swing freely about its point of suspension P_1 (Fig. 230). From P_1 allow a plumb line to hang freely above the surface of the lamina and mark the direction of this plumb line on the surface (1st plumb line). Then the centroid will fall somewhere along this line. Suspend the lamina from another point P_2 and mark the direction of the second plumb line, which will contain the centroid. The point at which the two plumb line directions intersect will be the required centroid.

EXPT. 21. *To find the position of the centroid in each of a series of triangular laminae.*

METHOD OF PROCEDURE. Cut out a series of three or more triangles of different shapes (as shown in Fig. 231) and with bases about 4 in.

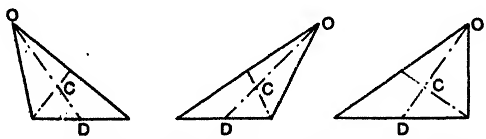


FIG. 231. Centroid of a triangle—at intersection of medians.

in length. Suspend each of these triangles from two points as described in Expt. No. 20 and find their centroids. Draw the medians of each triangle, that is, the lines joining the middle point of each side to the opposite vertex, and verify the following: **The centroid of a triangle is at the point of intersection of its medians, which is at a distance of one third of a median measured along this median from the side.**

The centroid of a section does not necessarily fall within the sec-

tion; for example, the centroid of many angle sections is outside the section. This may be verified by cutting out such a section (Fig. 232) in plywood or thin card and attaching a light wire or pin to its edge. The figure can now be made to balance, on a pencil top, at the point C.

Construction to obtain the centroid of a trapezium (Fig. 233). Draw a trapezium ABCD, and bisect the parallel sides AB and CD

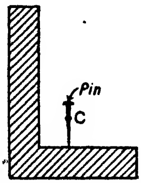


FIG. 232. Centroid outside the area.

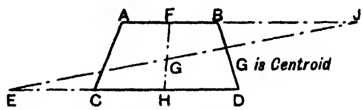


FIG. 233. Centroid of a trapezium.

at F and H. Join FH. Produce AB to J, making BJ = CD, and DC to E, making CE = AB. Join JE; then the point of intersection of FH and JE, that is, G, is the required centroid.

EXPT. 22. *To obtain the centroid of a semicircle.*

METHOD OF PROCEDURE. Cut out, from thick card, a semicircle of diameter about 5 in. (Fig. 234). Suspend this figure from two points in the manner described in Expt. 20 and find the position of the

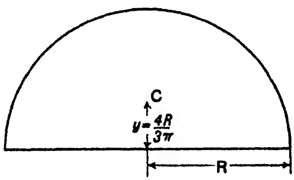


FIG. 234. Centroid of a semicircle.

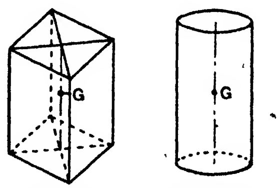


FIG. 235. Centre of gravity of cylinder and prism.

centroid. Verify, by actual measurement, that the **centroid of a semicircle is situated at a point C at a perpendicular distance, $\frac{4R}{3\pi}$, from the centre of its diameter.**

Centre of gravity of solids. The centre of gravity of a solid, of uniform density, which is symmetrical about an axis, is situated somewhere on this axis of symmetry. The position of this point for

some of the more common solids is indicated in the table. See also Figs. 235, 236 and 237.

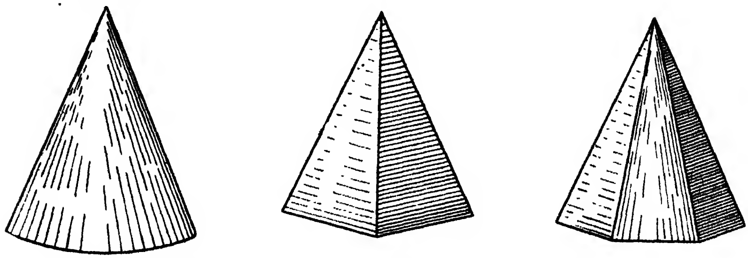


FIG. 236. Centre of gravity of right cones and pyramids is at $\frac{1}{4}$ of altitude measured from the base.

Solid	Position of centre of gravity
Cube, prism cylinder	At the mid point of the line joining the centroids of base and top.
Cone and pyramid	At a point $\frac{1}{4}$ of the length of the line joining centroid of base to apex, measured from base.
Hemisphere	At a point on a radius at right angles to base and $\frac{3}{8}$ radius from the base.

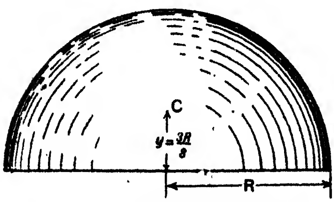


FIG. 237. Centre of gravity of a hemisphere.

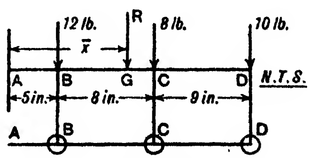


FIG. 238.

Resultant of parallel forces. The general method of determining the centroid, or centre of gravity, in cases where it cannot be obtained by considerations of symmetry, depends upon finding the resultant of a series of parallel forces. Suppose a light rod AD carries weights at B, C and D as shown in Fig. 238. Then each of these weights will be acting downwards and the centre of gravity of the rod and weights will be situated along the line of action of

their resultant, if the weight of the rod is neglected. Let the resultant R act at the centre of gravity G, when $R = 12+8+10 = 30$lb.

Then taking moments about A,

$$\bar{x} \times 30 = 5 \times 12 + 13 \times 8 + 22 \times 10$$
$$= 60 + 104 + 220$$
$$= 384.$$
$$\bar{x} = \tfrac{384}{30} = 12{\cdot}8$$

Then the centre of gravity is situated at a point G, 12·8 in. from A.

Centroid of composite figures. It is important, when dealing with beam sections, to be able to find the position of the centroid, and the following examples illustrate the methods used to obtain the centroid of some well-known beam sections.

Example 1. *A "T" section.* This section is symmetrical about OY, so that the centroid lies somewhere along the axis of symmetry OY (Fig. 239). Consider the section made up of two rectangles, the flange, of area $8 \times 2 = 16$ sq. in., and the web, of area $3 \times 6 = 18$ sq. in. Then the total area will be 34 sq. in.

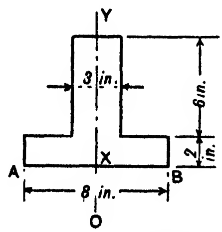


FIG. 239. T section.

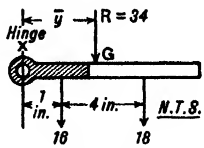


FIG. 240.

Suppose the section in the form of a thin lamina hinged at X along the line AB. Then the downward forces which tend to rotate the lamina about its hinge would be proportional to the areas of the flange and web, and would act at their respective centroids. A force R equal to the resultant of these forces applied upwards at the centroid of the T section, G, would then produce equilibrium.

Moments about the hinge X (Fig. 240).

$$16 \times 1 + 18 \times 5 = R \times \bar{y}$$
$$16 + 90 = 34\bar{y}.$$
$$\bar{y} = \tfrac{106}{34} = 3{\cdot}12.$$

The centroid is on the axis of symmetry and 3·12 in. from the line AB.

Example 2. *An unequal* "I" *section.* This composite section is made up of two flanges A and C together with the web B (Fig. 241).

Area of A $= 11 \times 2\frac{1}{2} = 27\frac{1}{2}$ sq. in.; centroid $1\frac{1}{4}$ in. from the base.

Area of B $= 2 \times 6 = 12$ sq. in.; centroid $5\frac{1}{2}$ in. from the base.

Area of C $= 8 \times 2 = 16$ sq. in.; centroid $9\frac{1}{2}$ in. from the base.

$$\text{Total area} = 16 + 12 + 27\tfrac{1}{2} = 55\tfrac{1}{2} \text{ sq. in.}$$

Let the centroid of the I section be at G a distance y from the base.
Moments about the base,

$$\begin{aligned} 55\tfrac{1}{2}\bar{y} &= 27\tfrac{1}{2} \times 1\tfrac{1}{4} + 12 \times 5\tfrac{1}{2} + 16 \times 9\tfrac{1}{2} \\ &= 34{\cdot}38 + 66{\cdot}0 + 152 \\ &= 252{\cdot}38. \\ y &= 4{\cdot}55. \end{aligned}$$

The centroid is on the axis of symmetry situated 4·55 in. from the base.

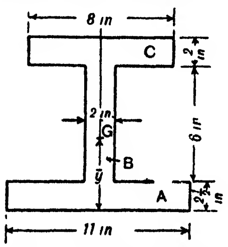


FIG. 241. Unequal I section.

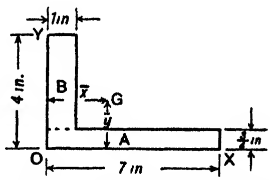


FIG. 242. Unequal angle section.

Example 3. *An unequal angle section.* This section has no axis of symmetry, and the centroid G (Fig. 242) will have to be specified according to its distances $\bar{x}$ and $\bar{y}$ from the two axes of reference OY and OX. The method of procedure is similar to that adopted for Examples 2 and 3, with the exception that moments will have to be taken about OX in order to find the distance $\bar{y}$, and about OY in order to find $\bar{x}$.

Area of A $= 7 \times \frac{3}{4} = 5\frac{1}{4}$ sq. in.; centroid at $\frac{3}{8}$ in. from OX and $3\frac{1}{2}$ in. from OY.

Area of B $= 3\frac{1}{4} \times 1 = 3\frac{1}{4}$ sq. in.; centroid at $2\frac{3}{8}$ in. from OX and $\frac{1}{2}$ in. from OY.

Moments about OX (*to find* $\bar{y}$),

$$\begin{aligned} \text{Total area} \times \bar{y} &= 5\tfrac{1}{4} \times \tfrac{3}{8} + 3\tfrac{1}{4} \times 2\tfrac{3}{8}. \\ 8\tfrac{1}{2}\bar{y} &= \tfrac{63}{32} + \tfrac{247}{32} = \tfrac{310}{32} = 9{\cdot}69. \\ \bar{y} &= \frac{9{\cdot}69}{8{\cdot}5} = 1{\cdot}14. \end{aligned}$$

Moments about OY (*to find* $\bar{x}$),

$$8\tfrac{1}{2}\bar{x} = 5\tfrac{1}{4} \times 3\tfrac{1}{2} + 3\tfrac{1}{4} \times \tfrac{1}{2}$$
$$= 18\tfrac{3}{8} + 1\tfrac{5}{8} = 20.$$
$$\bar{x} = \frac{20}{8\tfrac{1}{2}} = 2{\cdot}35.$$

The centroid is situated at a point 1·14 in. from OX and 2·35 in. from OY, measured at right angles to OX and OY respectively.

Example 4. *A shaft carries two pulleys* A *and* B *of weights* 46 *and* 20 *lb. respectively. Neglecting the weight of the shaft, find the position of a bearing between* A *and* B *in order that the pulleys may be supported at their combined centre of gravity.*

Let the bearing be at G, a distance of $\bar{x}$ from O (Fig. 243).

Moments about O,

Total weight at G = 66 lb.

$$66 \times \bar{x} = 20 \times 4.$$

$\bar{x} = \frac{80}{66} = 1{\cdot}21.$ **Ans. 1·21 ft. from O.**

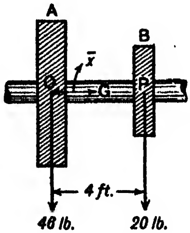


FIG. 243.

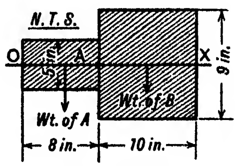


FIG. 244.

Centre of gravity of composite solids. Many of the solids familiar to the engineer—for example, stepped shafts, bolts and turned work—can be resolved into two or more of the common regular solids. The centre of gravity can then be found by using the method of parallel forces and moments; in much the same manner as the centroid of a lamina is found.

Example 1. *Find the centre of gravity of the steel punch shown in Fig.* 244.

This solid is composed of two cylinders, A and B, with a common axis of symmetry OX. The centre of gravity will lie along the line OX and at the point where the line of action of the resultant of the weights of A and B cuts OX.

The weights of A and B will be proportional to their respective volumes if the material is of uniform density.

$$\text{Volume of A} = \pi \times 8 \times \frac{5^2}{4} = 50\pi \text{ cu. in.}$$

$$\text{Volume of B} = \pi \times 10 \times \frac{9^2}{4} = 202{\cdot}5\pi \text{ cu. in.}$$

$$\text{Total volume} = 252{\cdot}5\pi \text{ cu. in.}$$

Let the centre of gravity be at a position $\bar{x}$ from O, then moments about O :

$$252{\cdot}5\pi \times \bar{x} = 50\pi \times 4 + 202{\cdot}5\pi \times 13.$$

Divide through the equation by π :

$$252{\cdot}5\bar{x} = 50 \times 4 + 202{\cdot}5 \times 13$$
$$= 200 + 2632{\cdot}5$$
$$= 2832{\cdot}5.$$
$$\bar{x} = 11{\cdot}22.$$ **Ans. 11·22 in. from O.**

Example 2. *A self-aligning cone bearing* (Fig. 245) *is to be pivoted on trunnions at its centre of gravity; find the position of the trunnions.*

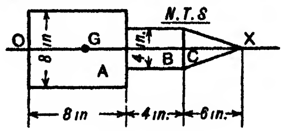


FIG. 245.

This solid consists of two cylinders A and B and a cone C with a common axis of symmetry OX.

Let the trunnions be situated at G a distance $\bar{x}$ from O, then the centres of gravity of the parts A, B and C respectively will be at distances 4 in., 10 in. and $(12 + \frac{1}{4} \times 6) = 13\frac{1}{2}$ in. from O.

$$\text{Volume of A} = \pi \times 4^2 \times 8 = 128\pi \text{ cu. in.}$$
$$\text{Volume of B} = \pi \times 2^2 \times 4 = 16\pi \text{ cu. in.}$$
$$\text{Volume of C} = \tfrac{1}{3}\pi \times 2^2 \times 6 = 8\pi \text{ cu. in.}$$
$$\text{Total volume} = 152\pi \text{ cu. in.}$$

Moments about O,

$$152\pi \times \bar{x} = 128\pi \times 4 + 16\pi \times 10 + 8\pi \times 13{\cdot}5.$$
$$152\bar{x} = 512 + 160 + 108.$$
$$152\bar{x} = 780.$$
$$\bar{x} = 5{\cdot}13.$$

The trunnions must be placed 5·13 in. from O.

Experimental methods of determining the centre of gravity of solids. (*a*) **Solids symmetrical about one plane.** A method employed to find the centre of gravity of this type of solid is as follows: Balance the solid as shown in Fig. 246 upon a long knife edge, and mark the position of the line upon the face of the solid. Take a second position of balance, and the intersection of the knife edge lines on the plane of symmetry is the position of the centre of gravity.

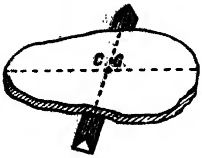


FIG. 246.

(*b*) **Solids symmetrical about two planes at right angles.** The centre of gravity of this type of solid may be determined by the general method of supporting in equilibrium about a knife edge. Fig. 247 shows the application of this method to common engineering examples.

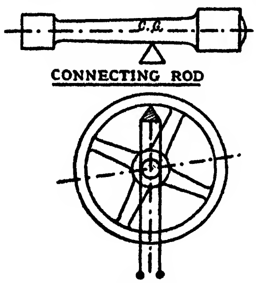


FIG. 247.

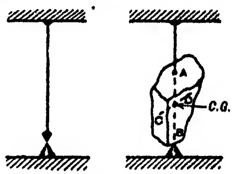


FIG. 248.

(*c*) **Solids with no axis or plane of symmetry.** The method employed for the determination of the centre of gravity of this type of solid is as follows (Fig. 248): Suspend the solid from a point A and arrange for a conical point to indicate the point B vertically under the point of suspension A. Take a second point of suspension C and obtain the point D vertically under C. Then the intersection of the lines AB and CD will indicate the position of the centre of gravity.

EXPT. 23. *To find the position of the centre of gravity of a bicycle.*

METHOD OF PROCEDURE. Suspend the bicycle from two hooks by means of spring balances and thus find its total weight. Attach a cord to each extension of the back hub and join the two cords to a spring balance as shown in Fig. 249, and arrange the cycle so that

the front wheel is resting upon the floor, between two strips of wood, which have been previously tacked to the floor. Carefully mark on the floor the length of the wheel base and measure this length; then read the spring balance supporting the back hub when the back wheel is just free of the floor.

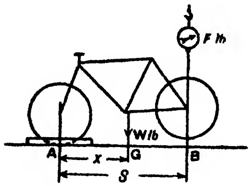


FIG. 249.

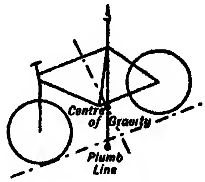


FIG. 250.

CALCULATIONS. Let the vertical line containing the centre of gravity be at a distance x from A.

Moments about A (Fig. 249),

$$F \times \text{wheel base } S = W \times x.$$

$$x = \frac{SF}{W}.$$

To complete the experiment suspend the cycle as shown in Fig. 250, and the point where the plumb line dropped from P cuts the original vertical line through G will be the centre of gravity. This experiment can be performed by suspending the bicycle from two separate points and taking the intersection of the plumb lines as the centre of gravity as in Expt. 20.

Example. *A car weighs 1700 lb., of which 920 lb. is carried on the back axle. If the wheel base is 11 ft. 8 in. find the position of the centre of gravity.*

Moments about A (Fig. 251),

$$1700x = 920 \times 11\tfrac{2}{3}.$$
$$1700x = 10733.$$
$$x = 6{\cdot}31.$$

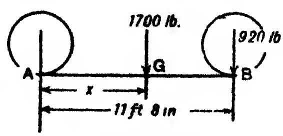


FIG. 251.

Centre of gravity is on a vertical line 6·31 ft. from the front axle.

Slinging and hoisting. It is important, when preparing to sling or hoist heavy loads, that the load shall be suspended from a point

vertically above its centre of gravity. For this reason complicated machine details and heavy castings are often marked to show the position of the vertical line through the centre of gravity. Alternatively, motors and turbine stator cases are fitted with an eye bolt which is situated in the plane containing the centre of gravity (Fig. 252).

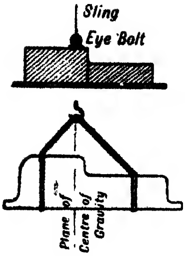


FIG. 252.

Example. *A case is known to have load concentrations as shown in Fig. 253. Find the position it should be marked for single slinging.*

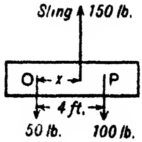


FIG. 253.

Moments about O, $150 \times x = 100 \times 4.$

$$x = \tfrac{400}{150} = 2\tfrac{2}{3}.$$

It should be marked $2\frac{2}{3}$ ft. from O.

To determine the position of the centre of gravity by means of the funicular polygon.

Example. *To find the position of the centre of gravity of the stepped shaft shown in Fig.* 254.

The weights of the several steps are proportional to their volumes.

$$\text{Volume of A} = \pi \times \tfrac{9}{4} \times 5 = 11\tfrac{1}{4}\pi \text{ cu. in.}$$
$$\text{Volume of B} = \pi \times \tfrac{25}{4} \times 7 = 43\tfrac{3}{4}\pi \text{ cu. in.}$$
$$\text{Volume of C} = \pi \times \tfrac{100}{4} \times 5 = 125\pi \text{ cu. in.}$$
$$\text{Total volume} = 180\pi \text{ cu. in.}$$

METHOD OF PROCEDURE. Draw the space diagram composed of the loads due to the respective weights of A, B and C acting through their centres of gravity. Set out the polar and vector diagram on a load line representing the weights of A, B and C and draw the funicular polygon. The terminal lines of the funicular polygon, when produced, intersect on the line of action of the resultant, which passes through the centre of gravity.

By graphical methods, centre of gravity is $12\frac{1}{4}$ in. from the left hand end.

VERIFICATION. Moments about Q,

$$180 \times \bar{x} = 11\tfrac{1}{4} \times 2\tfrac{1}{2} + 43\tfrac{3}{4} \times 8\tfrac{1}{2} + 125 \times 14\tfrac{1}{2}.$$
$$180\bar{x} = 28{\cdot}125 + 371{\cdot}875 + 1812{\cdot}5.$$
$$180\bar{x} = 2212{\cdot}5.$$
$$\bar{x} = 12{\cdot}29. \quad \textbf{Ans. } 12{\cdot}29 \textbf{ in. from Q.}$$

Stability. The position of the centre of gravity of a body standing upon a surface, or a foundation, or floating in a liquid, determines whether the body is in **stable, unstable,** or **neutral equilibrium.** It is well known that if the centre of gravity of a vehicle is too high, or that a ship carries an excessive deck cargo, the vehicle or ship is liable to capsize. The officer responsible for the stowage of

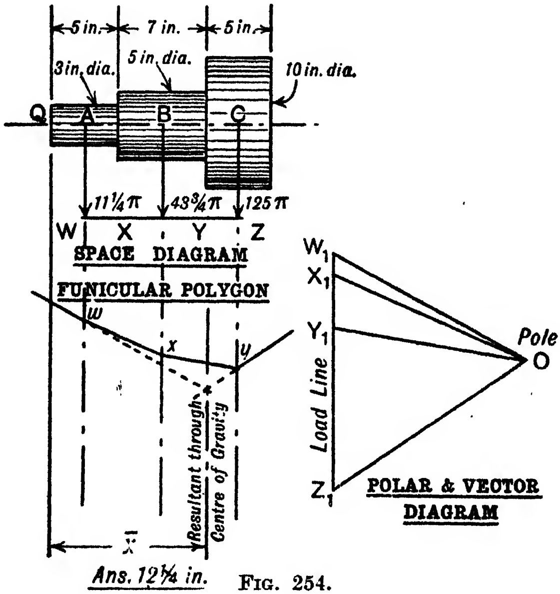


FIG. 254.

cargo in a ship knows from experience that heavy cargo must be stowed in the lower portion of the hold, in order to keep the centre of gravity of the vessel low. For a similar reason, if the ship carries a light, but bulky, cargo she must ship some heavy material as ballast. The driver of a heavy lorry which is loaded so that the centre of gravity is high persists in occupying the crest of the road, because a movement towards the slope of the camber of the road affects his steering and the stability of the lorry.

When a vehicle, or a body, is tilted, its weight exercises a moment which either tends to overturn the body or to right it, and the overturning or righting force passes through the centre of gravity of the body.

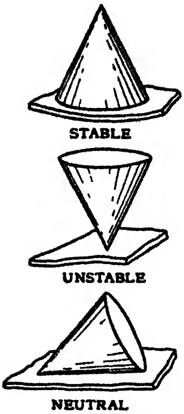


Fig. 255. Equilibrium.

A body is said to be in **stable equilibrium** when the moment of its weight acts as a righting moment and tends to restore it to its original position after a slight displacement from this position. If the moment is an overturning moment and tends to capsize the body, then the body is said to be in **unstable equilibrium.**

Another state of equilibrium is that known as **neutral equilibrium,** a condition in which any movement of the body still leaves it in equilibrium. For example, when a cone (Fig. 255) is resting on its base on a flat surface it is in stable equilibrium: if it is balanced upon its apex, it is in unstable equilibrium, and if it is allowed to lie on its curved surface it is in neutral equilibrium.

Condition to produce overturning. A body will overturn when the vertical line through its centre of gravity falls outside the boundary lines formed by joining the extreme points of its contact with the surface on which it rests. Fig. 256, diagram (*b*) shows the condition of stable equilibrium, where the vertical line through the centre of gravity falls on the inside of point A; in diagram (*c*) this line falls outside A and the vehicle will overturn since the wheels on side B are being lifted off the ground.

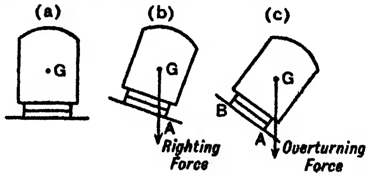


Fig. 256.

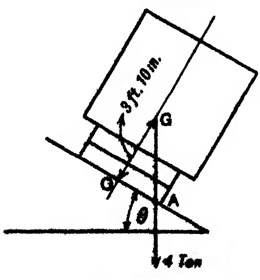


Fig. 257.

Example 1. *A road vehicle weighs, with its load, 4 tons and its centre of gravity is at a height of 3 ft. 10 in. above the track. To what angle may the vehicle be tilted before it is on the point of overturning? Track width = 5 ft.*

The vehicle will be at the point of overturning when the vertical through the centre of gravity G (Fig. 257) passes through A, the extreme point of the wheel base.

Then $$\tan\theta = \frac{OA}{OG} = \frac{30 \text{ in.}}{46 \text{ in.}} = 0{\cdot}6522.$$
$$\theta = \tan^{-1} 0{\cdot}6522 = 33° \ 7'.$$

Example 2. *Fig.* 258 *shows a crane post built on a radius of base of* 3 *ft. The weight of the post and its ballast is* 7 *tons. Find the radius at which a load of* ½ *ton being lifted would make the crane unstable. Calculate the righting moment when the load is raised at a radius of* 10 *feet.*

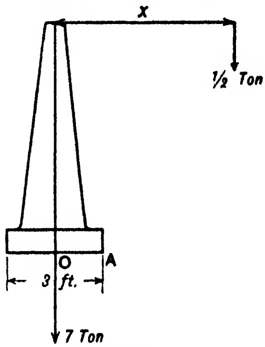


FIG. 258.

In order that the crane may be in unstable equilibrium the vertical through the resultant load must pass just outside A, the extreme edge of the base.

Resultant load $= 7\frac{1}{2}$ tons.

Moments about A,
$$7{\cdot}0 \times \tfrac{3}{2} = \tfrac{1}{2} \times (x - \tfrac{3}{2})$$
$$x = 22\tfrac{1}{2}.$$

Ans. Radius $= 22\frac{1}{2}$ ft.

When the load is at a radius of 10 ft.,

righting moment = unbalanced moment about A
$$= 7 \times 1\tfrac{1}{2} - (10 - 1\tfrac{1}{2})\tfrac{1}{2} = 10\tfrac{1}{2} - 4\tfrac{1}{4} = 6\tfrac{1}{4},$$

or **righting moment $= 6{\cdot}25$ tons ft.**

EXERCISES ON CHAPTER IX

1. Explain how the centroid of an irregular lamina may be found experimentally. Would this method be satisfactory for finding the centre of gravity, if the lamina was not of uniform density? Give reasons for your answer.

2. Draw a triangle of sides 4·25 in., 3·5 in. and 2·75 in. Find its centre of area and explain briefly the method you employ.

3. A sheet of plywood is cut to an irregular shape in order to be used as part of a moment board apparatus (Expt. 3). How would you determine the point at which it should be bored to take the fulcrum?

4. A wall is 11 ft. in height and is trapezoidal in section. The base is 5 ft., and the top 2 ft. 6 in., in width. Find the centre of area of the section if the left hand face is vertical.

5. A shaft carries two pulleys A and B of respective weights 32 lb. and 21 lb. with their centres 12 ft. apart. Find the position of a bearing

which is to be placed at (*a*) the centre of gravity of the pulleys neglecting the shaft, (*b*) the centre of gravity of the combination of shaft and pulleys, if the shaft is 2 in. in diameter and the material from which it is made weighs 0·28 lb. per cu. in.

6. A machine consists of two parts which give load concentrations of 14 cwt. and 9 cwt. situated 3 ft. 4 in. apart. Find the position in which an eye bolt must be attached to the machine in order that it could be lifted directly above its centre of gravity.

7. Find the position of the centre of area for each of the figures shown in Fig. 259.

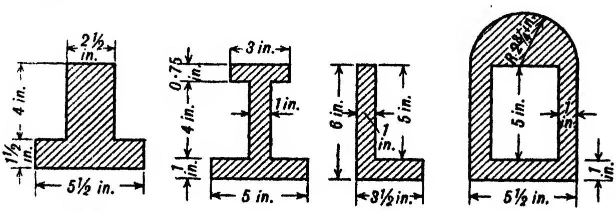


FIG. 259.

8. A steel plate 14 in. in diameter and 1 in. in thickness is reinforced by another circular plate 4 in. in diameter, ½ in. in thickness, riveted to its face so that the circumferences touch at one point. Find the position of the centre of gravity of the combination.

9. A connecting rod 14 in. between bearing centres is suspended horizontally by two spring balances A and B from points above the centres of the bearings. The readings of the balances are respectively 2·7 lb. and 3·9 lb. Find the position of the centre of gravity of the connecting rod.

10. Find the position of the centre of gravity for each of the solids shown in Fig. 260.

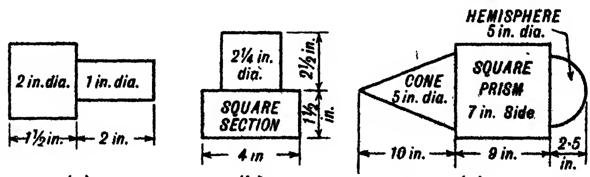


FIG. 260.

11. A hexagonal steel plate 1½ in. in thickness and 4 in. side is bored eccentrically with a 2 in. diameter hole placed 1 in. out of centre on a

diagonal. The hole is then plugged with a brass shaft 4 in. in length with equal overhang on each side of the plate. Find the position of the centre of gravity of the shaft and plate. Densities—steel 0·28, brass 0·32 lb. per cu. in.

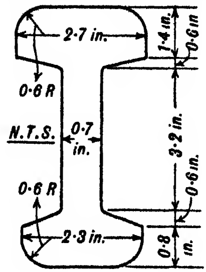


FIG. 261.

12. Use a graphical method to find the position of the centre of area of a semi-ellipse major axis 4·5 in., minor axis 3·5 in., the bisection of the ellipse to be made along the minor axis.

13. Find the centre of area of the section shown in Fig. 261.

14. A truck has a wheel base of 9 ft. 4 in. Its own weight is 3·1 tons equally distributed between front and back axles and it carries a machine of weight 5·4 tons, 3·7 tons of which falls on the back axle. Find the position of the centre of gravity for the loaded truck.

15. Explain the conditions of equilibrium known as stable, unstable and neutral. Illustrate your answers by showing how a cylinder could be arranged in each condition.

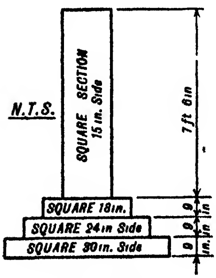


FIG. 262.

16. A box has a base 4 ft. 6 in. square and its centre of gravity is 3 ft. above the centre of the base. To what angle could the box be tilted before it was in danger of overturning ?

17. Find the weight and the position of the centre of gravity of the column and footings shown in Fig. 262. Concrete weighs $1\frac{1}{4}$ cwt. per cu. ft.

18. A column and footings (Fig. 263) weigh 4 tons and their centre of gravity is vertically above a point 2 feet from the edge of the base. What eccentric load could be carried at A, a distance of 3 ft. from the vertical containing the centre of gravity of the column, in order that the vertical through the combined centre of gravity may not fall outside the base ?

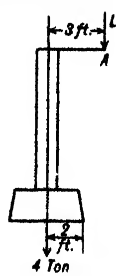


FIG. 263.

19. The axles of the driving and trailing wheels of a goods locomotive are respectively 7 ft. 10 in., 8 ft. 3 in., and 6 ft. apart, and the loads on them are, in order, 15·2, 16·3, 15 and 12·8 tons. Determine the horizontal distance of the centre of gravity of the locomotive from the front driving axle.

20. Describe methods of determining the centre of gravity of (*a*) a small flywheel, (*b*) a connecting rod, (*c*) a piston for an internal combustion engine, (*d*) a bell crank lever.

21. Part of the worm-shaft of a motor car rear axle was 6 in. long and was made in three lengths as follows : 3 in., $2\frac{1}{2}$ in., $\frac{1}{2}$ in. The

corresponding diameters were 1¼, 2, and 2½ in. Find the position of the C. of G. (U.L.C.I.)

22. The distribution of weights in a tramcar fully loaded is shown below; W = weight, x = height of centre of gravity from the rails.

Motors and truck - - - -	$x = 16{\cdot}5$ in.	$W = 10{,}300$ lb.
Controllers - - - - -	$x = 52$ in.	$W = 650$ lb.
Passengers (lower saloon) - -	$x = 59{\cdot}5$ in.	$W = 3{,}100$ lb.
Lower saloon body - - -	$x = 71$ in.	$W = 12{,}000$ lb.
Passengers (top deck) - - -	$x = 134$ in.	$W = 5{,}800$ lb.
Top cover, etc. - - - -	$x = 142$ in.	$W = 3{,}500$ lb.

Find the height of the centre of gravity of the tramcar above the rails. (U.L.C.I.)

23. In a Bedford two ton hand tipping lorry the centre of gravity of the two ton load, the centre of gravity of the tipping part of the lorry, and the axis of the lifting screw are respectively 27½ in., 25¾ in., and 80¾ in. measured horizontally from the fulcrum about which the truck tips. The screw is operated by a handle of 11½ in. radius turning a bevel gear wheel of 17 teeth which meshes with one of 27 teeth. The latter acts as a nut in raising the double lead screw of lead or pitch 1 in. If the unladen tipping portion of the lorry weighs 750 lb., find (*a*) the initial theoretical lifting force required along the screw axis, (*b*) the initial theoretical force tangential to the handle circle, (*c*) the initial efficiency of the tipping mechanism if the actual force required for (*b*) is 44 lb.

24. A heavy bar of uniform section is 12 ft. long. Find the position of its centre of gravity (*a*) when it is straight, (*b*) when it is bent into a right angle having legs each 6 ft. in length. (U.L.C.I.)

25. A locomotive stands on a bridge of 50 ft. span and the wheel loads of 10 tons, 10 tons, 16 tons, 14 tons, 14 tons are respectively distant 4 ft., 10 ft., 18 ft., 25 ft., and 32 ft. from the left end. Find the position of the centre of gravity of the loads, and the pressures on the end supports. (U.L.C.I.)

CHAPTER X

INTRODUCTION TO STRENGTH OF MATERIALS

It is noticeable that in old structures, such as bridges and buildings, the portions carrying load are often excessively strong, and in rare cases the margin of safety is too small. This is probably due to the absence, at the time of building, of reliable information concerning the behaviour of materials under load, and a natural tendency for the designer to adopt large dimensions in order to ensure safety.

At the present time, laboratory equipment is so far perfected that materials can be tested, and some estimate of their behaviour under loading conditions can be obtained. Thus the designer can, with a fair degree of certainty, anticipate the strength of any member which he proposes to employ in a structure.

Behaviour of metal under load. If a specimen of mild steel bar is loaded in tension with a steadily increasing load, until fracture occurs, the material will generally behave in the following manner. The application of the external load will be resisted by an internal resisting force due to the strength of the material itself, and while this resisting force is in operation the material is said to be in a state of **stress.** At the same time the dimensions, and shape, of the specimen will suffer alteration, and this alteration of size and shape is called **strain.** This strain is an essential condition associated with the internal stress; any stress must be accompanied by a corresponding strain. As the load is increased, a period is reached at which the internal resisting forces are unable completely to balance the external load, and the material becomes permanently distorted or strained. It is then said to have a **permanent set.** It is found that up to the period when permanent set first occurs the material is **elastic,** and if the load be removed the specimen will at once assume its original dimensions and shape, that is, those prevailing before loading. It is important to remember that a material does not *lose* its elasticity at any defined load, and even after it has

been loaded beyond complete elasticity it is capable of recovering partially its original dimensions when the load is removed.

If the loading is further continued the material will become **ductile,** that is, it becomes more plastic, and less elastic, or will draw out, gaining length with a corresponding reduction of cross-sectional area, until, at its weakest point, the cross-section will rapidly decrease, forming a **waist or stricture,** and fracture will occur at this waist. The load at this point is called the **ultimate load.**

Materials of a brittle character do not exhibit quite the same behaviour. Cast iron, for example, when tested to destruction in tension, shows very little, if any, indication of ductility, and fracture occurs almost immediately after the elasticity breaks down. On the other hand, the more ductile materials, such as soft copper and lead, have no well defined elastic period, and fracture occurs after the material has drawn out to a cross-section too small to resist the load.

If the test is carried out with the load applied as a compressive load the material will behave according to its nature. The ductile materials, such as soft brasses, copper and lead, will assume a shape similar to a barrel, and failure will occur by the bursting of the sides of this barrel-shaped solid. Cast iron will show no tendency to take this shape, but will fracture, either by splintering, or by breaking into two pieces across a plane at an angle to its axis. Mild steel will assume a barrel shape to a lesser extent but will ultimately fail across a plane inclined to its axis. It must be remembered that in any compression test the specimen must be short enough to prevent **buckling,** that is, failure due to the specimen bending under the load, a failure which occurs before that due to compression.

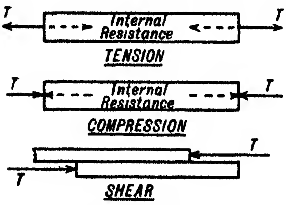


Fig. 264.

Methods of failure. The failure of structural materials may be due to a single stress or a combination of two or more stresses (Fig. 264).

A tensile stress is the stress set up in a material when it is internally resisting an axial pull.

A compressive stress is the stress set up in a material when it is internally resisting an axial thrust.

A shear stress is a stress due to the load producing a tendency for one portion, or layer, of the material to slide over another portion, or layer.

The members and pulley axle of an ordinary jib crane give practical examples of each type of stress. The tie is resisting a tensile load, the jib a compressive load, and the axle pin has a tendency to fracture by the piece in the bearing remaining and the centre portion breaking away, thus this axle is resisting shear.

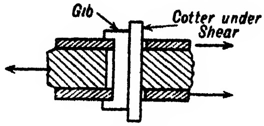


FIG. 265.

Other practical illustrations of these forms of stress can be found in (*a*) a connecting rod on the pulling stroke (tension), (*b*) a connecting rod on the thrust stroke (compression), (*c*) the cotter in the crosshead of an engine (shear) (Fig. 265).

Shearing is also the basis of failure in most of the cutting processes where one layer of the material is forced to slide over the other by the action of the cutting tool.

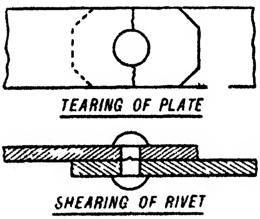


FIG. 266.

Similarly the failure of a riveted joint (Fig. 266) may be due to (*a*) the tearing of the plate (tension), or (*b*) the shearing of the rivets, or (*c*) the crushing of the rivets.

Flexure, or bending. When a material is subjected to bending loads the internal stresses set up are always complex, and if failure occurs it may be due to weakness in tension, compression or shear.

If a narrow piece of plywood is bent by hand it is readily seen that the top layer is subjected to tension, and the bottom to compression (Fig. 267). If a piece of adhesive paper is stuck to both top and bottom layers before bending, the top strip of paper will break and the bottom will be crinkled with the bending.

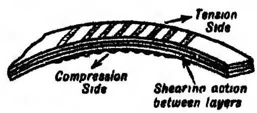


FIG. 267.

Thus bending sets up compressive and tensile stresses in the opposite outer layers of the material. Since these stresses are opposite in character, it appears reasonable to expect that somewhere between

the outer layers will be a layer which carries no compressive or tensile stress. This can be verified and this layer contains a plane of no stress, which is known as the **neutral plane of bending**; it generally passes through the centre of area of the cross-section (Fig. 268).

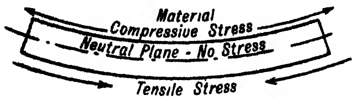


FIG. 268.

This variation of stress over the cross-section produces a condition in which one layer tends to slide horizontally over the other; in other words, there is a horizontal shearing action between the layers setting up a stress known as the **horizontal shear stress.**

If the material of the beam is considered near to the supports, these supports exert a reaction opposite in direction to the load and there is a tendency to shear through the vertical section. This tendency is greatest near to the supports (see Fig. 269) and is known as the **vertical shear.** The stress set up on a plane at right angles to the axis of the beam is thus called the **vertical shear stress.**

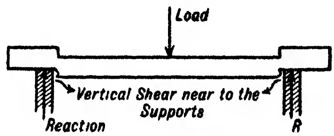


FIG. 269.

Thus a beam under simple bending action is liable to fail under the action of one, or more, of the following stresses: (*a*) tension in an outer layer, (*b*) compression in an outer layer, (*c*) horizontal shear, (*d*) vertical shear.

Torsion or twisting. When a material is twisted it is said to be under the action of **torsion**, and providing this torsion is not accompanied by bending, failure will occur by shearing, that is, the portion A (Fig. 270) sliding in a rotational manner over the section B. Actually in a torsion test the material does not generally fail across a plane at right angles to the axis of twisting, but this is the direction in which the simple shear takes action.

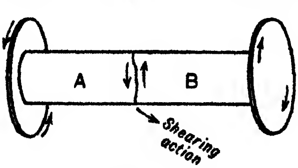


FIG. 270.

Combined bending and torsion. It is common in most cases of shafts transmitting power to have a combination of the stresses due to both bending and torsion, (*a*) bending between the bearings and

(*b*) the torsion due to the transmission. The treatment of combined bending and torsion is outside the scope of this book, but it should be remembered that the combination is often present and the stresses set up in the material are then those due to the combination of the bending and twisting effects.

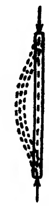


FIG. 271.

Failure by buckling. This type of failure is due to compression being exerted on a piece of material which is unduly long when compared with its cross-sectional dimensions. The material then tends to buckle or "double up" long before it has failed by any other method. This applies, particularly, to long struts and plates. It is due to the load not being axial after deflection has taken place, and the load producing bending action which increases the deflection (Fig. 271).

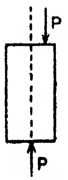


FIG. 272.

Eccentric loading. When a load is applied eccentrically, bending is induced and the material tends to fail due to a combination of direct and bending stresses. See Fig. 272.

Quantitative treatment of strength of materials. Measurement of stress. Stress is measured as an average or mean intensity of stress, a quantity usually referred to as the **stress**. This intensity of stress, or stress, is the load per unit area or $\dfrac{\text{load}}{\text{cross sectional area}}$.

Example 1. (*a*). *A steel bar 2 in. × 1½ in. in cross-section carries an axial tensile load of 20 tons. Find the intensity of stress.* (*b*) *If the bar tapers uniformly to a section 1 in. × ¾ in., find the maximum stress in the bar.*

$$(a)\ \text{Stress} = \frac{\text{load}}{\text{cross-sectional area}} = \frac{20}{2\times 1\frac{1}{2}} = \frac{20}{3} = 6\tfrac{2}{3}\ \textbf{tons per sq. in.}$$

$$(b)\ \text{Maximum stress} = \frac{\text{load}}{\text{least cross-sectional area}} = \frac{20}{1\times\frac{3}{4}} = 26\tfrac{2}{3}\ \textbf{tons/in.}^2.$$

Example 2. *The stress in a steel column of cross-section, outside diameter 5 in., inside diameter 4 in., is 8 tons per sq. in. Find the load carried by the column.*

$$\text{Cross-sectional area} = \frac{\pi}{4}(5^2-4^2) = \frac{\pi}{4}\times 9\times 1 = \frac{9\pi}{4}\text{sq. in.}$$

$$\therefore\ \text{Load} = \text{stress}\times\text{area of cross-section} = \frac{9\pi}{4}\times\frac{8}{1} = 18\pi = \mathbf{56{\cdot}52\ tons.}$$

Measurement of strain. Strain is measured as a fractional strain, that is, the strain per unit length or dimension. This fractional strain, or strain, is, for compression or tension, the ratio

$$\frac{\text{alteration in length}}{\text{original length}}.$$

Example 1. *A steel bar* 10 *ft. in length suffers an extension of* 0·23 *in. under a tensile load. Find the fractional strain.*

$$\text{Strain} = \frac{\text{extension}}{\text{original length}} = \frac{0{\cdot}23}{10 \times 12} = 0{\cdot}00192.$$

Example 2. *The fractional strain when a column* 6 *ft. in height carries a certain load is* 0·0015. *Find the amount by which the column shortens under the load.*

$$\begin{aligned}\text{Alteration in length} &= \text{fractional strain} \times \text{length}\\ &= 0{\cdot}0015 \times 6 \times 12 = 0{\cdot}108 \text{ in.}\end{aligned}$$

Modulus of elasticity. Some indication of the elastic nature of a material may be deduced from the relation between the strain produced and the stress set up while the material remains perfectly elastic. This relationship is known as the modulus of elasticity and is the ratio $\frac{\text{intensity of stress}}{\text{fractional strain}}$, or $\frac{\text{stress}}{\text{strain}}$, a quantity generally referred to as E. This quantity is frequently called **Young's modulus of elasticity, or Young's modulus,** after the man who first made use of the ratio.

Consistency of units. It is important in this, as in every branch of science, that the units employed throughout any process shall be consistent. Thus the modulus of elasticity is a stress in lb. per square inch, only when the stress used in the ratio is also in lb. per square inch and the fractional strain is the true ratio of quantities with like units, for example the extension in inches and the original length also in inches.

Compound bars. In working problems on compound or laminated bars of the same cross-section throughout, which are acting as ties or struts, the total load carried by each material can be calculated approximately by assuming that the longitudinal fractional strain is the same in each. See example, No. 141, p. 374.

Example 1. *Find the modulus of elasticity of steel if a bar* 10 *ft. in length,* $1\frac{1}{2}$ *in. in diameter stretches* 0·04 *in. under a load of* 8 *tons.*

$$\text{Area of cross-section} = \pi \times (\tfrac{3}{4})^2 = \frac{9\pi}{16} \text{ sq. in.}$$

$$\text{Stress} = \frac{\text{load}}{\text{area of cross-section}} = \frac{8 \times 16}{9\pi} \text{ tons per sq. in.}$$
$$= 4{\cdot}53 \text{ tons per sq. in.}$$

$$\text{Fractional strain} = \frac{\text{extension}}{\text{original length}} = \frac{0{\cdot}04}{10 \times 12} = 0{\cdot}0003333.$$

$$\text{Modulus of elasticity, } E = \frac{\text{stress}}{\text{strain}} = \frac{4{\cdot}53}{0{\cdot}0003333}$$
$$= \mathbf{13{,}580 \text{ tons per sq. in.}}$$

Example 2. *A cast iron column* 7 *ft. in length, external diameter* 6 *in., internal diameter* 4 *in. is known to contract* 0·007 *in. under a load. Find the load if* $E = 7000$ *tons per sq. in.*

$$\text{Area of cross-section} = \pi\{3^2 - 2^2\} = 5\pi \text{ sq. in.}$$

$$\text{Let load} = L \text{ tons, then stress} = \frac{L}{5\pi} \text{ tons per sq. in.;}$$

$$\text{strain} = \frac{0{\cdot}007}{7 \times 12} = \frac{0{\cdot}007}{84}.$$

$$E = \frac{\text{stress}}{\text{strain}} = \frac{L}{5\pi} \times \frac{84}{0{\cdot}007} = 7000 \text{ tons per sq. in.}$$

$$L \text{ in tons} = \frac{5\pi \times 0{\cdot}007 \times 7000}{84} = \mathbf{9{\cdot}16}.$$

Example 3. *A steel "I" section, to the dimensions shown in Fig.* 273, *carries a compressive stress of* 6 *tons per sq. in. Find the load and the contraction if* $E = 13{,}500$ *tons per sq. in. and the length is* 2 *ft.*

Area of cross-section

$$= 2(3 \times \tfrac{3}{4}) + \tfrac{1}{2} \times 2\tfrac{1}{2}$$
$$= 4\tfrac{1}{2} + 1\tfrac{1}{4} = 5\tfrac{3}{4} \text{ sq. in.}$$

$$\text{Load} = \text{stress} \times \text{area of cross-section}$$
$$= 6 \times 5\tfrac{3}{4} = \mathbf{34{\cdot}5 \text{ tons.}} \quad \textbf{Ans. } (a).$$

$$E = \frac{\text{stress}}{\text{strain}} = \frac{6}{x} \times 24 = \frac{144}{x}$$
$$= 13{,}500 \text{ tons per sq. in.}$$

$$x = \tfrac{144}{13500} = \mathbf{0{\cdot}0106}. \quad \textbf{Ans. } (b)\ \mathbf{0{\cdot}0106 \text{ in.}}$$

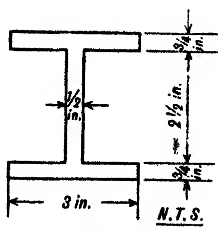


FIG. 273.

Ultimate stress. The maximum nominal stress set up in a material before fracture occurs is known as the ultimate stress and, according to the mode of failure, it is known as the **ultimate tensile stress, ultimate shear stress,** or **ultimate compressive stress.** Abbreviations for these stresses may be employed as follows: U.T.S., U.S.S., or U.C.S.

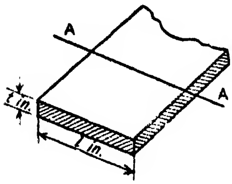


FIG. 274.

Failure by shearing. In order to cut the piece of metal shown in Fig. 274 across the plane AA it would be necessary to overcome the resistance offered by the material to shearing of this cross-section.

Thus, if the area of the section is $l \times t$ sq. in. and the U.S.S. is f_s tons per sq. in., the force required to shear the plate is

$$\mathbf{f_s \times l \,.\, t \text{ tons.}}$$

Example 1. *Find the least force required to shear a steel plate 2 in. wide, $\frac{1}{4}$ in. in thickness, if the U.S.S. = 26 tons per sq. in.*

$$\text{Area resisting shear} = 2 \times \tfrac{1}{4} = 0{\cdot}5 \text{ sq. in.}$$

$$\text{Force} = 0{\cdot}5 \times \text{U.S.S.} = 0{\cdot}5 \times 26 = \mathbf{13 \text{ tons.}}$$

Example 2. *A shearing machine is capable of exerting a force of 74 tons. Find the number of strips, each $1\frac{1}{2}$ in. in width and $\frac{1}{16}$ in. in thickness, which can be sheared in one operation, if the U.S.S. is 7 tons per sq. in.*

Let n be the number of strips, then

$$\text{area resisting shear} = n \times 1\tfrac{1}{2} \times \tfrac{1}{16} \text{ sq. in.}$$

$$\text{Force} = 7 \times n \times 1\tfrac{1}{2} \times \tfrac{1}{16} \text{ tons} = 74 \text{ tons.}$$

$$n = \frac{74 \times 16 \times 2}{21} = \mathbf{112.}$$

Punching. The process of punching holes in metal is a variation of failure by shearing. The material fails by shearing along the perimeter of the punched hole, and the area of metal resisting shear during punching is given by the perimeter × thickness of the metal.

Example 1. *Find the force required to punch a square hole of 2 in. side through plate $\frac{1}{4}$ in. in thickness, if the U.S.S. of the material is*

26 tons per sq. in. What is the mean intensity of compressive stress in the punch ?

Area resisting shear $= 4 \times 2 \times \frac{1}{4} = 2$ sq. in.

Force $=$ U.S.S. $\times$ area $= 26 \times 2 = 52$ **tons.**

Compressive stress in punch $= \frac{52}{4} = 13$ **tons per sq. in.**

Example 2. *Find the dimensions of the largest regular hexagonal hole which can be punched through plate* $\frac{1}{8}$ *in. in thickness, if the U.S.S. is* 8 *tons per sq. in. by a punch exerting* 50 *tons.*

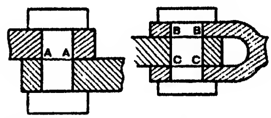
Let the side of the hexagon be S in., then

$$\text{area resisting shear} = 6 \times S \times \tfrac{1}{8} \text{ sq. in.}$$

$$\text{Force} = \text{area} \times \text{U.S.S.} = \frac{6S}{8} \times \frac{8}{1} = 50.$$

$$S = \tfrac{50}{6} = 8\tfrac{1}{3}. \quad \textbf{Ans. } 8\tfrac{1}{3} \textbf{ in. side.}$$

Single and double shear. When a piece of material is subjected to shearing action, the shearing may be taking place across one or two parallel planes, and the two actions are distinguished by the terms **single** and **double shear** (Fig. 275).



Single shear. Double shear.
Fig. 275.

In the case of single shear, the pin will fail across the plane AA, and thus the area resisting shear is equal to the cross-sectional area of the pin. When the action is double shear, the area resisting shear is twice the area of cross-section, because failure of the pin would have to take place simultaneously across the sections BB and CC.

Example 1. *Find the force required to fracture the rivets in the riveted lap joint shown in Fig.* 276. *Neglect the bending action. U.S.S. is* 25 *tons per sq. in.*

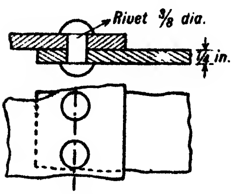


Fig. 276.

If bending is neglected, the joint will fail by shearing two rivets, which will be in single shear.

Area resisting shear

$$= 2\{\pi \times (\tfrac{3}{16})^2\} = 0{\cdot}221 \text{ sq. in.}$$

Force $=$ area $\times$ U.S.S.

$$= 25 \times 0{\cdot}221 = \mathbf{5{\cdot}525 \text{ tons.}}$$

Example 2. *Find the force required to fracture the rivets in the riveted butt joint shown in Fig. 277. U.S.S. = 25 tons per sq. in.*

This joint may fail (a) by shearing two rivets, (b) by tearing the plate. If the failure is by shearing the rivets, case (a) will be the weaker.

Each rivet is in double shear and is considered to offer twice the area to shear resistance. The Board of Trade rule takes the value as 1·75, and not 2, so as to allow for bending.

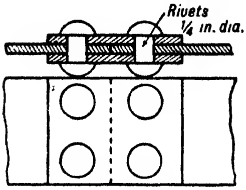


FIG. 277.

Area resisting shear (2 rivets)

$$= 2 \times 2 \times \pi \times (\tfrac{1}{8})^2 \text{ sq. in.} = \frac{\pi}{16} \text{ sq. in.}$$

$$\text{Force} = \text{area} \times \text{U.S.S.} = \frac{\pi}{16} \times 25 = \mathbf{4{\cdot}91\ tons.}$$

Example 3. *Find the necessary diameter of the pin in the forked joint in Fig. 275, if the stress due to shear is not to exceed 5 tons per sq. in. and the load is 20 tons.*

The pin is in double shear.

$$\text{Area resisting shear} = 2 \times \frac{\pi d^2}{4} \text{ sq. in.}$$

$$\text{Allowable load} = \text{area} \times \text{stress} = \frac{2\pi d^2}{4} \times 5 \text{ tons} = 20 \text{ tons.}$$

$$d = \sqrt{\frac{8}{\pi}} = 1{\cdot}595. \quad \textbf{Ans. 1·6 in.}$$

Experimental treatment of strength of materials. Hooke's law. Hooke, 1635-1703, carried out a series of experiments with springs and wires. He found that while a material remained fully elastic, the extension, or compression, was proportional to the load producing it. This is known as Hooke's law and can be expressed as follows: the ratio $\frac{\text{stress}}{\text{strain}}$ is constant during the perfectly elastic period.

Young's modulus. This ratio, $\frac{\text{stress}}{\text{strain}}$, has since been generally accepted by the name of Young's modulus of elasticity, and gives an indication of the elastic properties of the material.

EXPT. 24. OBJECT. *To verify Hooke's law for a spring.*

METHOD OF PROCEDURE. Hang the spring from a suitable support and carefully measure its length before loading. Place on the free

end of the spring a series of loads, which will not strain it beyond its elastic powers, and measure the length of the spring for each load. Remove the loads and measure the length of the spring at each stage, taking particular notice that the lengths correspond with the lengths during loading. The spring when fully unloaded should be restored to its original length, and this is a guarantee that the test has been carried out while the spring remained elastic. Calculate the extensions for each load and plot a graph of load v. extension.

OBSERVATIONS. Original length = 11·4 in.

Load, lb.	0	1	2	3	4
Length, loading, in.	11·4	11·9	12·4	12·9	13·4
Length, unloading, in.	11·4	11·9	12·4	12·9	—
Extension, in.	0	0·5	1·0	1·5	2·0

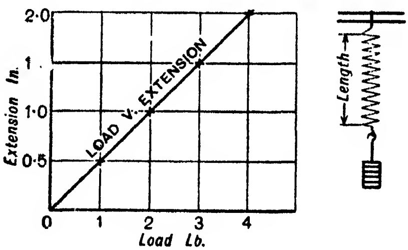


FIG. 278.

CONCLUSION. Because the graph of load v. extension is a straight line passing through the origin, the load is proportional to the extension produced, and Hooke's law is verified.

EXPT. 25. OBJECTS. (1) *To obtain an approximate verification of Hooke's law for india rubber.*

(2) *To find Young's modulus for india rubber.*

METHOD OF PROCEDURE. The loading in this experiment is carried out in a similar manner to that used in the spring experiment and the rubber is tested for elasticity by unloading in the same way. The value for the extension is obtained from the mean, or average, of the extensions measured at loading and unloading. When setting

up the experiment a small load is placed upon the hanger (Fig. 279), sufficient to remove the flexure or bends in the specimen, and this load is neglected. After this straightening load has been added, two lines are placed upon the rubber 12 in. apart and this distance

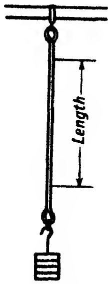


FIG. 279.

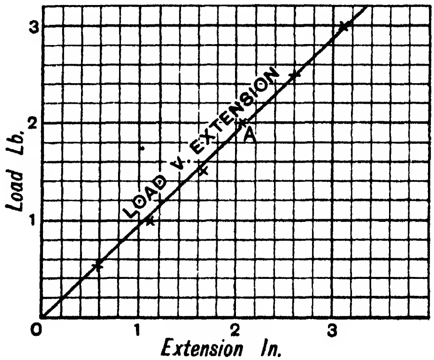


FIG. 280.

is the original or gauge length. The loading and unloading is then carried out, the extensions measured and a graph (Fig. 280) plotted of load *v.* mean extension. Young's modulus is then obtained from a point on the graph as shown under the derived results.

OBSERVATIONS. Diameter of test piece $= \frac{3}{8}$ in.
Cross-sectional area $= \pi \times (\frac{3}{16})^2 = 0{\cdot}11$ sq. in.
Gauge, or original length $= 12$ in.

Load, lb.	0	½	1	1½	2	2½	3
Length, loading, in.	12	12·5	13·1	13·6	14·0	14·6	15·1
Length, unloading, in.	12	12·6	13·2	13·7	14·1	14·6	—
Extension, loading, in.	0	0·5	1·1	1·6	2·0	2·6	3·1
Extension, unloading, in.	0	0·6	1·2	1·7	2·1	2·6	—
Mean extension, in.	0	0·55	1·15	1·65	2·05	2·6	3·1

Derived result. To find Young's modulus.

Select a point A on the graph.

At A, load = 2 lb., extension = 2·05 in.

$$\text{Stress} = \frac{\text{load}}{\text{area of cross-section}}$$

$$= \frac{2}{0{\cdot}11} \text{ lb. per sq. in.}$$

$$\text{Strain} = \frac{\text{extension}}{\text{original length}} = \frac{2{\cdot}05}{12} = 0{\cdot}171.$$

$$\text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}} = \frac{2}{0{\cdot}11 \times 0{\cdot}171}$$

$$= 106{\cdot}3 \text{ lb. per sq. in.}$$

Conclusions. The graph of load against extension is approximately a straight line passing through the origin, so that the extension is proportional to the load producing it (Hooke's law).

The value of Young's modulus for this specimen of indiarubber is 106·3 lb. per sq. in.

The value of the modulus for indiarubber is very variable and depends to a large extent upon the age of the specimen.

Expt. 26. Objects.

(1) *To verify Hooke's law for steel wire.*

(2) *To find the modulus of elasticity of steel.*

Apparatus. The apparatus consists of a steel wire of considerable length, secured to a rigid hook in the wall or ceiling (Fig. 281). The lower end is attached, by means of a terminal, to the sliding scale of a vernier, and the fixed scale of this vernier is fastened rigidly to a frame which is screwed to the bench, or alternatively supported by weighted wires from the ceiling. This fixed scale must be rigid in order that the extension of the wire under load may be registered by the sliding scale of the vernier.

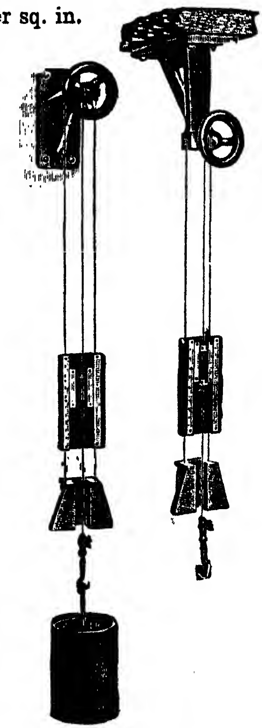


Fig 281.

(*Messrs. G. Cussons Ltd.*)

Method of Procedure. This experiment is conducted in a similar manner to the two previous experiments. A small load is placed upon the free end of the wire in order to remove the flexure, and this load is ignored. The terminal is then loosened and the sliding scale of the vernier adjusted to read zero. A series of loads is placed on the hanger and the corresponding extensions observed. These loads are then removed and the elasticity tested during the unloading period. The measurement of the original length is taken between the point where the wire emerges from the ceiling support and the terminal, that is, the length subjected to stretch. The value of Young's modulus is determined from a representative point on the load v. extension graph (Fig. 282) in the manner shown in the derived results.

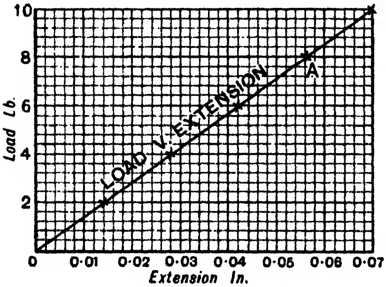


Fig. 282.

Observations. Diameter of wire $= 0{\cdot}024$ in.
Area of cross-section $= \pi \times 0{\cdot}012^2 = 0{\cdot}00045$ sq. in.
Original length $= 90$ in.

Load, lb.	0	2	4	6	8	10
Extension, loading, in.	0	0·014	0·028	0·042	0·056	0·07
Extension, unloading, in.	0	0·014	0·028	0·042	0·056	—

Derived results. To find Young's modulus.
Select a point on the graph A.

At A, load = 8 lb., extension = 0·056 in.

$$\text{Stress} = \frac{\text{load}}{\text{cross-sectional area}} = \frac{8}{0{\cdot}00045}.$$

$$\text{Strain} = \frac{\text{extension}}{\text{original length}} = \frac{0{\cdot}056}{90}.$$

$$E = \text{Young's modulus} = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{8}{0{\cdot}00045} \times \frac{90}{0{\cdot}056} = \mathbf{28{,}570{,}000 \text{ lb. per sq. in.}}$$

CONCLUSIONS. (*a*) Because the load *v.* extension graph is a straight line passing through the origin, the extension of the wire is proportional to the load producing it. This verifies Hooke's law. (*b*) The value of the modulus of elasticity for the steel of this wire is 28,570,000 lb. per sq. in.

EXPT. 27. A ductile material, annealed copper wire.

OBJECTS. (1) *To obtain a graph of load* v. *extension for an annealed copper wire.*

(2) *To examine the behaviour of this wire when tested to destruction by tension and to find the ultimate tensile stress.*

APPARATUS. The apparatus shown in Fig. 281 is employed for this experiment.

METHOD OF PROCEDURE. In this experiment arrangement must be made for a long fall of the load, and the extensions are measured with a scale instead of the vernier. In the first instance a relatively small load is placed on the wire, say 5 lb. and the extension measured. The increment of load is increased during the ductile period, until the period is reached when the cross-sectional area is rapidly decreasing, when the load is added in small increments until fracture occurs. It should be noted that this method of testing does not give an indication of the detail behaviour of such a material, but it shows, in a general manner, the behaviour of a ductile material, and gives a fair indication of the ultimate tensile stress.

OBSERVATIONS. Diameter of wire = 0·062 in.
Original length = 90 in.

Load, lb. - -	0	5	10	20	40	60	70	80	84
Extension, in. - -	0	1	2	3·5	8	13·5	18	25	32

DERIVED RESULT.

Ultimate tensile stress

$$= \frac{\text{load at fracture}}{\text{cross-sectional area}}$$

$$= \frac{84}{\pi \times 0{\cdot}031^2}$$

$$= 27{,}820 \text{ lb. per sq. in.}$$

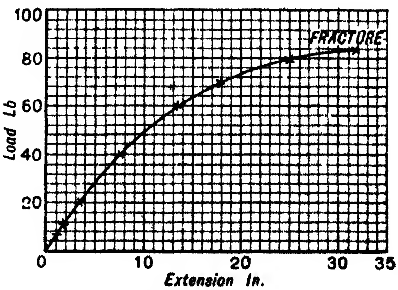


FIG. 283.

CONCLUSIONS. (*a*) For a ductile material the graph of load *v.* extension (Fig. 283) is a curve

up to the point of fracture, and no definite elastic period is indicated. Thus Hooke's law, which only holds during a period of complete elasticity, does not apply.

(*b*) The reduction of cross-sectional area is evident from the commencement of the test, and for this reason it is not possible to assess the stress at any point in the test unless measurements of diameter are taken periodically.

(*c*) The value of the ultimate tensile stress when the reduction of cross-sectional area is ignored, as is usual, is 27,820 lb. per sq. in.

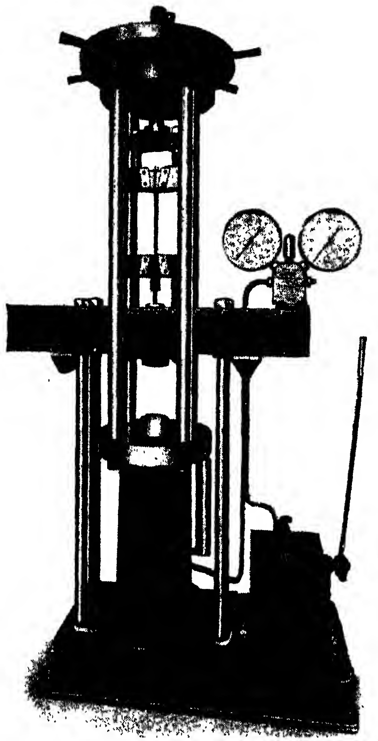


FIG. 284. 30,000 lb. Universal hydraulic testing machine.

Testing machines. For the testing of larger sizes of materials, machines capable of exerting greater forces are required. Such machines, fitted with accurate apparatus for the measurement of these forces, are called testing machines.

Fig. 284 shows a universal testing machine of 30,000 lb. maximum load with which tensile, compressive, bending and shearing tests can be undertaken. The diagram has been supplied by courtesy of Mr. A. Macklow Smith, the maker of the machine. Power is obtained by working the hand pump which forces oil under pressure into the hydraulic cylinder shown bolted to the base plate. The ram is thus raised and the force exerted on the specimen is recorded by the gauges, which are fixed to the transverse I beam, which is in turn bolted to the base plate. In the illustration a tensile specimen is shown fitted in the grips. The holders of the lower grips are bolted to the transverse beam while the upper holders are lifted by the moving ram. The large handwheel at the top

of the machine can be used to adjust the position of the grips and holders for convenience in fitting the specimen and giving a slight initial tension. Specimens can be compressed between the underside of the beam and the top of the ram flange, through special blocks. Axial loading, in tension and compression, is assured by spherical seatings for the holders. Transverse and shearing tests can also be carried out below the fixed beam. These machines are not as accurate as lever testing machines, because the load cannot be measured to the same degree of precision.

FIG. 285. Specimen for tensile test.

Preparation of specimens. Specimens for tensile tests are usually machined to the shape shown in Fig. 285. The ends are made larger to ensure that the specimen does not break in the grips. In the case of brittle materials it may be desirable to screw the ends of the specimens to obtain the best results. The specimen is accurately marked along its length with a light centre punch, and the marks are made 1 in. apart. The gauge length, or length tested, is often taken as 8 in. or 10 in., but the British Standards Institution also recommends test bars of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ sq. in. in cross-sectional area having respectively gauge lengths of 2 in., 3 in. and $3\frac{1}{2}$ in. These sizes are chosen so that comparative results may be obtained when smaller pieces of material of varying sizes have to be tested in tension. It may be noticed that the dimensions given satisfy the equation

$$\text{gauge length} = 4 \times \sqrt{\text{area of cross-section}} \text{ approximately.}$$

For compression tests the specimens must not be made too long or buckling will result. It is important that the ends be cut at right angles to the axis of the specimen to ensure that the load is applied axially. A length of $1\frac{1}{2}$ to 2 diameters will be found suitable.

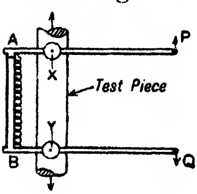


FIG. 286. Illustrating principle of extensometer.

The extensometer. This is an instrument used for measuring elastic tensile strain. The principle of action of one form of extensometer will be easily understood with the help of Fig. 286.

Two pairs of hardened steel set screws grip the specimen lightly but firmly at X and Y. The framed levers AXP and BYQ which carry these set screws are maintained a fixed distance apart at A and B by a spring and flexible pillar arrangement. When the tensile load is applied XY increases in length and so must PQ in the greater ratio $\frac{PA}{XA}$,

since AB is a fixed length. The extension of PQ is measured by some form of dial test indicator calibrated so that it registers the actual extensions at XY. The indicator is fixed to the arm BYQ at Q and works against a fixed frame pivoted at P. Extensions of XY can thus be measured to 0·0001 of an inch. A spherical cap at the top of the pillar AB fits against a spherical seating on the framed bar AXP. The spring ensures contact, and thus AB is a fixed length and yet sufficiently flexible to allow the framed levers freedom of movement. The complete instrument is shown in Fig. 287, thanks to the courtesy of the maker, Mr. A. Macklow Smith of Westminster. This extensometer works on a gauge length of 4 in. with a maximum reading of 0·1 in. The hook distance pieces shown hanging from the lower pair of set screws permit of ready adjustment to the gauge length of 4 in., but they must be removed before the load is applied.

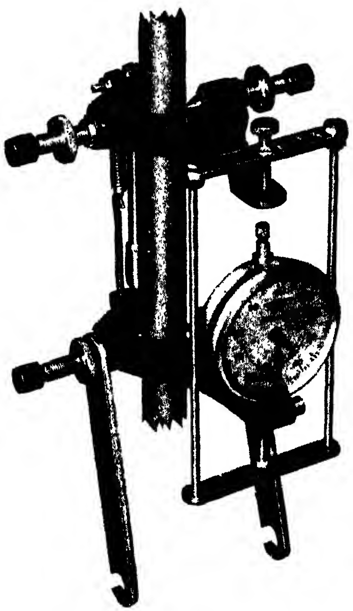


FIG. 287. Extensometer.

Load-extension diagram for mild steel. In a complete tensile test it is customary to measure the loads, and the corresponding extensions by the extensometer, and to plot a load-extension diagram similar to Fig. 288. Care must be taken, however, to see that the extensometer is removed before its maximum reading or before the specimen breaks. The larger extensions can be read by vernier attachments, or an autographic device might be used if available. The graph shows a typical diagram for mild steel. The elastic portion OA is a straight line through the origin showing that the load and extension are directly proportional, *i.e.* form a constant ratio. At A, there is a sudden change in the ratio of load to extension, and this point is referred to as the elastic limit. In this case it also marks the limit of proportionality, but the terms must not be regarded as synonymous. The stress where this sudden yielding of the specimen to the load is first noticed is called the **yield point,** and the calculation can be

made from the figures taken from point B on the curve. From point B an increase of load, when applied gradually and uniformly, causes an increase in the rate of stretching as the material becomes less elastic and more plastic. Point C, where the maximum load is

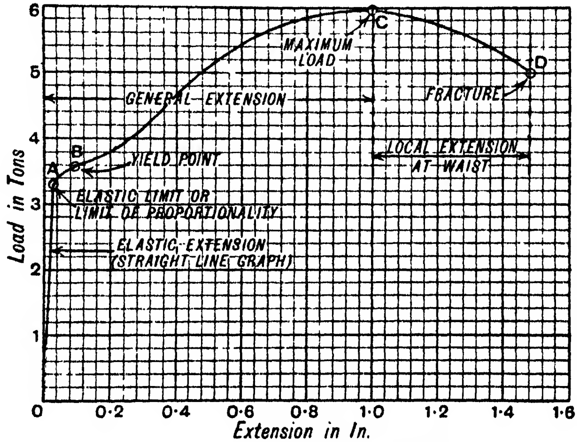


FIG. 288. Load-extension diagram.

recorded, also marks the end of the general extension and the commencement of the formation of a waist (Fig. 289). The cross-section becomes rapidly smaller in area as the waist forms, and a little less load is required to continue extending and finally fracture the specimen. Fracture occurs at D. At this point the actual breaking stress, $\frac{\text{load at D}}{\text{area of waist}}$, is the greatest stress to which the specimen is subjected.

In commercial work the original cross-sectional area is always taken for computing the stress and the maximum nominal stress, or ultimate tensile strength is the criterion of strength employed. It is vitally important that once a test has started the loading should be increased steadily and without shock until fracture occurs. Any stoppage and consequent resting of the material tends to alter its qualities.

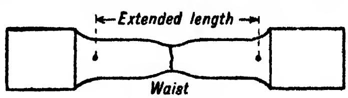


FIG. 289.

Nature of fracture. At the point of fracture, ductile materials, which break at a waist, generally reveal a circular crater on one side

of the break (Fig. 290 (*b*)) and a corresponding truncated cone on the other. Fibrous materials such as wrought iron (Fig. 290 (*a*)) show little if any waist, and the fracture is ragged and torn. Non-ductile materials such as cast iron (Fig. 290 (*c*)) rupture suddenly across a surface which is almost perpendicular to the direction of loading. The amount of extension is also considerably less. The last mentioned type of fracture is also characteristic of hard steels, although these carry a much higher breaking load than cast iron.

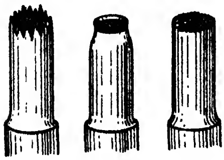


FIG. 290.

Factor of safety. From the foregoing it will be seen that the testing of a material beyond its elastic limit affords useful information as to its nature. Actually in practice a material should not be stressed beyond its elastic limit, and to ensure a margin of safety it is customary to obtain the safe load a bar can carry by dividing its breaking load by a factor of safety or

$$\text{factor of safety} = \frac{\text{maximum or ultimate stress}}{\text{safe stress}}.$$

High factors of safety are necessary if loads are live, variable, alternating or suddenly applied. Such loading tends to fatigue the material and cause it to fracture more readily.

Stress-strain diagrams. These diagrams are exactly the same shape as the load-extension diagrams since the cross-sectional area is considered constant for stress calculations and the gauge length is fixed in the calculation of strain. One diagram can be obtained from the other by a correct modification of the scales employed. The modulus of elasticity of a material tested in tension can easily be found from the loads and entensometer readings in the same manner as already described for a steel wire. The modulus is really the gradient of the elastic portion of the stress-strain graph.

Tensile commercial test on mild steel. In commercial work the yield point is often accepted as approximately the elastic limit since the latter is not easy to determine. In the case of hard and alloy steels and bronzes the yield point is non-existent and it is better to refer to the limit of proportionality. The yield point is the load per sq. in. when the measuring steel-yard first comes down against the stop for lever testing machines, or when the pressure pointer of a hydraulic testing machine falls back slightly, showing in each case a sudden alteration in the ratio of stress to strain.

The following are the measurements made in a commercial test, while the figures are typical for mild steel.

Original diameter of specimen	$=d_1$	$=$ 0·5 in.
Gauge length	$=l_1$	$=$ 8 in.
Final length	$=l_2$	$=$ 10·28 in.
Diameter at fracture	$=d_2$	$=$ 0·309 in.
Load at yield point	$=W_1$	$=$ 3·47 tons.
Maximum load	$=W_2$	$=$ 4·92 tons.

From these results the following calculations are made.

Original cross-sectional area $=0{\cdot}7854d_1^2=A_1=0{\cdot}196$ sq. in.

Area at fracture $=0{\cdot}7854d_2^2=A_2=0{\cdot}075$ sq. in.

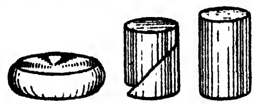
Percentage reduction in area $=\dfrac{A_1-A_2}{A_1}\times 100=61{\cdot}7.$

Percentage elongation $=\dfrac{l_2-l_1}{l_1}\times 100 \quad =28{\cdot}5.$

Yield point $=\dfrac{W_1}{A_1}=17{\cdot}7$ tons per sq. in.

Ultimate stress or strength $=\dfrac{W_2}{A_1}=25{\cdot}1$ tons per sq. in.

Compression tests. These are not as satisfactory as tensile tests for ductile materials. The latter barrel under large compressive loads (Fig. 291 (*a*)), which is in sharp contrast to the failure of brittle materials, such as cast iron (Fig. 291 (*b*)), that ultimately fail diagonally. Fig. 291 (*c*) shows the original size of each specimen. The value of Young's modulus is generally found to be the same in tension and compression.


(a) (b) (c)
FIG. 291.

EXERCISES ON CHAPTER X

1. Explain the terms stress, strain and elasticity. A piece of round bar $1\frac{1}{4}$ in. in diameter carries an axial load of $7\frac{1}{2}$ tons. Find the stress set up in the bar.

2. Find the maximum load a square section column of outside dimension 4 in. and $\frac{1}{2}$ in. thick can carry, if the direct stress is not to exceed 3 tons per sq. in.

3. If the fractional strain on a steel tie bar is not to exceed 0·002 and the permissible extension is 0·14 in., find the maximum length of the bar.

4. A cast iron strut, or column, 2 ft. in length and 4 in. diameter carries a load of 22 tons. Find (*a*) the intensity of stress, (*b*) the compression of the column if E is 8000 tons per sq. in.

5. A wire 0·024 in. in diameter and 94 in. in length is stretched 0·083 in. under a load of 12 lb.; find the value of Young's modulus for the material.

6. Explain the terms shearing, single shear, and double shear. Sketch examples from practice to show the existence of single and double shear.

7. A press is capable of exerting a force of 320 tons. What is the greatest thickness of plate through which a circular hole 11 in. in diameter may be punched with this machine? U.S.S. = 26 tons per sq. in.

8. A forked end carries an axis pin 1 in. in diameter which is in double shear. Find the ultimate and safe load for this pin if the U.S.S. of the material is 26 tons per sq. in. and the ratio between the ultimate and safe stress is to be 7 : 1.

9. A single riveted lap joint has rivet holes $\frac{1}{2}$ in. in diameter. The plates are $\frac{3}{16}$ in. in thickness and the U.S.S. of rivet material 25 tons per sq. in. while the U.T.S. of plate material is 30 tons per sq. in. Find the pitch required for equal strengths in shear and tension.

10. What is the nature of the stresses when buckling occurs? Give reasons why an excessively long strut is unsuitable in structural design.

11. Why is it important, in all structural work, to make sure that the load is centrally applied? Give some examples in which it is impossible to apply a central load.

12. State Hooke's Law and describe an experiment by means of which it is verified.

13. Why is it important commercially to determine the stress at which Hooke's Law fails? What is the name given to this stress, and what bearing has this stress upon the safe load which can be applied to a material?

14. State the objects of a test to destruction by tension upon a specimen of mild steel. Sketch a suitable test piece, and show a method of securing this test piece ready for test.

15. Sketch a characteristic load *v.* extension diagram for a mild steel specimen tested to destruction by tension. If this specimen is 1 sq. in. in cross-sectional area, mark a scale on both axes which will show, approximately, the behaviour of the material.

16. Explain the terms elastic period, ductile period and limit of proportionality when applied to a material under test.

17. How do cast iron, mild steel and a soft metal such as aluminium fail in compression?

18. In order to find the stress beyond which a material should not be loaded, what experiments would you perform, and how would you interpret your results and the graphs obtained from them ?

19. Sketch and describe a form of extensometer suitable for use with tension tests.

20. Outline the methods of failure, other than by tension and compression, and explain the nature of the stresses set up before failure.

21. A wire 0·038 in. diameter and 90 in. in length was stretched under a gradually increasing load, the extensions for the respective loads being as follows :

Load, lb. - -	0	5	10	15	20	25	27
Extension, in. -	0	0·017	0·033	0·05	0·067	0·082	0·089
Load, lb. - -	29	30	31	32	33	34	
Extension, in. -	0·095	0·099	0·104	0·11	0·121	0·15	

Plot the load extension graph using scales 1 in. to 10 lb. and 1 in. to 0·05 in. extension. Mark the elastic limit load on the graph and state its value. Find the force acting per sq. in. of sectional area and the extension per unit length when the total extension of the wire is 0·07 in. (U.L.C.I.)

22. Explain how force is measured and state what must be known about it in order that it may be represented graphically.

A wire of $\frac{1}{16}$ in. diameter and 10 ft. long is pulled with a force of 200 lb. and the increase of length is 0·102 in. Determine the ratio of the force per unit area to the extension per unit length. (U.L.C.I.)

23. State Hooke's Law. What do you understand by the *limit of proportionality of a material* ?

An iron tie bar is 40 ft. long, its section being 3 sq. in. Calculate the maximum load in pounds it can carry if its extension is not to exceed $\frac{1}{16}$ in. [Modulus of elasticity = 29,000,000 lb. per sq. in.] (U.L.C.I.)

24. The following data were recorded during an experiment in tension with a 0·035 in. diameter wire having a length of 110 in.

Load, lb. - -	0	10	12	14	16	18	20
Extension, in. -	0	0·041	0·046	0·056	0·064	0·074	0·082
Load, lb. - -	22	24	26	28	30	32	
Extension, lb. -	0·093	0·105	0·131	0·161	0·191	0·231	

Plot the load extension diagram and describe its characteristic features. Also estimate the stress at the elastic limit of the material in tons per sq. in. (U.L.C.I.)

25. Give the meaning of the terms "elastic" and "non elastic" deformation. Illustrate your answer by the behaviour of an iron or steel wire when loaded to destruction.

A steel wire 10 ft. long and 0·1 in. diameter stretched 0·152 in. when loaded with 300 lb. Find the ratio of the load per sq. in. to the extension per inch length. (U.L.C.I.)

26. What do you understand by the terms *strain*, *stress* and *modulus of elasticity* ? A tie rod 100 ft. long and 2 sq. in. sectional area is stretched by $\frac{3}{4}$ in. under a tension of 32,000 lb. What is the intensity of the stress, the strain, and the modulus of elasticity ? (U.L.C.I.)

27. To secure a tie rod for a roof truss, $3\frac{1}{2}$ in. wide and $\frac{5}{8}$ in. thick, three rivets each $\frac{3}{4}$ in. in diameter are used. If the rivets are in single shear and the load on the rod is 6 tons, calculate the mean shearing stress in the rivets and the tensile stress in the rod.

28. What is the meaning of the term *factor of safety* ? Calculate the diameter of a rod to take a load of 22 tons, (*a*) if the extension is not to exceed 0·005%, (*b*) if the factor of safety is 8 and the U.T.S. of the material of the rod 30 tons per sq. in. Modulus of elasticity $= 30 \times 10^6$ lb. per sq. in.

CHAPTER XI

FLEXURE AND FAILURE BY BENDING—BENDING MOMENT AND SHEAR FORCE DIAGRAMS FOR BEAMS—TORSION AND TORSION OF SHAFTS.

In the previous chapter some account has been given of the nature of the stresses induced when a material is subjected to bending, or flexure. If a ruler is held between the two hands and bent slightly, the impression is left with the holder that the material of the ruler is opposing his bending action and tending to straighten the ruler. Place the ruler against the edge of the bench and press with a force of say 1 lb. at a distance of 10 in. from the bench (Fig. 292) ; the ruler will bend and the bending will be resisted by the internal action of the material. The bending is caused by the force of 1 lb. at a distance of 10 in. from the bench, and the less the distance from the bench at which the force is applied, the less the bending action. In other words the bench behaves as a fulcrum, and the bending is caused by the force applied at a distance from the fulcrum, or the

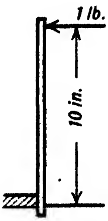


Fig. 292.

bending moment = force × distance,

and is equal to 1×10 lb. in.

The ruler will bend under the bending moment until equilibrium is obtained, that is, the internal elastic forces supply an equal and opposite moment to balance the bending moment; this opposition moment is called the **moment of resistance.**

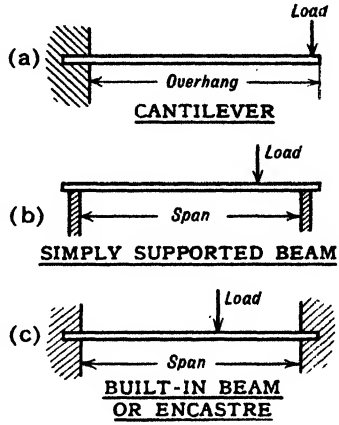


FIG. 293.

Thus, from this simple experiment an important relation is indicated, which is, that while a material remains elastic the moment of resistance is equal to the bending moment.

Cantilevers, beams and methods of loading. When a bar, carrying a load or loads inclined to its longitudinal axis, is firmly fixed at one end and the other end is free, the arrangement is known as a **cantilever** (Fig. 293 (*a*)). If the bar is supported at two or more places it is known as a **beam**, and when this beam is laid upon two supports (Fig. 293 (*b*)) it is said to be **simply supported.** The effect of building in the ends of a beam as shown in Fig. 293 (*c*) is to produce a **built in** or **encastré** beam, and when a single beam is carried upon more than two supports it is said to be a **continuous beam.** The treatment of encastré and continuous beams is outside the scope of this book, in which the mechanics of cantilevers and simply supported beams only will be considered.

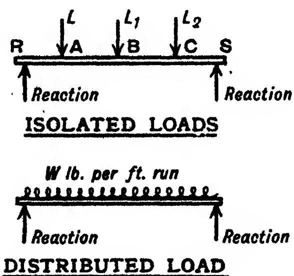


FIG. 294.

The loading of beams may consist of **isolated loads, or distributed loads** (Fig. 294), and many examples, drawn from practice, involve a combination of isolated and distributed loads. Further, in a more advanced treatment, beams and cantilevers may have rolling or moving loads. It is general practice, in the case of distributed loads, to specify the load as a number of lb., cwt. or tons per **foot run,** that is, per 1 foot length of the beam.

A portion of a machine or structure which is acted on by external forces, including loads and reactions, which are oblique to its longitudinal axis is called a **beam** or **cantilever** according to the number of supports. If the beam or cantilever is in equilibrium then the conditions of equilibrium already dealt with apply. Examples of beams and cantilevers may be found in practice in roof rafters, bridge girders, retaining walls, the keels of ships and deck structures.

Bending moment. *The bending moment at a cross-section of a beam is defined as the algebraic sum of the moments of all external forces taken on one side of the section considered.*

Again take a ruler, and arrange it as a cantilever from the edge of the table (Fig. 295). If a force of 2 lb. is applied 10 in. from AA the bending moment at AA is 2×10 or 20 lb. in., but if at the same time a further force of 1 lb. is applied in the opposite direction at B and 4 in. from AA, this opposite force will reduce the bending moment due to the 2 lb. force by an amount 1×4 lb. in.

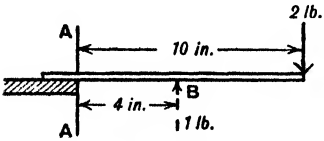


FIG. 295.

Thus the actual bending moment is the algebraic sum of the moments, that is

$$2 \times 10 - 1 \times 4 \text{ or } 16 \text{ lb. in.}$$

Suppose that it is required to find the bending moment about B; then on the right hand side of B a force of 2 lb. acts at a distance of 6 in. from B, and the bending moment about B is 2×6 or 12 lb. in.

Example 1. *Find the bending moments at* A *and* B *in the cantilever shown* (Fig. 296).

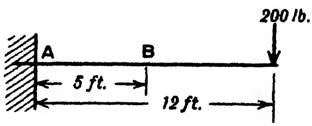


FIG. 296.

Bending moment at A $= 200 \times 12$
$= 2400$ lb. ft

Bending moment at B = moment of the force on one side of the section B
$= 200 \times 7 = 1400$ lb. ft.

Example 2. *Find the bending moments at* A *and* B *in the simply supported beam* (Fig. 297).

It is first necessary to obtain the reactions at the supports. As the

beam is centrally loaded each reaction will be one half the load, that is, 250 lb.

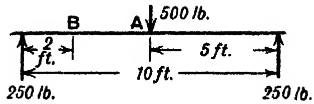


FIG. 297.

Bending moment at A.

Consider the right hand side of the section at A : the only force is one of 250 lb. with an anti-clockwise action, therefore,

$$\text{Bending moment at A in lb. ft.} = -250 \times 5 = -1250.$$

Bending moment at B.

On the right hand side of B there are two forces, (*a*) 500 lb. clockwise, (*b*) 250 lb. anti-clockwise.

$$\text{Bending moment at B in lb. ft.} = -8 \times 250 + 3 \times 500$$
$$= -2000 + 1500 = -500.$$

Sign convention. *The sign of a bending moment indicates the direction in which the moment operates, and, to be consistent, moments with a clockwise action are regarded as positive.*

Alternative solution to bending moment at B. Consider the left hand of the section at B ; then only one force is acting, 250 lb. in a clockwise direction, and the bending moment at B is $+250 \times 2$ or 500 lb. ft. ; this result is similar to that obtained for the right hand side of the section, but with an opposi' sign. Thus it is immaterial on which side of the section the moment is taken, as the sign merely indicates the direction of the moment but does not influence its magnitude.

Example 3. *Find the bending moments at* A *and* B *for the beam shown in Fig.* 298, *not centrally loaded.*

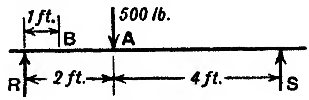


FIG. 298.

To find the reactions R and S.

Moments about R,

$500 \times 2 = S \times 6$.

$S = \frac{1000}{6} = 166{\cdot}7$ hence $R = 333{\cdot}3$.

$\therefore$ the reactions are 333·3 lb., and 166·7 lb.

$$\text{Bending moment at A in lb. ft.} = -S \times 4 = -166{\cdot}7 \times 4$$
$$= -666{\cdot}8.$$

$$\text{Bending moment at B in lb. ft.} = 500 \times 1 - S \times 5$$
$$= 500 - 166{\cdot}7 \times 5$$
$$= 500 - 833{\cdot}5 = -333{\cdot}5.$$

Treatment of uniformly distributed loads. When a beam carries a distributed load this load is reduced to the equivalent isolated load acting at the centre of gravity of the distributed load.

Example 1. *Find the bending moments for the beam shown in Fig.* 299 *at each of two sections situated* 5 *ft. and* 3 *ft. from the right hand support.*

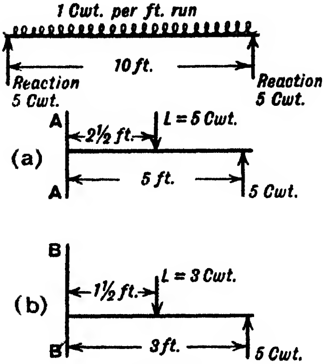


FIG. 299.

Reactions $= 10 \times 1 \times \frac{1}{2}$ or 5 cwt.

Consider the section AA (Fig. 299 (*a*)).

The equivalent isolated load on the right hand side of the section is 5 cwt. at a point $2\frac{1}{2}$ ft. from AA.

Bending moment in cwt. ft.

$$= 5 \times 2\tfrac{1}{2} - 5 \times 5 = 12\tfrac{1}{2} - 25$$
$$= -12\tfrac{1}{2}. \quad \textbf{Ans.} (a).$$

At the section BB (Fig. 299 (*b*)).

The equivalent isolated load is 3 cwt. at a point $1\frac{1}{2}$ ft. from BB.

Bending moment in cwt. ft.

$$= 3 \times 1\tfrac{1}{2} - 5 \times 3 = 4\tfrac{1}{2} - 15$$
$$= -11\tfrac{1}{2}. \quad \textbf{Ans.} (b).$$

Example 2. *Find the bending moments at* B *and* C *for the cantilever shown in Fig.* 300.

This is a combination of isolated and distributed load.

Bending moment at B in cwt. ft.

$$= 6 \times 3 + 8 \times 3 = 42. \quad \textbf{Ans.} (a).$$

Bending moment at C in cwt. ft.

$$= 3 \times 1\tfrac{1}{2} = 4\tfrac{1}{2}. \quad \textbf{Ans.} (b).$$

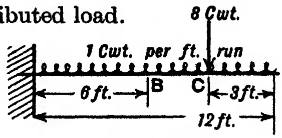


FIG. 300.

Bending moment diagrams. The foregoing examples have for their object the determination of the bending moment on a beam or cantilever at any point, with varied loading. It is now possible to study the change of bending moment throughout the length of the beam, or cantilever, and thus draw what is known as the **bending moment diagram.**

(*a*) *Consider a cantilever of length l ft. carrying an isolated load of W lb. at its free end.*

The bending moment (B.M.) at any point x ft. from the free end is Wx lb. ft. In equation form, B.M. $= Wx$, which is an equation of the form $y = mx$ where y and x are variables, and a form which is graphically represented by a straight line.

The maximum value of the B.M. is obtained when $x = l$ and the

bending moment diagram is a triangle as shown in Fig. 301 (*a*), in which the vertical ordinate represents the bending moment to scale.

For example, the bending moment at A can be obtained from the diagram by measuring the length of BB to the scale employed for Wl.

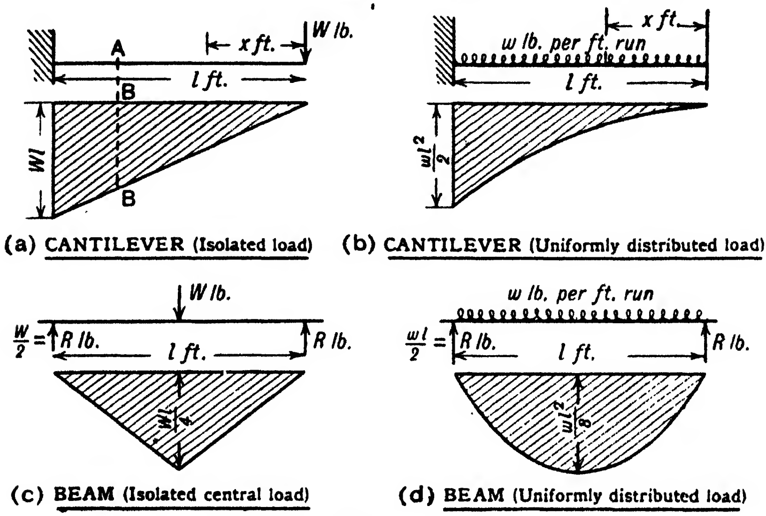


FIG. 301. Bending moment diagrams.

(*b*) *Consider the same cantilever carrying, instead of an isolated load, a uniformly distributed load of w lb. per foot run* (Fig. 301 (*b*)).

The bending moment at any point x ft. from the free end is

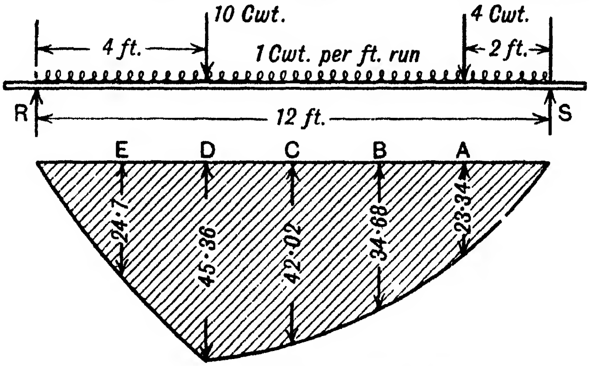
$$wx \times \frac{x}{2} \quad \text{or} \quad \frac{wx^2}{2} \text{ lb. ft.,} \quad \text{or} \quad \text{B.M.} = \frac{wx^2}{2},$$

when written in the form of an equation; this is an equation of the form $y = ax^2$, where y and x are variables, and is a form represented graphically by a curve, known as a parabola, and the bending moment diagram is a parabola with the maximum ordinate $wl^2/2$, that is, the bending moment when $x = l$ (Fig. 301).

Example. *To draw the bending moment diagram for the beam shown in Fig.* 302.

The bending moment diagram for a beam loaded in a complex manner may be obtained by two methods: (*a*) by calculating the bending

moments at a series of points along the length of the beam and plotting the values obtained as ordinates of the diagram ; (*b*) by drawing two bending moment diagrams (1) for the isolated loads, (2) for the distributed loads, and adding their ordinates to form a composite diagram.



BENDING MOMENT DIAGRAM

Fig. 302.

Method (*a*). Calculate the B.M. at intervals of 2 ft. from S. For the reactions R and S take moments about R,

$$4 \times 10 + 4 \times 10 + 12 \times 6 = 12 \times S$$

$$152 = 12S; \quad \mathbf{S = 12{\cdot}67,\ R = 13{\cdot}33.}$$

$\therefore$ reactions are 13·33 cwt. and 12·67 cwt.

Distance from S in ft.	Calculation of bending moment	B.M.
0	——	0
2	$2 \times 1 - 12{\cdot}67 \times 2$ cwt. ft.	$-23{\cdot}34$
4	$4 \times 2 + 4 \times 2 - 12{\cdot}67 \times 4$ cwt. ft.	$-34{\cdot}68$
6	$6 \times 3 + 4 \times 4 - 12{\cdot}67 \times 6$ cwt. ft.	$-42{\cdot}02$
8	$8 \times 4 + 4 \times 6 - 12{\cdot}67 \times 8$ cwt. ft.	$-45{\cdot}36$
10	$10 \times 5 + 10 \times 2 + 4 \times 8 - 12{\cdot}67 \times 10$ cwt. ft.	$-24{\cdot}7$
12	$12 \times 6 + 10 \times 4 + 4 \times 10 - 12{\cdot}67 \times 12$ cwt. ft.	0

Method (*b*). Draw the bending moment diagrams for the isolated loads and the distributed loads, separately, and combine them to form the composite bending moment diagram. In the following calculations the sections A, B, C, D and E are respectively 2, 4, 6, 8 and 10 ft. from **S.**

Isolated loads. For the reactions R and S.

Moments about R:

$$10 \times 4 + 4 \times 10 = 12S$$

$$12S = 80; \quad \mathbf{S} = 6\tfrac{2}{3} \text{ so that } \mathbf{R} = 7\tfrac{1}{3}.$$

$\therefore$ reactions are $7\frac{1}{3}$ cwt. and $6\frac{2}{3}$ cwt.

B.M. at A	$-2 \times S \quad = -13\frac{1}{3}$ cwt. ft.
B.M. at D	$4 \times 6 - 8S = -29\frac{1}{3}$ cwt. ft.

Distributed load. The reactions due to this load are equal **and** of amount $wl/2 = 6$ **cwt.**

B.M. at A	$2 \times 1 - 6 \times 2 \quad = -10$ cwt. ft.
B.M. at B	$4 \times 2 - 6 \times 4 \quad = -16$ cwt. ft.
B.M. at C	$6 \times 3 - 6 \times 6 \quad = -18$ cwt. ft.
B.M. at D	$8 \times 4 - 6 \times 8 \quad = -16$ cwt. ft.
B.M. at E	$10 \times 5 - 6 \times 10 = -10$ cwt. ft.

The B.M. at each support is zero.

The final diagram from this method will be found to be identical with that obtained by method (*a*). If preferred the diagrams can be drawn separately, one above and one below the base line.

Shearing force and shearing force diagrams. It has been shown that one of the stresses set up in a beam under conditions of bending is the vertical shear stress. The shearing force producing the stress may be calculated for a system of loading, and diagrams known as **shearing force diagrams** may be drawn to show the variation of this shearing force throughout the length of the beam.

If all the loads and reactions are at right angles to the plane of the beam, the shearing force at any section can be defined as the algebraic sum of the forces acting upon one side of the section.

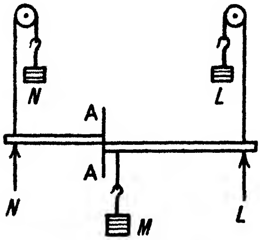


FIG. 303.

Fig. 303 gives an idea of the nature of this vertical shearing force. The tendency to shear along AA is brought about by the combined action of the forces L and M, and the shearing force is $M-L$, which is the algebraic sum of the forces on the right hand side of the section AA. It follows that the shearing force on the left hand side of AA, that is, N, must be equal to $M-L$.

Example. *Calculate the shearing force at each of the sections* A, B *and* C *in the beam loaded as shown in Fig. 294, where L, L_1 and L_2 are 5, 3 and 6 cwt. respectively, $RA=2$ ft., $RB=6$ ft., $RC=11$ ft., and span $=14$ ft.*

Reactions R and S.

Moments about R: $5\times 2+3\times 6+6\times 11=14S$,

$$14S=94,$$

$$S=6\tfrac{5}{7} \text{ so that } R=7\tfrac{2}{7}.$$

$\therefore$ reactions are $7\frac{2}{7}$ cwt. and $6\frac{5}{7}$ cwt.

S.F. at A -	$3+6-6\frac{5}{7}$ cwt.	$2\frac{2}{7}$ cwt.
S.F. at B -	$6-6\frac{5}{7}$ cwt.	$-\frac{5}{7}$ cwt.
S.F. at C -	$-6\frac{5}{7}$ cwt.	$-6\frac{5}{7}$ cwt.

The sign convention employed in these calculations is downward forces positive, upward forces negative on the right hand side of a section, and the reverse on the left hand side of the section.

NOTE.—If the shear force is calculated at a position slightly to the left of A, B or C, the loads acting at these points affect their respective shear forces, so that in passing across the point A from right to left the shearing force alters from $+2\frac{2}{7}$ cwt. to $+2\frac{2}{7}+5=7\frac{2}{7}$ cwt., and becomes equal to the reaction R.

Fig. 304 shows the shearing force diagrams corresponding to the bending moment diagrams for the beams loaded as shown in Fig. 301.

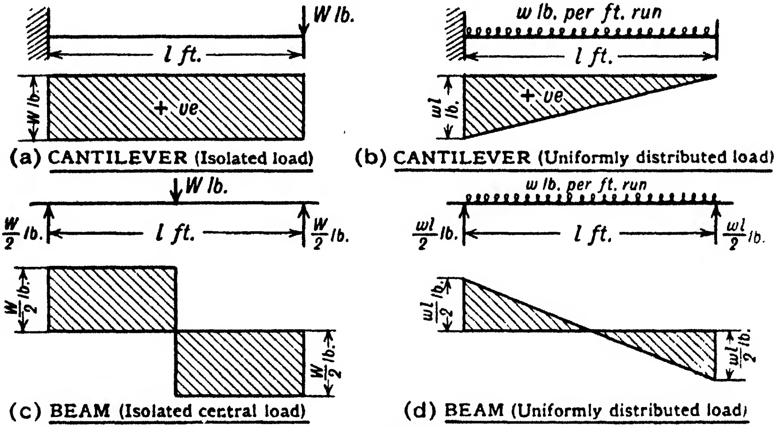


FIG. 304. Shearing force diagrams.

Example. *Draw the bending moment and shearing force diagrams for the beam loaded as shown in Fig.* 305.

For the reactions, take moments about R :

$$7S = 6 \times 2 + 5 \times 4 + 3 \times 6 + 8 \times 2 = 66 \text{ cwt. ft.}$$

$\mathbf{S} = 9\frac{3}{7}$, and $\mathbf{R} = 12\frac{4}{7}$, that is, the reactions $= 12\frac{4}{7}$ cwt. and $9\frac{3}{7}$ cwt.

At	On R.H. Side	B.M.	On R.H. Side	S.F.
A	$-S \times 1$	$-9\frac{3}{7}$	$-S$	$-9\frac{3}{7}$
B	$+3 \times 2 - S \times 3$	$-22\frac{2}{7}$	$-S + 3$	$-6\frac{3}{7}$
C	$5 \times 2 + 3 \times 4 + 4 \times 1 - S \times 5$	$-21\frac{1}{7}$	$-S + 5 + 3 + 4$	$2\frac{4}{7}$
D	$5 \times 1 + 3 \times 3 + 2 \times \frac{1}{2} - S \times 4$	$-22\frac{5}{7}$	$-S + 5 + 3 + 2$	$\frac{4}{7}$
E	$6 \times 1 + 5 \times 3 + 3 \times 5 + 6 \times 1\frac{1}{2} - S \times 6$	$-11\frac{4}{7}$	$-S + 6 + 5 + 3 + 6$	$10\frac{4}{7}$

Bending moments are in cwt. ft., shearing forces in cwt.

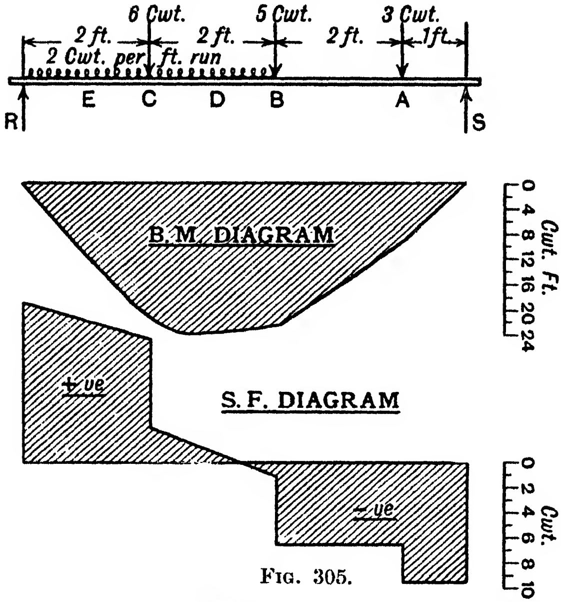


FIG. 305.

NOTE.—At the supports B.M. is zero, S.F. at $R = 12\frac{4}{7}$ cwt., S.F. at $S = -9\frac{3}{7}$ cwt.

Graphical determination of shearing force and bending moment. It is possible to draw the S.F. and B.M. diagrams for any system of loading by the methods outlined in the following example (Fig. 306).

METHOD OF PROCEDURE. Draw to scale the space diagram of the beam loading and letter it according to Bow's notation. Then draw the load line $A_1B_1C_1D_1$, and at a distance, preferably an integral distance, from this line mark a pole O. Join OA_1, OB_1, OC_1, and OD_1. The S.F. diagram is drawn by projecting horizontally from the points $A_1B_1C_1D_1$ in the load line to the projections of the spaces A, B, C, D. The zero shear line is obtained, after the B.M. diagram is drawn, by drawing a line through O which meets the load line in E_1 and is parallel to the closing line *pt* of the B.M. diagram. Then A_1E_1 represents the reaction R and D_1E_1 the reaction S, and the zero shear line is the horizontal through E_1. To obtain the B.M. diagram

draw, in the projected space A, a line pq parallel to OA_1, and from q one parallel to OB_1, until the polygon is completed by the closing line pt. Then the vertical height of this diagram measured at any

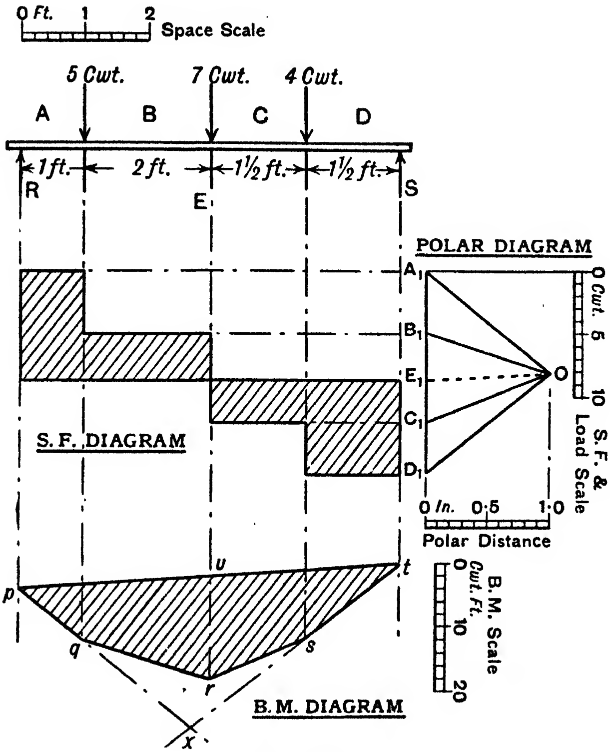


FIG. 306.

point is the bending moment at this position on the beam measured to some scale. To find the scale of the bending moment diagram, the scales of the space and polar diagrams are combined, so that :

Bending moment scale in cwt. ft. = space diagram scale × load line scale × polar distance, all taken in ft. cwt. units.

Scale example. If the space diagram is drawn to a scale of 1 in. = 2 ft., the load line to a scale of 1 in. = 10 cwt., and the polar distance is 1 in.,

then the bending moment diagram scale is 1 in. $=2\times10\times1$ cwt. ft., that is, 1 in. $=20$ cwt. ft.

Measurements of bending moment must always be made parallel to the loads. For example, *vr* represents the maximum B.M. to which the beam is subjected.

$$vr=0{\cdot}8 \text{ in., representing } 0{\cdot}8\times20 \text{ or } \mathbf{16 \text{ cwt. ft.}}$$

Check by calculation: B.M. at $v=1\frac{1}{2}\times4-3S$, $S=D_1E_1=7\frac{1}{3}$ cwt.; therefore B.M. $=6-22$ or $-$**16 cwt. ft.**

Distribution of stress in beams. It has been shown that the bending moment causing the bending of a beam is accompanied by a corresponding moment provided by the resisting forces tending to overcome the bending. This moment is known as the **moment of resistance,** and while the material composing the beam remains perfectly elastic, it follows that the

moment of resistance is equal to the bending moment.

Consider the cross-section of a rectangular beam of breadth b in. and depth d in. (Fig. 307 (a)).

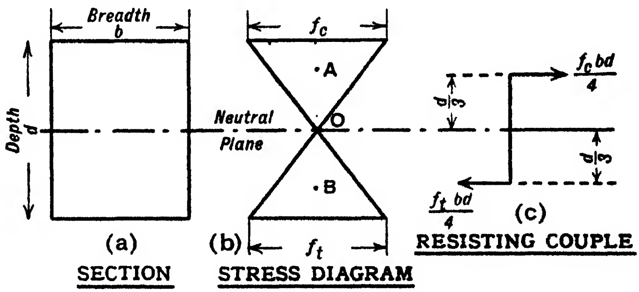


Fig. 307.

Along the top layer a compressive stress equal to f_c will be set up by the bending, and at the bottom layer a tensile stress f_t will be established. These stresses decrease towards the neutral plane of bending where the stress becomes zero, so that the stress diagram (Fig. 307 (b)) shows the distribution of stress over the cross-section. The compressive stress may be taken as concentrated at A, the centroid of the triangle representing the compressive stress, and is

equivalent to a force of $\frac{f_c}{2} \times \frac{1}{2}$ (area of section), that is, $\frac{f_c \times bd}{2 \times 2}$ lb. acting at a distance OA $= \frac{2}{3} \times \frac{d}{2}$ from the neutral plane. Similarly the tensile stress may be considered concentrated in a force of $\frac{f_t \times bd}{2 \times 2}$ lb. at B where OB $= \frac{2}{3} \times \frac{d}{2}$ from the neutral plane.

Thus the moment of resistance is represented by the couple shown in Fig. 307 (*c*) in which the forces are respectively $\frac{f_c bd}{4}$ and $\frac{f_t bd}{4}$, and the arm of the couple is $\frac{d}{3} \times 2$.

If the tensile and compressive stresses due to bending are equal, that is, $f_c = f_t = f$, the **moment of resistance** in lb. in. $= \frac{fbd}{4} \times \frac{2d}{3} = \frac{fbd^2}{6}$.

Therefore, for a rectangular section the bending moment = moment of resistance $= f \times \frac{bd^2}{6}$.

The quantity by which the stress has to be multiplied in order to give the moment of resistance is called the **modulus of section** and often written Z.

Thus, when f is the maximum stress

bending moment = moment of resistance = f × modulus of section.

Example 1. *Calculate the maximum stress set up in a beam of cross section,* 3 *in. in width,* 5 *in. in depth, if the maximum bending moment is* 12 *tons ft.*

$$\text{Modulus of section} = \frac{bd^2}{6} = \frac{3 \times 5^2}{6} = 12{\cdot}5 \text{ in.}^3 \text{ units.}$$

$$\text{Bending moment} = 12 \text{ tons ft.} = 144 \text{ tons in.}$$

$$\text{B.M.} = f \times Z, \quad 144 = f \times 12{\cdot}5.$$

$$f = 11{\cdot}52. \quad \textbf{Ans. } 11{\cdot}52 \textbf{ tons per sq. in.}$$

NOTE.—In all calculations of this type, consistency of units must be preserved, and the bending moment be reduced to tons in. in order to obtain the stress in tons per sq. in. with a modulus of section in in.^{3} units.

Example 2. *Find the greatest central load which may be carried by a joist 9 in. in depth, $4\frac{1}{2}$ in. in width and 10 ft. span, in order that the stress may not exceed $\frac{1}{2}$ ton per sq. in.*

Let the load be L tons.

Bending moment at the centre in tons ft. $=\frac{LS}{4}=\frac{L\times 10}{4}=2\frac{1}{2}L.$

$=30L$ tons in.

Modulus of section Z in in.^{3} $=\frac{bd^2}{6}=\frac{4\frac{1}{2}\times 81}{6}=60{\cdot}75.$

$$\text{B.M.}=fZ=\tfrac{1}{2}\times 60{\cdot}75=30{\cdot}375.$$

$30L=30{\cdot}375$, $L=1{\cdot}01$. **Ans. 1·01 ton.**

Reinforcement of beams. Very few materials possess an equal strength in compression and tension, so that it frequently becomes necessary to strengthen a beam in the portion which has to resist tension. For example, cast iron and concrete are relatively strong in compression but very weak in tension, and cast iron beams are strengthened on the tension side. Thus the position of the neutral plane of bending is lowered and, in the case of flanged girders a larger area is given to the tension flange (Fig. 308 (*a*)).

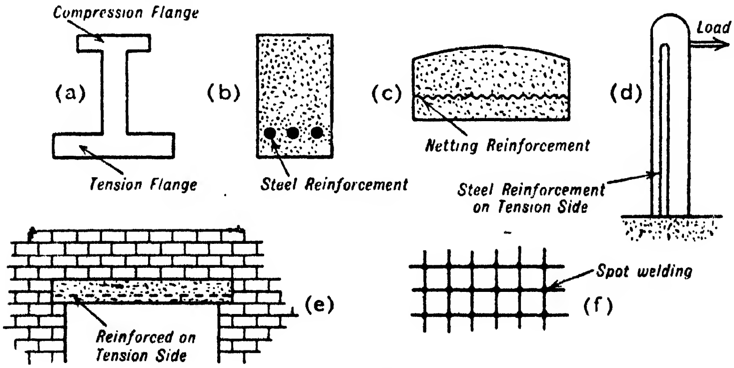


FIG. 308.

Fig. 308 (*b* to *e*) shows methods of reinforcing concrete to strengthen the tension side. In the case of an ordinary beam (*b*) and (*e*), steel rods are cast into the concrete on the tension side and these steel rods take the majority, if not all, the stress due to tension. It is often the practice to reinforce the base of concrete roads with wire

netting or a network of steel wires, spot welded at the joints (*e*) and (*f*). Posts which have to resist a bending action are often reinforced by a steel rod, bent back upon itself to a U shape and inserted on the tension side of the concrete (*d*). Composite beams of concrete and steel are possible because these materials expand at nearly equal rates when the temperature rises.

Standard rolled steel sections. The modern demand for structures framed in steel has led to the steel manufacturers producing standard sections (Fig. 309), which are of shapes lending themselves to ready use in built up structures. The makers provide tables giving the Z or modulus of section value for each of these sections,

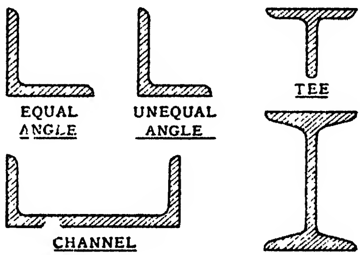


Fig. 309.

and the dimensions have so far been standardised that it is seldom necessary to depart from the stock sizes carried by the makers.

Treatment of an I section. The importance of this section calls for a special reference to its general treatment. If the radii of the fillets are ignored, it is general practice to consider that the flanges take the stresses due to bending, that is tension and compression, while the web takes the shear.

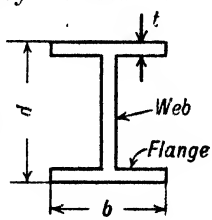


Fig. 310.

If f tons per sq. in. is the stress in the flanges due to bending and the area of each flange is bt sq. in.

then the force on each flange $= fbt$ tons,

and the moment of resistance $= fbt \times \frac{d}{2} \times 2$ tons in.

$= f \times$ modulus of section.

Modulus of section = btd in.³ units approximately.

Example. *Find the uniformly distributed load which may be carried by an I section girder, of span 12 ft., if $b=4$ in., $d=6$ in., and $t=\frac{5}{8}$ in. when the maximum allowable stress is not to exceed 5 tons per sq. in.*

$$\text{Modulus of section } Z \text{ in in.}^3 = btd = 4 \times \tfrac{5}{8} \times 6 = 15.$$

Let L = load in tons per ft. run and S the span.

$$\text{Maximum bending moment} = \frac{LS^2}{8} \text{ tons ft.} = \frac{L \times 144^2}{8} \text{ tons in.}$$

Then
$$\frac{L \times 144^2}{8} = fZ = 5 \times 15.$$

$$L \text{ in tons per ft. run} = \frac{5 \times 15 \times 8}{144 \times 144} \quad \text{or} \quad \frac{25 \times 2240}{864} = 64{\cdot}8 \text{ lb. per in. run.}$$

Ans. 777·8 lb. per ft. run.

Experimental treatment of bending.

EXPT. 28. OBJECT. *To compare the deflection of the free end of a cantilever with the load producing it.*

APPARATUS. A strip of spring steel about 26 in. in length, 1 in. wide and $\frac{1}{10}$ in. thick, a weight hanger, weights and a scale. The end of the strip is clamped by two fly-nuts and a plate to a suitable stand (Fig. 311), securely fastened, or weighted. A slight notch is made in each edge of the strip near to the free end, and a wire loop passed around the notched portion to carry the load and hanger. The deflection is obtained by reading the difference in the heights of the free end of the cantilever above the bench before and after loading.

OBSERVATIONS. Length of cantilever unsupported = 24 in.

Load in lb. -	0·1	0·2	0·3	0·4	0·5	0·6	0·7
Deflection, in.	0·18	0·36	0·54	0·71	0·89	1·07	1·25

GRAPH AND DIAGRAM OF APPARATUS.

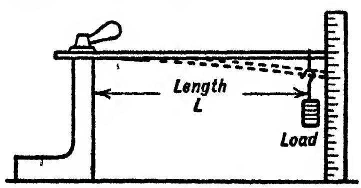


FIG. 311.

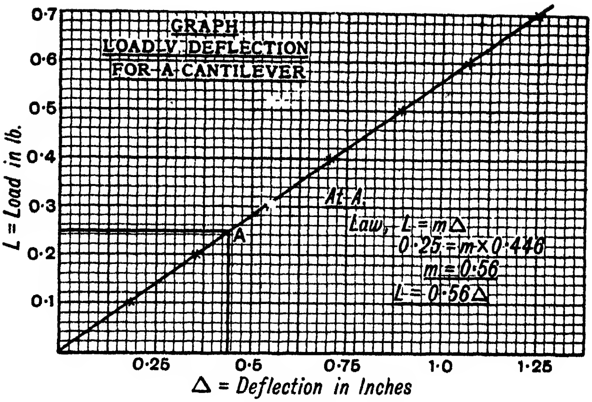


FIG. 312.

CONCLUSIONS. The deflection of a cantilever is proportional to the load and the law connecting load and deflection for this cantilever is $L = 0{\cdot}56\Delta$ (Fig. 312).

EXPT. 29. OBJECTS. (*a*) *To obtain a graph showing the relationship between the deflection of the centre of a beam and the bending moment producing it, when the beam is bent to a circular arc.*

(*b*) *To find the law connecting the deflection and the bending moment under this condition of loading.*

NOTE.—The theory of simple elastic bending is founded upon premises which assume that the beam bends to the portion of a circle, if there is no shearing force and the bending moment is uniform throughout its length. The ordinary method of loading a beam between two supports gives a variable bending moment and the arc of curvature is not a portion of a circle. For this experiment the bending moment throughout the beam between the supports is constant, and this is achieved by supporting the beam freely at two supports A and B (Fig. 313), and supplying the bending moment by two loads L at equal distances from the supports on the outside. Thus the constant B.M. is $L \times x$ lb. ft.

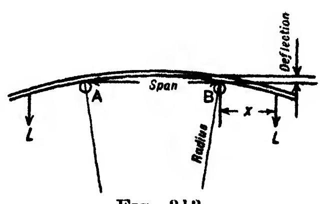


FIG. 313.

APPARATUS. A convenient apparatus for this experiment is a lathe bed and two supports consisting of cylinders of steel or knife

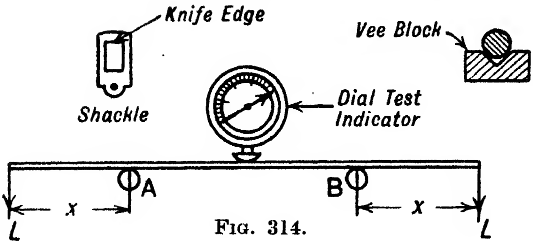


Fig. 314.

edges supported by vee-blocks placed on the lathe bed. The deflection is measured directly by means of a dial test indicator and the load is applied to shackles of known weight (Fig. 314). The specimen may consist of a length of bright steel bar $\frac{5}{8} \times \frac{3}{8}$ in. in cross-section and about 40 in. in length.

Observations.

Load, lb., L -	5	10	15	20	25	30
Deflection, in. -	0·027	0·054	0·082	0·109	0·137	0·16
Distance, x in. -	4	4	4	4	4	4
Bending moment, $L \times x$ lb. in. -	20	40	60	80	100	120

Graph and conclusions. The deflection of a beam, simply supported and loaded to bend to the arc of a circle, is proportional to the bending moment. The law connecting B.M. and deflection in this particular case is $\mathbf{M} = 740\Delta$ (Fig. 315).

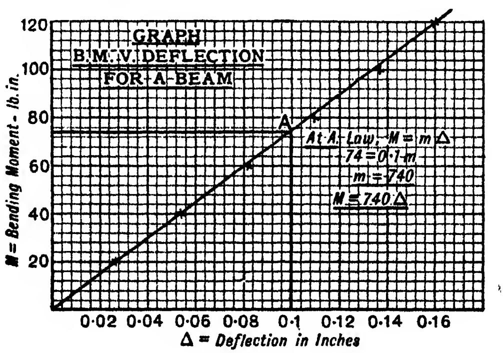


Fig. 315.

Stiffness of a beam or cantilever. The stiffness of a beam or cantilever depends upon the deflection, and one of the conclusions which may be deduced from the previous experiments is that the stiffness is inversely proportional to the deflection, which in turn is directly proportional to the bending moment. For beams of different sizes the deflection is proportional to the cube of the length, and inversely proportional to the breadth and the cube of the depth.

EXPT. 30. OBJECTS. (*a*) *To test to destruction by bending a series of wooden beams, simply supported and centrally loaded.*

(*b*) *To obtain graphs showing the relation between deflection and load for these beams.*

(*c*) *To find the modulus of rupture for each of the timbers from which the beams are made.*

APPARATUS. Beam testing machine. The load is applied by means of a screw and handwheel through a spring balance which records the central load (Fig. 316). This upward load draws the beam at each end against steel rods which are passed through holes in the bottom cross framing and so form the beam supports. The upward movement of the beam centre, or the deflection, is taken up by a light cord, which is weighted after being coiled around a spindle; the latter moves a pointer over a calibrated dial.

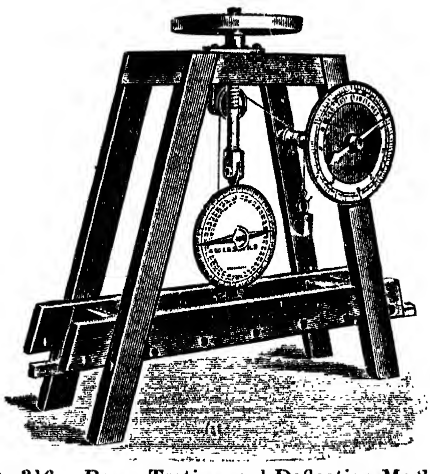


FIG. 316. Beam Testing and Deflection Machine. (Messrs. G. Cussons Ltd.)

Observations. Dimensions of the beams :

span = 2 ft., breadth = 1 in., depth = 1 in.

Material -	20	60	100	140	180	220	260	300	340	360	Load, lb.
Oak - -	0·02	0·08	0·16	0·25	0·34	0·44	0·55	0·69	0·85	0·94	Deflection, in.
Material -	20	60	120	140	160	200	240	280	290	—	Load, lb.
American Whitewood	0·03	0·12	0·25	0·31	0·36	0·48	0·66	0·90	1·04	—	Deflection, in.
Material -	20	60	100	140	180	200	220	230	240	—	Load, lb.
Mahogany -	0·04	0·14	0·26	0·38	0·53	0·61	0·72	0·80	0·89	—	Deflection, in.
Material -	20	40	60	80	100	120	140	160	180	190	Load, lb.
Yellow Deal	0·06	0·12	0·17	0·24	0·30	0·37	0·45	0·54	0·64	0·73	Deflection, in.

Graphs.

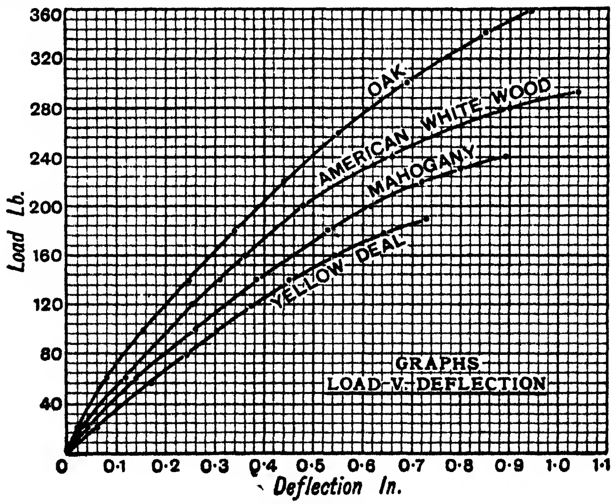


Fig. 317.

Conclusions from the graph. The graph of load v. deflection, in each case, is a curve, so that the deflection is not proportional to the load (see Fig. 317).

Derived results. In this type of test it is customary to assume that the beam remains elastic until fracture, and according to this

assumption the bending moment at fracture $= Z \times$ stress. The stress calculated from this expression is not the true stress but may be used for comparative purposes between beams made of different materials and is known as the **modulus of rupture.**

Material	Ultimate load, lb.	Ultimate B.M., lb. in.	Modulus of section	Modulus of rupture, lb. per sq. in.
Oak - - -	360	2160	$\frac{1}{6}$ in.^{3}	12,960
American Whitewood	290	1740	$\frac{1}{6}$ in.^{3}	10,440
Mahogany - -	240	1440	$\frac{1}{6}$ in.^{3}	8,640
Yellow Deal - -	190	1140	$\frac{1}{6}$ in.^{3}	6,840

NOTE.—Timber as a material is of very variable strength, and every result must be taken as applying to the particular specimen considered.

Relation between linear dimensions and strength of a beam. Consider a simply supported beam of span S ft., depth d in., and breadth b in. carrying a central load of W lb.

$$\text{Bending moment} = \frac{WS}{4} \text{ lb. ft.} = \frac{3WS}{1} \text{ lb. in.}$$

Since B.M. = modulus of section $Z \times$ stress f,

$$\text{stress } f = \frac{\text{B.M.}}{Z} \quad \text{and} \quad Z = \frac{bd^2}{6},$$

so that

$$f = \frac{3WS \times 6}{bd^2} = \frac{18WS}{bd^2}.$$

The strength of the beam depends upon the stress set up in its cross-section. From the above equation, it will be seen that the stress is proportional to the span and inversely proportional to the breadth and square of the depth. This may be stated as follows:

The strength of a beam is proportional to the breadth, and to the square of the depth, but is inversely proportional to the span.

Example. *A beam of 10 in. span, 1 in. in breadth and 2 in. deep, will safely carry a central load of 300 lb. Find the safe load of a beam of similar material 30 in. span, 2 in. in breadth and 6 in. in depth.*

Effect of increased dimensions on the safe load :

(*a*) Span trebled, safe load $= \frac{1}{3} \times 300$ lb.

(*b*) Breadth doubled, safe load $= 2 \times \frac{1}{3} \times 300$ lb.

(*c*) Depth trebled, safe load $= 3^2 \times 2 \times \frac{1}{3} \times 300$ lb.

Safe load = 1800 lb.

Torsion of shafts. When a shaft is subjected to torsion, or twisting, one portion is secured and the other portion is acted upon by a true couple (Fig. 46). If the **twisting moment** or **torque** is not the sole action of a couple, bending will result. If the loading of a shaft in pure torsion is continued, the ultimate cause of failure will be shearing; but if bending is also taking place, this shearing will be accompanied by tension or compression along the line of the shaft axis, or parallel to it.

EXPT. 31. OBJECTS. (*a*) *To compare the angle of twist of a shaft with the torque producing it, the length of shaft under twist remaining constant.*

(*b*) *To compare the angle of twist with the length of shaft under twist when the torque is maintained constant.*

(*c*) *To compare the angle of twist with the diameter of various shafts of the same material, when the torque and length are constant.*

APPARATUS. Torsion testing machine. A simple form is shown in Fig. 318. The torque is applied at the wheel rim in the form of a couple and the shaft is passed through the boss of this wheel and gripped by three symmetrically placed set screws. Movement at the other end of the shaft is prevented by gripping the shaft in the frame of the machine in a similar way. The angle of twist is measured by a radial pointer clamped to the shaft and moving over a concentric

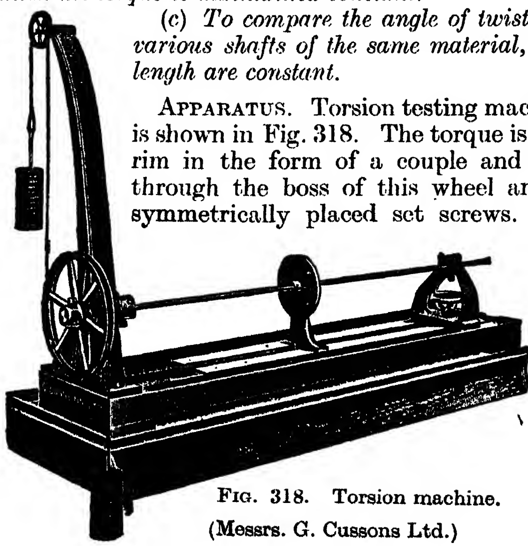


FIG. 318. Torsion machine.
(Messrs. G. Cussons Ltd.)

scale marked in degrees. The length under twist is measured between the planes containing the fixing set screws and the pointer screw.

OBSERVATIONS.

Expt. (*a*) Diameter of shaft $= \frac{3}{8}$ in.
Length of shaft $= 45$ in.
Material of shaft—steel.
Diameter of torque wheel $= 10$ in.

Load, lb.	2·2	4·4	6·6	8·8	11	13·2
Twisting moment or torque, T lb. in.	22	44	66	88	110	132
Angle of twist, θ degrees	2	4	6	8	10	12
Ratio, $\frac{T}{\theta}$	11	11	11	11	11	11

Expt. (*b*) Diameter of shaft $= \frac{3}{8}$ in.
Constant torque $= 200$ lb. in.
Material of shaft—steel.

L = length under twist, in.	0	5	15	25	35	45
θ = angle of twist, degrees	0	$2\frac{1}{2}$	7	12	17	21
Ratio, $\frac{L}{\theta}$	—	2·0	2·14	2·08	2·06	2·14

Expt. (*c*) Material of shaft—steel.
Length of shaft $= 40$ in.
Constant torque $= 50$ lb. in.

Diameter of shaft, D in.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$
Angle of twist, θ degrees	25	11	5	3	$1\frac{1}{2}$	1
$\frac{1}{\theta}$	0·04	0·091	0·20	0·333	0·667	1
D^4	0·0039	0·0095	0·0198	0·0366	0·0625	0·100

GRAPHS.

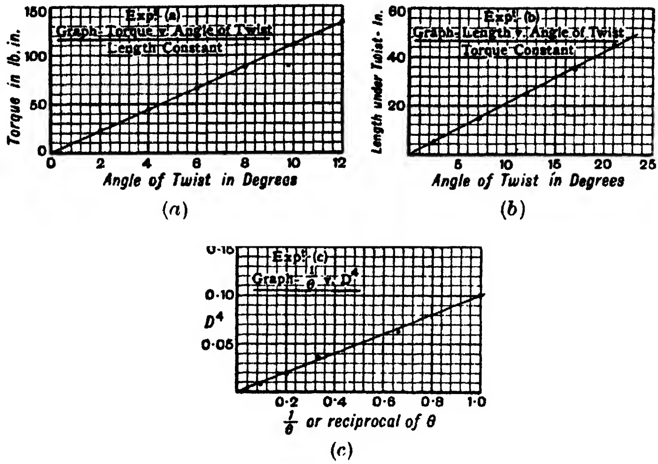


FIG. 319.

CONCLUSIONS. From Expt. (*a*) the graph of torque *v*. angle of twist is a straight line through the origin, and therefore, *torque is proportional to angle of twist.*

From Expt. (*b*) the graph of length under twist *v*. angle of twist is a straight line through the origin, therefore *length under twist is proportional to the angle of twist.*

From Expt. (*c*) the graph of the (diameter)^4 *v*. the reciprocal of the angle of twist is a straight line through the origin, therefore *the (diameter)^4 is inversely proportional to the angle of twist.* It should be noted that these conclusions provide the conditions governing the stiffness of shafts. The stiffness of a shaft is inversely proportional to the angle of twist.

EXERCISES ON CHAPTER XI

1. Draw diagrams to show the stresses set up in a beam subjected to a simple bending action.

2. What is meant by the neutral plane of bending? Show in a diagram the position of this plane for a rectangular section, and draw the stress diagram for this section if the maximum stress is 5 tons per sq. inch.

3. A cantilever is 13 ft. in length and carries an end load of 520 lb.; calculate the bending moment at the support and 5 ft. from the support.

4. Explain the term shearing force when applied to a loaded beam. Sketch the shearing force diagram for (*a*) a cantilever with an end load, (*b*) a beam simply supported and centrally loaded, (*c*) a beam simply supported and uniformly loaded throughout its length.

5. A cantilever 20 ft. in length carries loads of 10, 7 and 4 cwt. at distances 3, 9 and 11 ft. from its support respectively. Draw the shearing force and bending moment diagram for the cantilever, and state the maximum value of each.

6. A beam of 10 ft. span, simply supported at its ends, carries a concentrated central load of 14 cwt. and a uniformly distributed load of 1 cwt. per ft. run. Draw the bending moment and shearing force diagrams, and state the maximum value for each.

7. A steel bar, $1\frac{1}{2}$ in. in width, $\frac{3}{4}$ in. in depth, is loaded at the centre of a span of 2 ft. 6 in. Calculate the greatest stress in the section when the load is 200 lb.

8. What load could the bar in Question 7 carry as a centrally loaded beam, simply supported, if the maximum stress is not to exceed 5 tons per sq. in.?

9. An I section in which the top flange is $5 \times \frac{3}{4}$ in., the bottom flange $8 \times 1\frac{1}{4}$ in., and the web $9 \times \frac{1}{2}$ in., is used as a beam. Find the position of the neutral plane of bending.

10. Find the modulus of section for an I beam of flanges $6 \times \frac{3}{4}$ in. and web $5 \times \frac{1}{2}$ in. Use the approximate rule.

11. An I section of breadth 6 in., equal flanges of $\frac{3}{4}$ in. in thickness and overall depth 12 in., is to carry a central load as a beam of 10 ft. span. Find the maximum load if the stress is not to exceed 4 tons per sq. in.

12. A beam of span 12 ft. carries two loads of 4 tons and 3 tons respectively 5 ft. and 8 ft. from one support. Draw the shearing force and bending moment diagrams by the graphical method, and state the maximum shearing force and bending moment and the reactions of the beam.

13. Make sketches of some of the more important standard rolled steel sections.

14. How can the strengths and stiffnesses of two beams be compared?

15. Describe how cast iron and concrete beams may be strengthened to resist bending.

16. Calculate the modulus of rupture for a specimen of cast iron if its modulus of section is 0·15 in.^{3} units and the central breaking load is 1·14 tons on a span of 18 in.

17. A beam of 10 ft. span, 3 in. in breadth and 8 in. in depth, will safely carry a central load of 1 ton. What is the safe central load of a similar beam of 20 ft. span, 4 in. in breadth and 10 in. deep?

18. What do you understand by the terms angle of twist of a shaft and stiffness of a shaft ? What is the relation between them ? State in what way the angle of twist is affected by changes of diameter, length upon which the twist is measured, and the torque applied.

19. Describe an experiment to show that, if the length and diameter of a shaft are kept constant, the torque applied is directly proportional to the angle of twist produced.

CHAPTER XII

MOTION—LAWS OF MOTION—MOMENTUM—IMPULSE AND FORCE.

WHEN a body changes its position in a certain time it is common to refer to the rate of change of position as the **speed** of the body. For example, a car may change its position a distance of 2 miles in 10 minutes, in which case the speed would be 12 miles per hour.

The term velocity must not be confused with speed, because velocity is a vector quantity, that is, it depends upon both speed and direction, whereas speed is purely a scalar quantity. An illustration of this distinction may be taken from a car circling a track at 60 miles per hour. The speed remains at 60 m.p.h. throughout, but the velocity is constantly changing direction and thus is not fully defined as 60 m.p.h., but must be stated as 60 m.p.h. in a certain direction (Fig. 320).

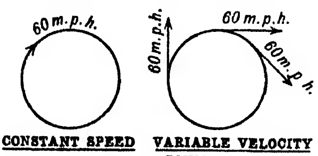


FIG. 320.

The motion of the centre of gravity of a body or a particle may be

(a) in a straight line without change of speed,
(b) in a straight line with increase or decrease of speed, that is with acceleration or retardation,
(c) in a curve or irregular path, with or without linear acceleration or retardation.

Cases (b) and (c) can be shown to be the result of the action of a force upon the body.

Definitions. **Velocity** *is the rate of change of position* and is expressed by the ratio $\frac{\text{distance covered}}{\text{time taken}}$, with an additional reference to the direction in which the motion takes place.

Uniform velocity *is velocity which does not vary however small the interval of time.*

For example, if a train were travelling at 60 m.p.h., in order that the velocity may be uniform, the train would have to travel at 1 mile in *any* minute, $\frac{1}{60}$ mile in *any* second and $\frac{1}{6000}$ mile in *any* $\frac{1}{100}$ second and so on, no matter how small the interval of time considered.

Acceleration *is the rate of change of velocity* and is expressed by the ratio $\frac{\text{change of velocity}}{\text{time taken}}$. This is also a vector quantity and reference must be made to the direction in which the acceleration is taking place.

Uniform acceleration *is acceleration which does not vary however small the interval of time.*

Retardation *is the name given to the reverse process to acceleration, that is, negative acceleration or a reduction of velocity.*

Units and measurement. The measurement of velocity and acceleration, and the units employed are very important features of the ensuing work. It is advisable to employ the **foot, pound, second (F.P.S.) units** for all calculations; that is, distances in feet, forces in pounds and times in seconds. For example, a car moving in a straight line is retarded from 60 m.p.h. to 40 m.p.h. in 4 sec. The velocity at the commencement of the observation is 60 m.p.h., or the distance covered in 60 miles in a time of 1 hour. The change of velocity is 20 m.p.h. (60–40 m.p.h.) and the time taken for this change is 4 sec., so that the retardation is $\frac{\text{change of velocity}}{\text{time taken}}$, which is 20 miles per hour per 4 sec. This can be expressed as 5 miles per hour per second, or in F.P.S. units,

$$\frac{5 \times 5280}{3600} \text{ ft. per sec. per sec.} = 7\tfrac{1}{3} \text{ ft. per sec. per sec.,}$$

which is sometimes written $7\frac{1}{3}$ ft. per sec.^{2}.

Graphical representation. If a graph is drawn of velocity against time, during any period of motion, the area under this graph will represent the space passed over (Fig. 321). This may be verified from the definition of velocity; that is $v = s/t$ or $s = vt$ where s is the space, v the velocity and t the time. When F.P.S. units are

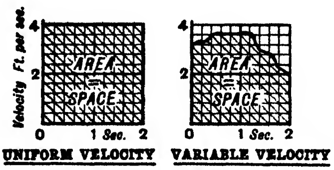


Fig. 321.

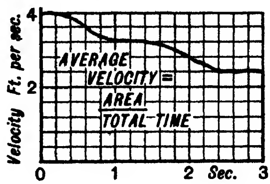


Fig. 322.

employed for velocity and time, the resulting area will represent the space in feet. This has a useful application when it is required to find the average velocity over a given time. If a graph of velocities at any instant against time is plotted (Fig. 322), the area under this graph when divided by the total time will give the average velocity.

An important conversion. It is frequently required to express a velocity of miles per hour in feet per second.

$$60 \text{ miles per hour} = 1 \text{ mile per minute.}$$
$$= \tfrac{5280}{60} \text{ or } 88 \text{ ft. per second.}$$

Thus, **60 miles per hour is equivalent to 88 feet per second,**

or, **15 miles per hour = 22 feet per second.**

Derivation of the formulae for uniformly accelerated motion in a straight line. Consider the motion of a body possessing a uniform acceleration in a straight line.

In F.P.S. units, let

s be the space passed over in ft.,
u the initial velocity in ft. per sec.,
v the final velocity in ft. per sec.,
a the acceleration in ft. per sec.^{2}.
t the time in seconds.

(a) Suppose the body starts from rest and receives an acceleration of 1 ft. per sec.^{2} for 3 sec.

The velocity time diagram is shown in Fig. 323.

The area under the graph = space $=\frac{v}{2}\times t$, so that

$$s=\frac{vt}{2} \quad \text{and} \quad v=at.$$

(*b*) If the body has an initial velocity of 2 ft. per sec. and receives an acceleration of 1 ft. per sec.² for 3 sec.

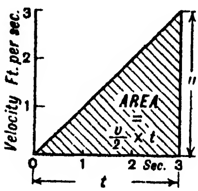


FIG. 323.

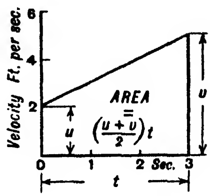


FIG. 324.

The velocity time diagram is shown in Fig. 324.

The area under the graph

$$=\text{space}=\left(\frac{u+v}{2}\right)\times t, \text{ or average velocity} \times \text{time},$$

so that $$s=\left(\frac{u+v}{2}\right)\times t. \quad\ldots\ldots(1)$$

and $$v=u+at. \quad\ldots\ldots(2)$$

Substituting the value of v in (2) for v in (1),

$$s=\frac{u+v}{2}\times t=\frac{u+u+at}{2}\times t=\frac{2ut+at^2}{2},$$

and $$s=ut+\tfrac{1}{2}at^2. \quad\ldots\ldots(3)$$

Squaring both sides of (2),

$$v^2=(u+at)^2=u^2+2uat+a^2t^2$$

$$=u^2+2a(ut+\tfrac{1}{2}at^2),$$

and since in (3) $$s=ut+\tfrac{1}{2}at^2,$$

$$v^2=u^2+2as. \quad\ldots\ldots(4)$$

Summary of the formulae :

$$(1)\ s = \left(\frac{u+v}{2}\right)t.$$

$$(2)\ v = u + at.$$

$$(3)\ s = ut + \tfrac{1}{2}at^2.$$

$$(4)\ v^2 = u^2 + 2as.$$

Use of the formulae. These formulae will, with proper selection, provide the solution to most problems involving uniformly accelerated motion, and the following examples will explain the methods adopted to determine the one unknown quantity when the other quantities are known.

Example 1. *A vehicle moving at 15 m.p.h. is uniformly accelerated to a speed of 30 m.p.h. Find the acceleration and the distance travelled during acceleration if the time is 10 sec.*

Given :

$u = 15$ m.p.h. $= 22$ ft. per sec.

$v = 30$ m.p.h. $= 44$ ft. per sec.

$t = 10$ sec.

To find : a the acceleration and s the space.

Method. Search for a formula which includes the above given quantities and possesses but one unknown quantity, which is the quantity required.

For a,

$$v = u + at.$$

$$44 = 22 + a \times 10.$$

$a = 2{\cdot}2$. **Ans. (1) 2·2 ft. per sec.^{2}.**

For s,

$$s = \left(\frac{v+u}{2}\right)t = \frac{44+22}{2} \times 10 = 330.$$ **Ans. (2) 330 ft.**

Example 2. *An electric train is accelerated at 1 ft. per sec.^{2}. Find the distance travelled before a speed of 45 m.p.h. is reached from an initial speed of 15 m.p.h.*

Given :

$a = 1$ ft. per sec.^{2},

$u = 22$ ft. per sec.,

$v = 66$ ft. per sec.

To find s :

$$v^2 = u^2 + 2as,$$

$$66^2 = 22^2 + 2 \times 1 \times s,$$

$$s = \frac{66^2 - 22^2}{2} = 1936$$ **Ans. 1936 ft.**

Use of the formulae in retardation examples. It should be remembered that retardation can be regarded as a negative acceleration, and in retardation examples the formulae may still be employed if the acceleration term is regarded as negative.

Example. *The following figures are taken from the data sheet for a light car. Find the average retardation and time of braking under these conditions :* 30 *m.p.h. to rest in* 32 *feet*

Given : $u = 30$ m.p.h. $= 44$ ft. per sec.,
$v = 0$ or the velocity at rest,
$s = 32$ ft.

To find a and t :

For a : $v^2 = u^2 - 2as$,

$0 = 44^2 - 2 \times a \times 32$,

$a = -\frac{484}{16} = -30\frac{1}{4}$. **Ans.** ($a$) **$30\frac{1}{4}$ ft. per sec.^{2}.**

For t : $s = \left(\frac{v+u}{2}\right)t$,

$32 = \left(\frac{44+0}{2}\right)t$,

$t = \frac{32}{22} = 1\frac{5}{11}$. **Ans.** ($b$) **$1\frac{5}{11}$ sec.**

Motion under gravity. When a body is acted upon by the attractive force of the earth and drawn towards the earth's centre the body is said to be moving under gravity.

It can be shown that all bodies small in comparison with the earth, whatever their mass, are influenced in practically the same manner by this force of gravity, and if dropped in a vacuum from the same height will reach the ground level together.

If this experiment is tried in air the amount of the air resistance varies and the result may not completely verify the objectives. The body is accelerated towards the earth's centre with an acceleration which varies at different places on the earth's surface, due to variation in distance from the earth's centre, but in London the acceleration produced by gravity is 32·18 ft. per sec.^{2}.

This value is frequently taken as 32·2 and sometimes as 32, and is referred to as "**g**", the acceleration due to the earth's pull. Since g is, within practical limits, a uniform acceleration, the formulae can be used for gravity calculations if the value of g is substituted in them for the acceleration term a.

Example 1. *A bomb is dropped from a height of* 10,000 *ft. Neglecting air resistance, calculate* (a) *the time of fall and* (b) *the velocity of impact with the ground.*

Given : $s = 10{,}000$ ft., $a = g = 32{\cdot}2$ ft. per sec.^{2}.

To find t and v, remembering that the initial downward velocity u is zero.

For t :
$$s = ut + \tfrac{1}{2}gt^2,$$
$$10000 = 0 + \tfrac{1}{2} \times 32{\cdot}2 \times t^2,$$
$$t = \sqrt{\frac{10000}{16{\cdot}1}} = 24{\cdot}92. \quad \textbf{Ans. } (a)\ \textbf{24·92 sec.}$$

For v :
$$v^2 = u^2 + 2gs,$$
$$v^2 = 2 \times 32{\cdot}2 \times 10000,$$
$$v = 802{\cdot}5. \quad \textbf{Ans. } (b)\ \textbf{802·5 ft. per sec.}$$

Example 2. *Water is projected vertically at* 100 *ft. per sec. Neglecting air resistance, find* (a) *the greatest height of the jet,* (b) *the time for the projected water to return to the level of projection.*

It is necessary before attempting this example to be clear on two points :

(*a*) The greatest height is reached when the vertical velocity has become zero.

(*b*) Since the upward motion of the jet is retarded by gravity, and the downward motion accelerated by gravity, the time of ascent is equal to the time of descent, and the velocity at the return to the level of projection is equal to the velocity of projection, air resistance being neglected.

Given : $u = 100$ ft. per sec., $v = 0$ and $a = -g$.

To find s and t :

For s :
$$v^2 = u^2 - 2gs,$$
$$0^2 = 100^2 - 64{\cdot}4s,$$
$$s = \frac{10000}{64{\cdot}4} = 155{\cdot}2.$$

For t :
$$v = u - gt,$$
$$0 = 100 - 32{\cdot}2t,$$
$$t = \frac{100}{32{\cdot}2} = 3{\cdot}106.$$

Ans. greatest height = 155·2 ft., time = 6·21 sec.

Momentum is defined as the quantity of motion possessed by a body. It is given by the *product of mass and velocity* and is thus dependent upon each of these quantities. In this way it is possible for a body of small mass, and large velocity, to have a great momentum; a rifle bullet with a mass of about $\frac{1}{2}$ oz. and leaving a rifle with a velocity of 1000 ft. per sec. has a relatively high momentum.

Inertia is the sluggishness or resistance which a body offers to starting from rest or to change of velocity. The mass of a body is a measure of its inertia.

Newton's laws of motion. Approximately three centuries ago Sir Isaac Newton formulated the laws of motion, which although capable of modification in view of later theories, still provide the basis upon which engineers study the relation of force to motion.

Law 1. The *law of inertia* states that a body will remain in a state of rest or uniform motion in a straight line until it is compelled to change that state by being acted upon by an impressed force.

Law 2. The *law of change of momentum* states that the rate of change of momentum is, if correct units are employed, equal to the impressed force.

Law 3. The *law of reaction* states that every action has an equal and opposite reaction.

Consider the first law of motion; this law gives the definition of force, which states that force is that which changes, or tends to change, a state of rest, or of uniform motion in a straight line.

The second law of motion provides the method of measuring forces by their effect on motion, and indirectly gives the relation between mass and weight.

Suppose a body of mass m lb. has its velocity reduced from V_1 to V_2 ft. per second in a time of t sec.

Then the momentum of the body has changed from that due to V_1 ft. per sec. to that due to V_2 ft. per sec. and momentum is the product of mass and velocity.

So that 1st momentum $= mV_1$ units,

2nd momentum $= mV_2$ units,

change of momentum $= m(V_1 - V_2)$,

and rate of change of momentum $= \dfrac{m(V_1 - V_2)}{t}$.

Now if the definition of acceleration is examined,

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time taken}} = \frac{V_1 - V_2}{t},$$

and $$\textbf{rate of change of momentum} = \frac{\mathbf{m(V_1 - V_2)}}{\mathbf{t}} = \textbf{mass} \times \textbf{acceleration},$$

which according to Newton's second law is equal to the **force.**

Unit of momentum. The unit of momentum is the momentum possessed by a body of mass 1 lb. moving at a velocity of 1 ft. per sec., and this unit is referred to as a lb. ft. per sec. unit of momentum, and sometimes called a *poundem.*

Relation between mass and weight. It has been shown that weight is the measure of the earth's pull on a body. The earth produces an acceleration of g on any body free to move towards its centre, so that the force due to the earth's pull, that is, the *weight of a body, is its mass times its acceleration g.*

$$\text{Weight of a body} = \text{mass} \times \text{acceleration } g,$$

and $$\textbf{mass} = \frac{\textbf{weight}}{\textbf{g}}.$$

Summary of the results :

$$\textit{Force} = F = \textit{rate of change of momentum} = \textit{mass} \times \textit{acceleration}.$$

$$\textit{Weight} = \textit{mass} \times g.$$

Hence the engineer's unit of mass is $\dfrac{1 \text{ lb. weight}}{g}$ in lb.

The engineer's unit of momentum is the momentum possessed by a body of weight 1 lb. moving with a velocity of 1 ft. per sec. and is equal to $1/g$ lb. ft. per sec. units.

Change of momentum is referred to as **impulse** $= Ft = m(V_1 - V_2)$.

Example 1. *Find the average braking force when a car of weight* 1500 *lb. has its velocity reduced from* 60 *to* 30 *m.p.h. in a time of* 4 *sec.*

This example admits of alternate solutions.

(*a*) Rate of change of momentum = force.

$$\left.\begin{aligned} \text{1st momentum} &= \frac{1500 \times 88}{g} \\ \text{2nd momentum} &= \frac{1500 \times 44}{g} \end{aligned}\right\} \text{change} = \frac{1500 \times 44}{g}.$$

$$\text{Force} = \text{rate of change of momentum} = \frac{1500 \times 44}{gt} \text{ lb.}$$

$$= \frac{1500 \times 44}{32 \cdot 2 \times 4} = \mathbf{512 \cdot 4 \text{ lb.}}$$

(*b*) Mass × acceleration = force.

For the retardation: $v = u - at$, $44 = 88 - 4a$, $a = 11$ ft. per sec.^{2}.

$$\text{Force} = \text{mass} \times \text{acceleration} = \frac{1500}{g} \times 11 \text{ lb.} = \mathbf{512 \cdot 4 \text{ lb.}}$$

Example 2. *Find the additional draw bar pull required to accelerate a train of weight* 500 *tons from a speed of* 15 *to* 60 *m.p.h. in* $1\frac{1}{2}$ *minutes.*

For acceleration:

$$v = u + at,$$
$$88 = 22 + 90a,$$
$$a = \tfrac{11}{15} \text{ ft. per sec.}^2.$$

$$\text{Force} = \text{mass} \times \text{acceleration} = \frac{500}{g} \times \frac{11}{15} \text{ tons} = \frac{5500}{15g} = \mathbf{11 \cdot 39 \text{ tons.}}$$

Example 3. *Find the acceleration produced on a car of weight* 10,000 *lb. by an accelerating force of* 120 *lb.*

$$\text{Force} = \text{mass} \times \text{acceleration}.$$

$$120 = \frac{10000}{g} \times a.$$

$$a = \frac{120 \times 32 \cdot 2}{10000} = \mathbf{0 \cdot 3864.} \quad \textbf{Ans. } \mathbf{0 \cdot 3864 \text{ ft. per sec.}^2.}$$

Example 4. *A jet of water moving at* 80 *ft. per sec. suddenly impinges on a flat plate and the whole of its velocity is destroyed in* $\frac{1}{10}$ *sec. Calculate the pressure on the plate in lb. per sq. in.*

Since 1 cu. ft. of water weighs 62·3 lb., then weight of water striking the plate per sec. per sq. in. of plate

$$= \frac{62 \cdot 3 \times 80}{144} \text{ lb.}$$

Rate of loss of momentum = force = pressure on 1 sq. in. of plate

$$= \frac{62 \cdot 3 \times 80 \times v}{g \times 144 \times t} = \frac{62 \cdot 3 \times 80^2 \times 10}{32 \cdot 2 \times 144}$$

Pressure on plate = **860 lb. per sq. in.**

NOTE.—When a body loses the whole of its momentum in an infinitely small interval of time, the force causing this loss is said to be an **impulsive force.** This condition is not obtainable in practice, although the interval is often very small.

Collision of inelastic bodies. The **principle of conservation of momentum** states that *when two inelastic bodies collide, the total momentum after the collision is equal to the sum of the momenta before collision, and is in the direction of the motion of the body with greater momentum.*

Example 1. *The hammer of a pile driver weighs* 5 *cwt. and falls on the head of a pile from a height of* 12 *ft. Find the speed with which pile and hammer tend to move if the pile weighs* 1 *ton.*

Velocity of hammer at impact in ft. per sec. $= \sqrt{2gh} = \sqrt{2g \times 12} = 27{\cdot}8$.

Momentum of hammer in cwt. ft. per sec. units $= \dfrac{wv}{g} = \dfrac{5 \times 27{\cdot}8}{32{\cdot}2}$.

Let the combined velocity after impact be V_1, then

$$\text{momentum after impact} = \frac{(W+w)V_1}{g} = \frac{(20+5)V_1}{32{\cdot}2}.$$

$$\frac{25V_1}{32{\cdot}2} = \frac{5 \times 27{\cdot}8}{32{\cdot}2},$$

$V_1 = 5{\cdot}56$. Ans. 5·56 ft. per sec.

Example 2. *Two trucks, A and B, A of weight* 15 *tons and velocity* 10 *ft. per sec., B of weight* 20 *tons and velocity* 4 *ft. per sec., collide. Find the velocity after impact* (a) *if A overtakes B,* (b) *if A and B approach from opposite directions.*

$$\text{Momentum of } A = \frac{15 \times 10}{g} \text{ tons ft. per sec. units.}$$

$$\text{Momentum of } B = \frac{20 \times 4}{g} \text{ tons ft. per sec. units.}$$

$$(a) \quad \frac{15 \times 10}{g} + \frac{20 \times 4}{g} = \frac{35 \times V}{g}.$$

V in ft. per sec. = 6·57 in the direction of B.

$$(b) \quad \frac{15 \times 10}{g} - \frac{20 \times 4}{g} = \frac{35V}{g}.$$

V in ft. per sec. = 2 in the direction of A.

Addition and subtraction of velocities. It has been shown that velocity is a vector quantity, with specified magnitude and direction; it can therefore be treated in a similar way to a force, that is, represented by a straight line of appropriate direction. The adding

and subtracting processes for velocities can then be performed, using the equivalent of the parallelogram of forces known as the **parallelogram of velocities.**

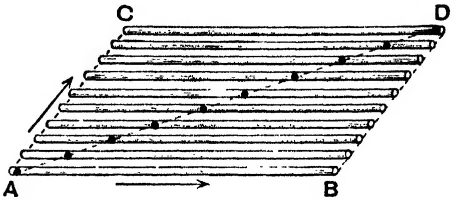


FIG. 325.

If a glass tube is allowed to rest on a horizontal table, and a ball is placed in the end of the tube as shown in Fig. 325; then if the ball is projected along the tube at the same time as the tube is rolled along the table, the resultant motion of the ball will be along the line AD, the diagonal of the parallelogram whose adjacent sides represent the component motions.

Relative motion. *The relative velocity of one body to another is the velocity of the former as it appears when viewed from the latter.* The mechanics of mechanisms, turbines and water wheels depends to a large extent upon a knowledge of relative velocity.

For example, if two trains A and B (Fig. 326) moving respectively on parallel tracks at 60 and 20 m.p.h. pass in opposite directions, a passenger in either A or B will see the other train pass him at a velocity equal to the sum of the velocities of A and B, that is, 80 m.p.h.

If, on the other hand, A overtakes and passes B, a passenger in A will see B receding at a velocity equal to the difference in the velocities of A and B, that is, 40 m.p.h. Thus the relative velocity of B to A will be -40 m.p.h., and the motion of A as viewed from B, or the relative velocity of A to B, will be $+40$ m.p.h.

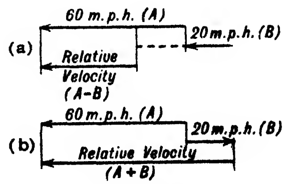


FIG. 326.

Consider two cars A and B, A moving at 60 m.p.h. in an easterly direction,

B moving at 40 m.p.h. in a northerly direction (Fig. 327). If it is required to find the relative velocity of A to B, B must be brought to rest by giving it an opposite velocity of 40 m.p.h. Then if this opposite velocity is compounded with the actual velocity of A,

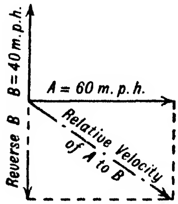


FIG. 327.

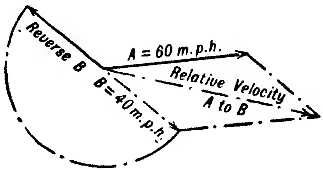


FIG. 328.

and the resultant found, this resultant is the relative velocity of A to B. A further examination of this construction shows the resultant to be the resultant velocity of the velocity A and a velocity which is equal and opposite to that of B, or the vector difference between the velocities of A and B. Fig. 328 shows the relative velocity of A to B when the motion of A and B is along lines not at right angles.

Example 1. *A jet of water moving at 100 ft. per sec. strikes a vane of a water wheel which is moving at 50 ft. per sec. Find the relative velocity of the jet to the vane.* (Fig. 329.)

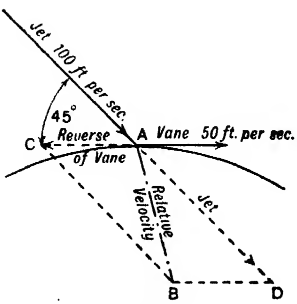


FIG. 329.

Let AC, to scale, be the reverse velocity of the vane, and AD, to scale, the actual velocity of the jet. Then AB, the diagonal of the parallelogram, represents in magnitude and direction the relative velocity of the jet to the vane.

Ans. 73·6 ft. per sec. at 75° to the motion of the vane.

It is interesting to note that the vane must be so shaped that the water is allowed to move along the direction of this relative velocity after impact, without shock and without further contact with the vane.

Example 2. *The pin* A *receives two motions, one of* 50 *ft. per sec. tangential to the circle and the other radially outwards at* 20 *ft. per sec. If this pin is fitted to a block* B *in a mechanism, find the direction in which the slide* CD *must be set.* (Fig. 330.)

Since A is subjected to two velocities, it will move along the direction of the resultant of these two velocities. The diagonal of the parallelogram of velocities gives the direction of this resultant motion, and the slide CD must be set in this direction.

$\theta = 21^\circ\ 48'$ to the tangent to the circle.

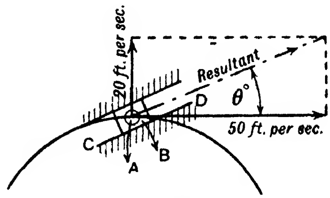


FIG. 330.

Angular velocity. It is often necessary to express a velocity in angular measure, that is, radians per second. The angle known as the **radian** is the angle subtended at the centre of a circle by an arc which is equal to the radius, so that one revolution is equal to 2π radians and one radian $= 360^\circ/2\pi =$ **57·3 degrees approximately.** Angular acceleration is also measured in radians per second per second. *The formulae on p. 258 can be used for angular motion if radians are substituted for feet.*

Example. *Express a speed of* 100 *revolutions per minute in radians per second.*

$$100 \text{ revs. per min.} = 100 \times 2\pi \text{ rad. per min.}$$

$$= \frac{100 \times 2\pi}{60} \text{ rad. per sec.} = \mathbf{10{\cdot}47 \text{ rad. per sec.}}$$

An angular velocity is written as ω (*omega*), so that **ω in this example is 10·47 rad. per sec.** Angular acceleration is written α (alpha).

Motion in a circle. A body moving with uniform speed in the same plane and in a circular path has a velocity of continuously changing direction. Thus in order to maintain a body in a circular path some force is required, sufficient to produce the change of direction of the velocity, or acceleration. This force is known as the **centripetal force,** and it acts radially towards the centre of the circle.

Consider a body moving at uniform speed V ft. per sec. in a circle of radius r ft. (Fig. 331), then in an angle θ the velocity changes, in direction only, from V_1 to V_2 and the change of velocity is given by

AB in the vector diagram, and acts towards the centre of the circle. This change of velocity, when divided by the time, t sec., taken for the body to move through the angle θ, will be the acceleration which

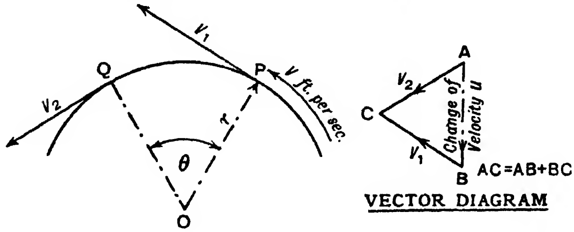


FIG. 331.

is produced by the force constraining the body to move in a circular path, or the *centripetal force.*

Let the change of velocity of AB be u ft. per sec., then

$$\text{the acceleration} = \frac{u}{t} \text{ ft. per sec. per sec.}$$

Now if t and θ are made very small the sector OPQ may be regarded as a triangle, and this triangle is similar to the triangle ACB, that is:

$$\frac{\text{AC or BC}}{\text{AB}} = \frac{r}{Vt}.$$

But AC and BC $= V$,

$$\therefore \quad \frac{V}{u} = \frac{r}{Vt} \text{ and } \frac{u}{t} = \frac{V^2}{r} \text{ ft. per sec.}^2.$$

$$\textbf{Centripetal acceleration} = \frac{V^2}{r} \textbf{ ft. per sec.}^2,$$

and the $\quad$ **centripetal force** = mass × acceleration $= \dfrac{WV^2}{gr}$ **lb.**

Centrifugal force. This is the name given to the force which balances, or is the equilibrant of, the centripetal force, which is necessary for curvilinear motion. Thus when the body is constrained to move in a circular path the centrifugal force is equal to the centripetal force but of opposite direction. Unless this radially inward or centripetal force be provided, the body will cease to move along its circular path and move off in a straight line along a tangent. Centrifugal or centripetal force may be conveniently expressed in angular measure.

$$\text{Angular velocity} = \frac{\text{linear velocity}}{r} \text{ or } \omega = \frac{V}{r},$$

then $$\text{centripetal force} = \frac{WV^2}{gr} = \frac{\mathbf{W}\omega^2\mathbf{r}}{\mathbf{g}}.$$

If the angular speed is N revs. per min.,

$$\omega \text{ in rad. per sec.} = \frac{2\pi N}{60} = \frac{\pi N}{30},$$

and $$\text{centripetal force} = \frac{W\pi^2N^2r}{30^2 \times g} = \mathbf{0{\cdot}000341WrN^2}.$$

Example 1. *The vanes of a turbine each weigh $\frac{1}{18}$ lb., and are attached to a rotor of 20 in. radius. Find the centripetal force acting upon each vane when the whole rotor speed is 2000 r.p.m.*

$$\text{C.F.} = 0{\cdot}000341\, WrN^2 \text{ lb.}$$
$$= 0{\cdot}000341 \times \tfrac{1}{18} \times \tfrac{20}{12} \times 2000^2 \text{ lb.}$$
$$= \mathbf{126{\cdot}3 \text{ lb.}}$$

Example 2. *The centripetal force acting upon each ball of a governor = 114 lb. Find the speed of rotation if the radius of the ball centre is 8 in. and each ball weighs 4 lb.*

Let N be the speed in r.p.m.,

$$\text{C.F.} = 0{\cdot}000341\, WrN^2,$$

$$N = \sqrt{\frac{114 \times 12}{0{\cdot}000341 \times 4 \times 8}} = \mathbf{354}. \quad \textbf{Ans. 354 r.p.m.}$$

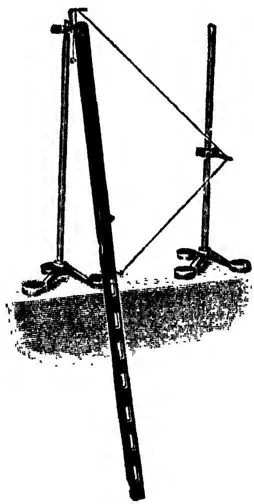


FIG. 332.
(*Messrs. G. Cussons, Ltd.*)

EXPT. 32. OBJECT. *To find a value for "g", the acceleration due to the earth's pull.*

APPARATUS. This consists of a light lath, about 4 ft. in length, freely supported upon a ball bearing (Fig. 332). One edge of the lath is blackened, and by means of a length of cotton passing over pulleys or metal rods to alter the direction of pull, a counterbalance ball is used to pull the lath from the vertical position (Fig. 332). The metal rod immediately above the ball is arranged so that the ball is in the same plane as the lath and that the inside point of the ball is vertically above the blackened edge of the lath.

METHOD OF PROCEDURE. Before connecting the cotton and ball, the lath is allowed to swing freely and 20 oscillations, that is, 20 complete backward and forward swings, are timed. From this the average time of $\frac{1}{4}$ of an oscillation, or the time for the lath to reach its mid position, can be found. Next the ball is attached and its surface chalked, and its position of equilibrium marked on the blackened edge of the lath. Now if the cotton is burned near the hook on the lath, the falling ball will strike and mark the black edge of the lath, when the lath has made $\frac{1}{4}$ of a complete oscillation. By measurement of the distance h between the two chalk marks on the lath, the fall of the ball can be obtained for a time equal to $\frac{1}{4}$ the time of oscillation of the lath, and g can be calculated.

OBSERVATIONS AND DERIVED RESULTS.

Time of 20 oscillations of the lath $=36$ sec.

$t=$ time of $\frac{1}{4}$ oscillation of the lath $=0{\cdot}45$ sec.

Fall or distance $h=3$ ft. 3 in. $=3{\cdot}25$ ft.,

then from $s=ut+\frac{1}{2}gt^2$ where $u=0$,

$$g \text{ in ft. per sec.}^2=\frac{2s}{t^2}=\frac{6{\cdot}5}{(0{\cdot}45)^2}=32{\cdot}1.$$

EXPT. 33. **The Fletcher trolley.**

OBJECTS. (*a*) To show that a force produces an acceleration upon a mass.

(*b*) To verify the relationship between the moving force and the mass moved, when the acceleration is uniform.

APPARATUS. The Fletcher trolley consists of a long plane set perfectly level, over which runs a trolley mounted on three free running wheels. The mass of the trolley can be increased or decreased by adding or removing a number of equal weights (Fig. 333).

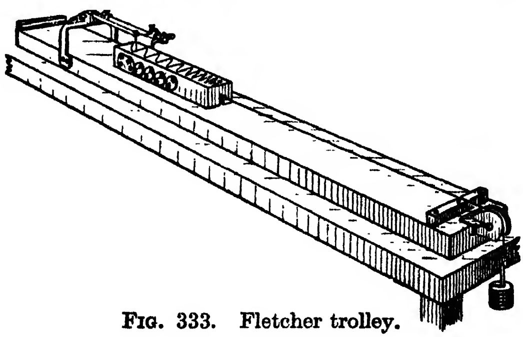


FIG. 333. Fletcher trolley.

Above the trolley is a bridge carrying a vibrator, timed to $\frac{1}{5}$ second per oscillation and fitted with a brush inking device at its free end. A strip of paper can be pinned to the top face of the trolley and a trip piece is fitted to hold the vibrator end displaced from its mid position. Arrangements can be made to release the vibrator at the instant that the trolley commences to move along the plane, and thus an inked record of the motion of the trolley is obtained upon the paper strip.

NOTE.—The Fletcher trolley may be obtained in a more complex form for collision experiments, but for the purposes of this book a simple trolley suffices.

METHOD OF PROCEDURE. It is first necessary, after removing the detachable weights, to assess the friction between the trolley and the plane. This is done by finding the force required to cause the trolley to move along the plane without gain of speed. Next the strip is attached, and a moving force of 1 lb. applied, after the vibrator has been set and the brush inked. Upon releasing the trolley the moving force will cause it to accelerate along the plane and the strip will give an inked record of the motion. This process should be repeated for each additional added weight, that is, when the mass moved has been increased. A set of four or more strips are thus obtained and these can be analysed as described in the next paragraph.

OBSERVATIONS AND DERIVED RESULTS. Draw on the strip (Fig. 334) the centre line OP, and mark a point Q so that OQ is the space passed over in a certain number of complete or half oscillations.

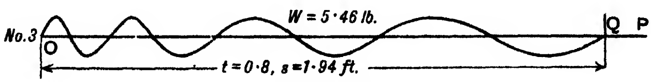


FIG. 334.

Since 1 oscillation is $\frac{1}{5}$ sec., the time for the distance may be calculated. Measure OQ in inches and reduce the distance to feet, and note the weight or mass of the trolley together with the added masses used for this strip. Then for this uniformly accelerated motion $u=0$, t and s are known, so that the acceleration $a=2s/t^2$ (from $s=\frac{1}{2}at^2$) may be found. Next find the value of the product of the total mass moved and the acceleration, remembering that the total mass moved is made up of the mass of the trolley, the mass of the moving force, and the mass of the force used to overcome friction.

Expt. No.	F lb. moving force	f lb. effect of friction	Mass moved, lb.	Mass moved $\}m$, engineer's units	s ft., space	t sec., time	$a = \frac{2s}{t^2}$, acceleration, ft. per sec.^{2}	m.a.
1	1·0	0·04	3·84	$\frac{3·84}{g}$	2·08	0·7	8·49	1·02
2	1·0	0·05	4·65	$\frac{4·65}{g}$	1·76	0·7	7·18	1·04
3	1·0	0·06	5·46	$\frac{5·46}{g}$	1·94	0·8	6·06	1·03
4	1·0	0·07	6·27	$\frac{6·27}{g}$	2·18	0·9	5·39	1·05

CONCLUSIONS. (*a*) In each case the strip shows that the force produced an acceleration upon the mass.

(*b*) It is found, within practical limits, that the calculated value of mass moved in engineer's units times the acceleration produced is equal to the moving force.

EXPT. 34. **Ribbon Atwood machine.**

OBJECTS. (1) *To show that a force produces an acceleration.*

(2) *To find the acceleration produced upon a mass by the application of a force.*

(3) *To verify the formulae for uniformly accelerated motion.*

APPARATUS. The ribbon type of apparatus (Fig. 335) has a light, well-balanced pulley fitted to a horizontal axle running on ball bearings. The rim of the pulley carries a tape to the ends of which are attached equal weights, which serve as the moving masses. An accelerating force is applied to one of these masses in the

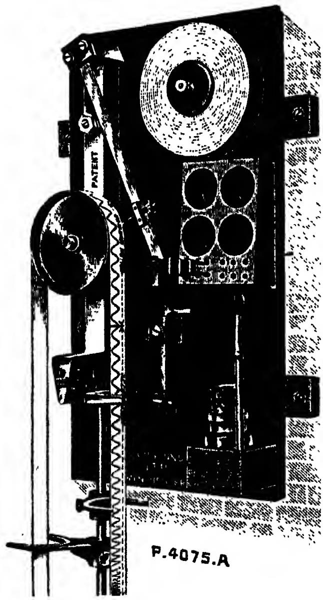


FIG. 335. Ribbon Atwood machine. (*Messrs. G. Cussons Ltd.*)

shape of a light, fork-shaped rider, which can be lifted free of the mass by means of a lower platform attached to a rod extending downwards from the machine frame. Timing is achieved by a vibrator which makes a complete vibration in $\frac{1}{5}$ sec., and carries an inked brush which leaves a wavy line on the tape as the mass falls. An arrangement is made to allow the falling mass to be supported on a hinged platform, and the release of this platform is synchronised with the release of the vibrator.

METHOD. The tape is arranged to support the two weights, and an extension carried as an endless tape from one mass to the other. Arrange the right hand mass on the top platform and add the rider, when a trial drop should be attempted to make sure that the rider is removed by the lower platform without fouling. The experiment can then be performed, and the resulting tape analysed in the way shown below. The motion can be divided into two parts: (1) the uniformly accelerated motion, before the rider is removed; and (2) the uniform motion period, after the rider is removed.

OBSERVATIONS AND DERIVED RESULTS.

Time of 1 oscillation of the vibrator $= 0 \cdot 2$ sec.

Distance through which accelerating force is applied $= 18 \cdot 4$ in. $= 1 \cdot 534$ ft.

Initial velocity $= 0$.

Acceleration period. To find the acceleration a, it is first required to determine the final velocity v. This is found by considering that portion of the tape which is beyond the point at which the rider, or accelerating force, was removed.

Uniform velocity period.

Space passed over $= 20$ in. $= 1 \cdot 667$ ft.

Time taken $= 9 \times 0 \cdot 2$ sec. $= 1 \cdot 8$ sec.

Final velocity $v = \dfrac{1 \cdot 667}{1 \cdot 8}$ or **0·926 ft. per sec.**

Then, in the acceleration period,

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

$$= \left(\frac{v-u}{t}\right) = \frac{0 \cdot 926 - 0}{\text{time in acceleration period}}.$$

Time in acceleration period $= 16 \times 0 \cdot 2$ sec. $= 3 \cdot 2$ sec.

$$\text{Acceleration} = \frac{0 \cdot 926}{3 \cdot 2} = \mathbf{0 \cdot 289\ ft.\ per\ sec.^2}$$

Summary of observed results.

Time of acceleration $= t = 3{\cdot}2$ sec.
Initial velocity $= u = 0$.
Final velocity $= v = 0{\cdot}926$ ft. per sec.
Space passed over $= s = 1{\cdot}534$ ft.
Acceleration $= a = 0{\cdot}289$ ft. per sec.^{2}.

To verify the formulae.

(a) $s = \left(\dfrac{u+v}{2}\right)t = \dfrac{0+0{\cdot}926}{2} \times 3{\cdot}2$ or **1·482 ft.,**

and the **observed result = 1·534 ft.**

(b) $v = u + at = 0 + 0{\cdot}289 \times 3{\cdot}2$ or **0·925 ft. per sec.,**

and the **observed result = 0·926 ft. per sec.**

(c) $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 0{\cdot}289 \times 3{\cdot}2^2$ or **1·48 ft.,**

and the **observed result = 1·534 ft.**

(d) $v^2 = u^2 + 2as = 0 + 2 \times 0{\cdot}289 \times 1{\cdot}534$ or $0{\cdot}8866$,

$v = \sqrt{0{\cdot}8866}$ or **0·942 ft. per sec.,**

and the **observed result = 0·926 ft. per sec.**

Conclusions. (1) It is found that the distance between the crests of successive waves, that is, the distance covered in 0·2 sec., steadily increases until the rider, or accelerating force, is removed, when the distances become equal, thus showing that the velocity becomes uniform when the force is removed.

(2) The calculated values of the term on the left of each equation is approximately equal to the observed value in the experiment. The slight differences are due to bearing friction and to the fact that the pulley itself has had to be moved, or accelerated.

Note.—This apparatus can be used for the determination of g.

EXERCISES ON CHAPTER XII

1. What do you understand by the following terms: Velocity, acceleration and uniform acceleration? Distinguish between speed and velocity, and give an example of where velocity changes without change of speed.

2. A graph is drawn of the speed of a car against the time. If the speed scale is 1 in. = 20 ft. per sec. and the time scale is 1 in. = 2 sec. what does the area under the graph, which is 22·7 sq. in., represent?

3. The following values of velocity and time were taken during the acceleration test of a car. Find the average velocity and the total space passed over by the car.

v ft. per sec.	12	11·1	13·4	15·7	18	21·1	24	27
t sec. - -	0	1	2	3	4	5	6	7

4. A sliding piece in a mechanism is brought to rest from a speed of 20 ft. per sec. in a distance of 3·1 feet under uniform retardation. Calculate (a) the time taken, (b) the retardation.

5. A locomotive and train have an acceleration of 0·6 ft. per sec. per sec. Find the time taken and the space required to attain a speed of 60 miles per hour from one of 10 miles per hour.

6. A hammer is brought to rest in a distance of 3 inches and a time of $\frac{1}{100}$ sec. Find its velocity of impact and retardation.

7. A pile driver is hoisted to a height of 14 ft. above the pile head and then allowed to fall. Find the velocity of impact and the time taken to fall.

8. Find the velocity of striking when a bomb is dropped from a plane at a height of 12,000 ft. Neglect air resistance and assume the bomb to drop vertically.

9. If a field gun were set with its piece (or barrel) vertical and a projectile projected vertically at 1600 ft. per sec., find the height to which the shell would ascend before commencing to return to earth, and the time of this ascent. Neglect air resistance.

10. If the shell in Question 9 were fitted with a fuse to explode 25 seconds after projection, find the height at which explosion would occur.

11. If a bomb were dropped from an airship at a height of 10,000 ft. and was required to explode at a height of 100 ft., what time fuse would have to be fitted? Neglect air resistance.

12. Express a velocity of 20 ft. per second on the rim of a flywheel 4 ft. in diameter, (a) as radians per second, (b) as revolutions per minute.

13. A truck of weight 6000 lb., moving at 30 m.p.h., is brought to rest in a distance of 20 feet. Find the average force exerted by the brakes.

14. The makers of a certain car claim that it will attain a speed of 60 m.p.h. from rest in a distance of 1100 ft. Find the average acceleration, and the average accelerating force exerted by the engine if the weight is 1520 lb.

15. A striker weighing 2 lb. makes contact at 25 ft. per second with a block of weight 5 lb. Find the speed of striker and block if they are free to move off together after collision.

16. A 12 H.P. saloon car weighing $27\frac{1}{2}$ cwt. accelerates in top gear from 10 to 30 m.p.h. in $10\frac{1}{2}$ seconds. Calculate the mean acceleration in ft. per sec. per sec. and the average force producing it. What distance would the car cover in $10\frac{1}{2}$ seconds ?

17. A 10 H.P. Standard saloon car weighing 21 cwt. with 2 people in it takes 13 sec. in top gear to increase its speed from 10 to 30 m.p.h. and also 13 sec. from 30 to 45 m.p.h. Calculate the mean acceleration during each period of 13 sec. and the mean force acting. If this car is brought to rest from 40 m.p.h. in 65 ft., find the mean retardation and retarding force.

18. A pole piece weighing 60 lb. is attached by two bolts placed radially, with the rotor of an alternator rotating at 600 r.p.m. The centre of gravity of the pole piece is 20 in. from the axis of the alternator, and the least cross-sectional area of the bolts is $1\frac{1}{4}$ sq. in. Find the tensile stress in the bolts.

19. During shunting operations two wagons moving in the same direction, of weights 25 and 12 tons and speeds of 10 and 15 ft. per sec. respectively, become linked together by an automatic coupling. What is the common velocity after impact ?

20. An electric train weighing 120 tons is subjected to a maximum tractive force of 11,200 lb. after allowing for tractive resistance. How long will the train take to reach a speed of 30 m.p.h., and what is the average acceleration ?

21. Find the force on the shaft bearings, if a 400 lb. pulley on the shaft has its centre of gravity $\frac{1}{10}$th inch from the shaft axis, when the speed is 300 r.p.m.

22. When a train weighing 300 tons is travelling along a level straight track at 52 m.p.h., steam is suddenly shut off and in 15 sec. the speed has dropped to 48 m.p.h. What is the average force per ton resisting motion ?

23. If a machine-gun fires 500 bullets per minute, each weighing 0·4 oz., with a horizontal muzzle velocity of 2660 ft. per sec., find the average force exerted upon the gun. Neglect the momentum of the gases.

24. A forging hammer weighing $\frac{1}{2}$ ton and moving at 48 ft. per sec. is brought to rest in 0·003 sec. Calculate the average force of the blow.

25. State Newton's Laws of Motion and specify the units usually employed.

26. During a laboratory experiment it was found that a body moved over distances of 3 in. and 5 in. respectively in two consecutive periods of $\frac{1}{10}$ sec. each. Find the acceleration of the body in ft. per sec. per sec. If the body weighed 15 lb., what would be the magnitude of the accelerating force ? (U.L.C.I.)

27. What do you understand by the term " momentum " ? A force of 25 lb. in excess of the force necessary to overcome resistance moves a body weighing 960 lb. from rest over a horizontal plane. Determine the velocity of the body after the force has been acting for 15 sec. (U.L.C.I.)

28. A parachutist is falling at a speed of 20 m.p.h. across a wind blowing at 15 m.p.h. Find the angle of drift and his actual speed of descent.

29. A vessel is moving N.E. at 20 knots and a target is being towed at 10 knots due S. Find the relative velocity of the vessel to the target.

30. Find the vector sum of two velocities of respectively 20 and 12 ft. per sec. when their lines of action have an angle of 120° between them.

31. Find the acceleration of the crank pin of an engine if the crank is 15 in. radius and makes 270 r.p.m.

32. Suppose a boy weighing 100 lb. clings to the wing of an aeroplane which is rounding a horizontal curve of radius $\frac{2}{3}$ mile at 240 m.p.h. Find the horizontal force with which the boy must cling to a stay on the aeroplane in order to remain on it.

33. What change of momentum is produced when a force of 1 ton acts on a body for $\frac{1}{100}$th second ?

34. The draw bar pull exerted by an engine on a train weighing 450 tons when it is travelling at 71 m.p.h. is 2·35 tons. Find the resistance to the motion of the train in lb. per ton weight and the rate at which work is being done measured as draw bar horse-power. (U.L.C.I.)

35. A train is reduced in speed from 28 m.p.h. to 25 m.p.h. in 520 yds., after which it is brought to rest in 2 minutes. Determine the average retardation in ft. per sec. per sec. during each stage. (U.L.C.I.)

36. An electric passenger lift for a several storied building acquires its maximum speed of running, namely 360 ft. per min., in 1·4 sec. from rest. Determine the average acceleration and the height the lift rises during acceleration. (U.L.C.I.)

37. Define *acceleration*. A load of 3 tons is raised by a chain and is given a starting acceleration of $\frac{1}{2}$ ft. per sec. per sec. Determine the initial pull in the chain.

If the acceleration be continued for 6 sec., find the final velocity and the distance travelled in this time. (U.L.C.I.)

CHAPTER XIII

ENERGY—FORMS OF ENERGY—ENERGY OF TRANSLATION AND ROTATION—FLYWHEELS—SIMPLE HARMONIC MOTION AND THE SIMPLE PENDULUM

A BODY may be capable of doing work, in falling from a height, or by virtue of the momentum it possesses ; for example, the hammer of a pile driver can do work on the pile by falling from a position above the pile head. Further, a jet of water moving at a high speed may impinge on the vane of a water wheel and thereby lose some of its momentum, thus performing work on the wheel. This ability to perform work is called **energy,** and energy is defined **as the capacity a body possesses for doing work.**

This energy may exist in many forms ; for instance, a current passing in a conductor possesses electrical energy and is capable of performing work during its passage. A mixture of petrol vapour and air in the cylinder of a petrol engine possesses chemical energy, and upon explosion this chemical energy is converted to heat energy, which in turn is converted to mechanical energy and supplies the driving force on the piston, thus performing work :

Chemical energy	$\longrightarrow$	Heat energy	$\longrightarrow$	Mechanical energy
$\downarrow$		$\downarrow$		$\downarrow$
Petrol and air		Heat from explosion		Work done on piston

Other types of energy may be mentioned in the form of the energy possessed by a sound wave, and the energy absorbed by a wireless receiver, which may be referred to as radio energy.

Principle of conservation of energy. This principle states that the types of energy are interchangeable, and although energy can neither be created nor destroyed, one form may be converted to another form, or forms, without loss.

Consider the case of a hydro-electric plant, in which water is stored in a reservoir at a high level, and by virtue of this high position can do work in falling to a lower level. This energy is the

energy due to position, and is referred to as **potential energy.** Now if this water is carried by a pipe line to the lower level, its potential energy is changed to the energy due to velocity or the **kinetic energy,** and with this form of energy the water is allowed to strike the vanes of a water turbine or Pelton wheel. Work is done on this wheel and it receives kinetic energy of rotation. At this stage the mechanical energy is converted to electrical energy by coupling the turbine to an electric generator, or dynamo, which generates electrical energy. This electrical energy is in the form of a current carried at a high voltage, and may be distributed by cables to positions where electrical power is required. In these positions changes of the form of energy may take place ; for example, if the current is used to operate fires or lamps, the energy is converted to heat or light ; or some of it may be used to drive motors, and thus be converted to mechanical energy. In the event of any portion being employed to work buzzers or sirens, the energy is converted to sound, or in an electro-plating works the electrical energy may revert to chemical energy and produce a deposit of metal.

Inefficiency in the exchange of energy. Although the principle of conservation of energ indicates that energy may be converted to another type without loss, it is not possible in actual practice to convert one type of energy completely to another. Part of the energy is used to do work in overcoming resistances in the course of the transformation ; for example, some is converted to heat in overcoming the friction of bearings, or the turbulence and friction in pipe lines, while some has to be supplied to meet the friction losses in transmission gears. Most of the energy lost is transformed to heat and is dissipated into the surrounding atmosphere, or used to heat the lubricant in the bearings.

Joule's equivalent, known as the **mechanical equivalent of heat,** is the rate of exchange between heat and mechanical energy, and was determined experimentally by the physicist Joule in a manner described in the section on heat and heat engines. The value of this equivalent is known as 1 *British thermal unit* and *is equal to 778 ft. lb. of mechanical work,* where the British thermal unit is the heat required to raise the temperature of 1 pound of water 1 degree Fahrenheit.

Example. *One pound of a certain class of coal is known to be capable of generating* 12,500 *British Thermal Units of heat. Find the theoretical horse-power of an engine served by a boiler using* 100 *lb. of this coal per hour. If the overall efficiency of this plant is* 12%, *find the actual H.P.*

$$\text{Heat value of coal} = \frac{12{,}500 \times 100}{60} \text{ B.Th.U. per min.}$$

$$\text{Mechanical value} = \frac{12{,}500 \times 100 \times 778}{60} \text{ ft. lb. per min.}$$

$$\text{Theoretical H.P.} = \frac{12{,}500 \times 100 \times 778}{60 \times 33{,}000} = 491{\cdot}2.$$

$$\text{Actual H.P.} = \tfrac{12}{100} \times 491{\cdot}2 = 58{\cdot}94.$$

NOTE.—This example gives some idea of the large losses associated with the conversion of heat to mechanical energy.

Potential energy (P.E.) is the form of mechanical energy a body possesses by virtue of its position.

Suppose a weight of 2000 lb. were raised to a height of 10 ft., then 20,000 ft. lb. of work would be required to raise the weight. If this weight were allowed to fall 10 ft. it could do 20,000 ft. lb. of work in falling, and when at the greater height it is said to possess a potential energy of 20,000 ft. lb.

Kinetic energy (K.E.) is the form of mechanical energy a body possesses by virtue of its velocity.

Consider the weight of 2000 lb. falling a distance of 10 ft. During the fall the weight loses potential energy and acquires velocity and therefore kinetic energy, and the loss of potential energy is equal to the gain of kinetic energy, if losses are ignored, so that when it reaches the end of the 10-ft. fall its energy is wholly kinetic.

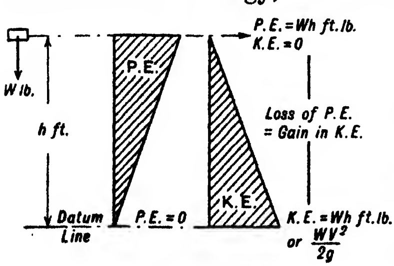


FIG. 336.

Relation between potential and kinetic energy. A weight W lb. at a height of h feet possesses potential energy $= Wh$ ft. lb. Let the weight fall a distance h ft. to the datum level, when the whole of this P.E. will be converted to K.E. At any intermediate point in the fall (Fig. 336) the K.E. possessed by the weight must

be equal to the P.E. given up, because no work is done in falling. An examination of Fig. 336 will show the increase of K.E. as the P.E. is lost, until at datum level the whole of the P.E. becomes K.E., except for the losses against air resistance.

Velocity at datum level, from the formula

$$V^2 = u^2 + 2gh \text{ where } u = 0, \text{ is } \sqrt{2gh},$$

that is, $$V = \sqrt{2gh} \text{ and } h = \frac{V^2}{2g}.$$

Substituting this in the expression for P.E.,

$$\text{P.E.} = \text{K.E.} = Wh = \frac{WV^2}{2g},$$

and $$\mathbf{K.E.} = \frac{\mathbf{WV^2}}{\mathbf{2g}}.$$

This expression may be verified by algebraical manipulation of the velocity formula $V^2 = u^2 + 2gh$.

Multiply throughout by $\frac{W}{2g}$, then

$$\frac{WV^2}{2g} = \frac{Wu^2}{2g} + Wh;$$

and since $u = 0$,

$$\frac{\mathbf{WV^2}}{\mathbf{2g}} = \mathbf{K.E.} = \mathbf{P.E.} = \mathbf{Wh}.$$

If u is not zero,

$$Wh = \text{work done} = \frac{WV^2}{2g} - \frac{Wu^2}{2g} = \text{change of K.E.}$$

Example 1. *What is the available energy when* 400 *tons of water are stored in a reservoir in which the centre of gravity of the water is* 300 *ft. above the inlet level of a water turbine? Find the theoretical velocity of the water at turbine level.*

(*a*) Available energy in ft. tons $= Wh = 400 \times 300 = \mathbf{120{,}000}$.

(*b*) Velocity in ft. per sec. $= \sqrt{2gh} = \sqrt{2g \times 300} = \mathbf{139}$.

Example 2. *The displacement of H.M.S. "Hood" is* 41,000 *tons. What is her K.E. at a speed of* 30 *knots ?*

1 knot $= 6080$ ft. per hr.

Velocity $= 30$ knots $= \dfrac{30 \times 6080}{3600}$ or $50\frac{2}{3}$ ft. per sec.

$$\text{K.E. in ft. tons} = \frac{WV^2}{2g} = \frac{41{,}000 \times 152 \times 152}{64 \times 9} = \mathbf{1{,}645{,}000.}$$

Example 3. *Find the velocity of a car of weight* 1500 *lb. if the K.E. is known to be* 100 *ft. tons.*

100 ft. tons $= 224{,}000$ ft. lb.

$$\text{K.E.} = \frac{WV^2}{2g} = \frac{1500V^2}{64 \cdot 4} = 224{,}000.$$

$$V = \sqrt{\frac{64 \cdot 4 \times 224{,}000}{1500}} \text{ or } 98 \cdot 06 \text{ ft. per sec.} = \mathbf{66 \cdot 86 \text{ m.p.h.}}$$

The energy equation. Since energy is the capacity a body possesses for performing work, any change in the energy content of a body is accounted for by either (*a*) work done on the body or (*b*) work done by the body. In other words,

change of energy = work done,

and rate of change of energy = power.

Consider a body at a height h ft. with a velocity of V ft. per sec. and of weight W lb., its total energy is the sum of its potential and kinetic energies, and is $Wh + \dfrac{WV^2}{2g}$ ft. lb. Now if this body falls to a level of x ft. its new potential energy is Wx ft. lb., and if its new velocity is V_1 ft. per sec., the total energy is $Wx + \dfrac{WV_1^2}{2g}$ ft. lb., and the energy equation becomes :

(*a*) *If no work is done in falling,*

$$\mathbf{Wh + \frac{WV^2}{2g} = Wx + \frac{WV_1^2}{2g}} ;$$

(*b*) *If work is done in falling,*

$$\mathbf{Wh + \frac{WV^2}{2g} = Wx + \frac{WV_1^2}{2g} + \text{work done.}}$$

This expression of the energy equation is very important, and may be expressed in general terms by the equation :

Potential energy + kinetic energy + work done = constant.

Example 1. *A car of weight 1700 lb. has its speed reduced from 60 m.p.h. to 30 m.p.h. on a level track in 4 sec. Find (a) the total work done by the brakes, (b) the H.P. absorbed by the brakes.*

$$\text{1st K.E.} = \frac{WV^2}{2g} = \frac{1700 \times 88^2}{2g} \text{ ft. lb.}$$

$$\text{2nd K.E.} = \frac{WV_1^2}{2g} = \frac{1700 \times 44^2}{2g} \text{ ft. lb.}$$

From the energy equation : change of K.E. is equal to the work done, so that

$$\frac{1700}{2g}(88^2 - 44^2) \text{ ft. lb.} = \text{work done.}$$

$$\text{Work done} = \mathbf{153{,}300 \text{ ft. lb.}}$$

$$\text{Rate of doing work} = \frac{153{,}300}{t} \text{ ft. lb. per sec.}$$

$$= \frac{153{,}300 \times 60}{4 \times 33{,}000} \text{ H.P.} = \mathbf{69{\cdot}7 \text{ H.P.}}$$

Example 2. *A water wheel receives water from a height of 30 ft., and the water leaves the wheel with a velocity of 10 ft. per sec. Find the theoretical H.P. of the wheel if the supply of water is 700,000 gallons per hour.*

Velocity of water at wheel level $= \sqrt{2gh} = \sqrt{2g \times 30} = 43{\cdot}95$ ft. per sec.

At entry to the wheel K.E. per pound of water

$$= \frac{WV^2}{2g} \quad \text{or} \quad \frac{1 \times 43{\cdot}95^2}{2g} \text{ ft. lb.}$$

At exit from the wheel K.E. per pound of water

$$= \frac{WV_1^2}{2g} \quad \text{or} \quad \frac{1 \times 10^2}{2g} \text{ ft. lb.}$$

From the energy equation : change of K.E. = work done.

$$\text{Work done per pound of water} = \frac{43{\cdot}95^2}{2g} - \frac{10^2}{2g} \text{ ft. lb} = \mathbf{28{\cdot}44 \text{ ft. lb.}}$$

The wheel receives 700,000 gallons per hour, that is,

$$\frac{700{,}000 \times 10}{60} \text{ lb. per min.}$$

$$\text{H.P.} = \frac{28{\cdot}44 \times 700{,}000 \times 10}{60 \times 33{,}000} = \mathbf{100{\cdot}6.}$$

Example 3. *Find the K.E. of the "Flying Scotsman" when the total weight is* 500 *tons and the speed* 75 *m.p.h. What average braking force would be required to stop the train in* 1000 *ft., and how many B.Th.U. of heat would be produced during the braking action? If all this heat could be utilized, find the quantity of water at* 32° *Fah. which could be heated to boiling point by its use.*

$$75 \text{ m.p.h.} = 1\tfrac{1}{4} \times 88 \text{ or } 110 \text{ ft. per sec.}$$

$$\text{K.E. in ft. tons} = \frac{WV^2}{2g} = \frac{500 \times 110 \times 110}{64 \cdot 4} = \mathbf{93{,}960.}$$

$$\begin{aligned} \text{Work done in braking} &= \text{K.E. to be dissipated as heat} \\ &= \text{force} \times \text{distance} \\ &= F \times 1000 = 93{,}960. \quad \textbf{Force} = \mathbf{93 \cdot 96 \text{ tons.}} \end{aligned}$$

$$\text{Heat generated} = \frac{93{,}960 \times 2240}{778} \text{ or } \mathbf{270{,}500 \text{ B.Th.U.}}$$

Heat required to raise 1 lb. of water from 32° F. to 212° F.

$$= 180 \text{ B.Th.U.}$$

$$\text{Quantity of water} = \frac{270{,}500}{180} \text{ or } \mathbf{1503 \text{ lb.}}$$

Conversion of electrical to mechanical energy. The *watt* is the unit of electrical power, and is the power developed in a circuit when a current of 1 *ampere* flows under a pressure of 1 *volt*, thus

current in amperes × pressure in volts = power in watts.

The kilowatt (k.W.) is a larger unit of power, employed where the number of watts would be an inconveniently high number, and is 1000 watts.

The conversion of the electrical to mechanical units of power is made on the assumption that

$$\mathbf{1 \text{ horse power} = 746 \text{ watts} = 0 \cdot 746 \text{ k.W.,}}$$

or

$$\mathbf{1 \text{ kW.}} = \frac{1000}{746} = \mathbf{1 \cdot 34 \text{ H.P.}}$$

The Board of Trade Unit (*B.T.U.*) *of energy* is 1 kW. maintained for 1 hour, or 1 kilowatt hour.

Example 1. *Find the H.P. of a motor which takes a current of* 15 *amperes at* 230 *volts.*

$$\text{Power in watts} = 230 \times 15 = \mathbf{3450.}$$

$$\text{H.P.} = \frac{3450}{746} = \mathbf{4 \cdot 625.}$$

Example 2. *An electric motor working on a 230 volt supply is required to operate a hoist which raises $\frac{1}{4}$ ton at a rate of 100 ft. per min. Find the current taken by the motor if its efficiency is 90% and that of the hoist 45%.*

Mechanical H.P. $= 560 \times 100 \div 33{,}000.$

Motor output, H.P. = Mechanical H.P. $\times$ 100/45.

Motor input, H.P. = Motor output, H.P. $\times$ 100/90 = 4·19.

$$\text{Current in Amperes} = \frac{4{\cdot}19 \times 746}{230} = 13{\cdot}59.$$

Example 3. *An electric tramcar weighs 6 tons, and is driven up an incline of 1 in 75 at a steady speed of 10 miles per hour. Find the H.P. of the motors if the frictional and other resistances are steady and amount to a force of 120 lb. What current would be consumed, and what would be the cost of running per mile on a 460 volt supply at $\frac{1}{2}$d. per B.T.U.? Neglect losses due to inefficiency of motors and transmission.*

Mechanical work done :

(*a*) to overcome resistance at 10 m.p.h.

$$= 120 \times \frac{52{,}800}{60} \text{ or } 105{,}600 \text{ ft. lb. per min.};$$

(*b*) to climb the incline at 10 m.p.h.

$$= 6 \times 2240 \times \frac{52{,}800}{60} \times \frac{1}{75} \text{ or } 157{,}696 \text{ ft. lb. per min.}$$

$$\text{H.P.} = \frac{105{,}600 + 157{,}696}{33{,}000} = 7{\cdot}98.$$

$$\text{Current consumed} = \frac{7{\cdot}98 \times 746}{460} = 12{\cdot}94 \text{ amp.}$$

$$\text{Cost per mile} = 7{\cdot}98 \times \frac{0{\cdot}746}{10} \times \frac{1}{2} = 0{\cdot}298 \text{ pence.}$$

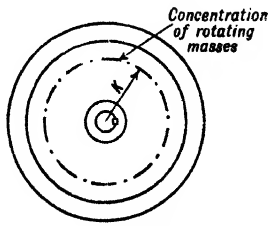


FIG. 337.

Kinetic energy of rotation. When a body is rotating the whole of the rotating mass may be considered concentrated at a certain radius known as the **radius of gyration.** Consider a flywheel of weight W lb. and radius of gyration K ft., revolving at a speed of N revs. per min. (Fig. 337), then the velocity at the radius of gyration

$$= \frac{2\pi K \times N}{60} \text{ ft. per sec.} = V,$$

and the

$$\text{K.E. in ft. lb.} = \frac{WV^2}{2g} = \frac{W \times 4\pi^2 K^2 N^2}{3600 \times 2g}.$$

From this the constants may be extracted and evaluated, thus,

$$\frac{4\pi^2}{3600 \times 2g} = 0{\cdot}00017,$$

and the $$\text{K.E.} = 0{\cdot}00017\text{WK}^2\text{N}^2.$$

The quantity $WK^2 \div g$ is termed the **moment of inertia** of the fly-wheel in Engineers units, and is the capacity of a rotating body to possess momentum.

Example 1. *Find the K.E. of a flywheel of weight* 8000 *lb. and radius of gyration* 2·3 *ft. if speed is* 60 *revs. per min.*

$$\text{K.E.} = 0{\cdot}00017WK^2N^2$$
$$= 0{\cdot}00017 \times 8000 \times 2{\cdot}3^2 \times 60^2 \text{ ft. lb.} = \mathbf{25{,}900 \text{ ft. lb.}}$$

Example 2. *A rolling mill has a flywheel of weight* 8 *tons, and radius of gyration* 2·1 *ft. When the work is passed through the rolls the speed of the flywheel is reduced from* 60 *to* 45 *r.p.m. in* 2 *sec. Find the H.P. absorbed.*

$$\text{Work done} = \text{change of K.E.}$$
$$\text{1st K.E.} = 0{\cdot}00017 \times 8 \times 2{\cdot}1^2 \times 60^2 \text{ ft. tons.}$$
$$\text{2nd K.E.} = 0{\cdot}00017 \times 8 \times 2{\cdot}1^2 \times 45^2 \text{ ft. tons.}$$
$$\text{Change of K.E.} = 0{\cdot}00017 \times 8 \times 2{\cdot}1^2(60^2 - 45^2) \text{ or } 9{\cdot}446 \text{ ft. tons.}$$
$$\text{H.P.} = \frac{60 \times \text{rate of change of K.E.}}{33{,}000} = \frac{9{\cdot}446 \times 2240 \times 60}{2 \times 33{,}000} = \mathbf{19{\cdot}24.}$$

Example 3. *Each ball of a fly press weighs* 40 *lb. and the ball arm operates a screw of* $1\frac{1}{2}$ *in. pitch, to which is attached a punch. Find the average force on the metal being punched if the radius of the ball arm is* 2 *ft.* 4 *in. and the balls move* $\frac{1}{3}$ *of a revolution in* $\frac{1}{2}$ *sec. Efficiency of the screw* = 30%.

$$\text{K.E. of balls in ft. lb.} = \frac{WV^2}{2g} = \frac{80}{2g} \times \left(\frac{2\pi \times 2{\cdot}33 \times 2}{3}\right)^2.$$

$$\text{Stroke of punch in } \frac{1}{2} \text{ sec.} = \frac{1}{3} \times 1\frac{1}{2} = \frac{1}{2} \text{ in.}$$

Let P be the force in lb., then

$$P \times \frac{1}{2} \times \frac{1}{12} = \frac{80}{2g} \times \left(\frac{2\pi \times 2{\cdot}33 \times 2}{3}\right)^2 \text{ or } P = \mathbf{2842.}$$

But efficiency is 30%, therefore actual force is

$$\frac{30}{100} \times 2842 = \mathbf{853 \text{ lb.}}$$

Expt. 35. A horizontal flywheel.

Objects. *To determine for a given flywheel:*

(*a*) *The loss of energy per revolution, due to friction.*
(*b*) *The kinetic energy at 1 revolution per minute.*
(*c*) *The radius of gyration.*

Apparatus. A heavy flywheel is set to rotate in a horizontal plane (Fig. 338). The wheel is rotated by a weight attached to a cord wound around the axle of the flywheel. This axle is fitted with a dowel, and the end of the cord with an eye which automatically frees itself from the dowel when the cord is unwound. The cord is carried from the axle to the weight over a suitable guide pulley, and the apparatus is set so that the weight is capable of, at least, a 6-feet fall.

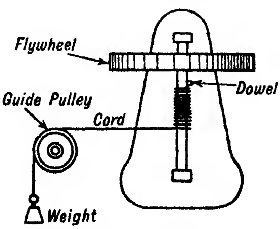


Fig. 338. Flywheel.

Method of Procedure. The weight is allowed to fall about 6 feet, when it reaches the floor and the cord automatically releases itself from the flywheel axle. Neglecting friction the potential energy possessed by the weight at the commencement of the fall will then have been converted to kinetic energy: (*a*) the K.E. of the weight due to its velocity, (*b*) the K.E. of rotation given to the flywheel.

The energy equation is:

$$P.E.\ of\ weight = K.E.\ of\ weight + K.E.\ of\ flywheel,$$

in which

P.E. of weight = weight × distance the weight falls.

K.E. of weight $= W_1V^2/2g$, where W_1 is the weight and V is the velocity of the weight at the ground.

This velocity, V, is double the average velocity of fall, and is expressed by $2 \times h/t$, where h is the distance of fall in feet and t the time in sec. Hence

$$K.E.\ of\ flywheel = W_1h - \frac{W_1V^2}{2g}.$$

Observations are now taken of the total number of revolutions of the wheel from start to rest, namely N_1; and the number of revolutions of the wheel to the fall of the weight are obtained from $(h/\pi d) = n$, where d is the diameter of the flywheel axle plus rope diameter. The retardation of the flywheel takes place over $(N_1 - n)$ revolutions, and during this retardation the K.E. is given up to

overcome friction, so that *the average energy absorbed per revolution due to friction is* $\dfrac{K.E.\ of\ flywheel}{N_1 - n}$.

To find the K.E. of the flywheel at 1 rev. per min. its speed in revolutions per minute at the commencement of retardation, or when the weight reaches the floor, must be found. If the total time of the flywheel to rest is T sec., the retardation period is $T - t$ sec. and the retardation revolutions are, as before, $N_1 - n$ revolutions. The maximum speed is thus twice the average speed during retardation, which is $2\left(\dfrac{N_1 - n}{T - t}\right)$ revs. per sec. or $120\left(\dfrac{N_1 - n}{T - t}\right)$ revs. per min., and the

$$K.E.\ of\ the\ flywheel\ at\ 1\ rev.\ per\ min.\ is\ \frac{K.E.\ of\ flywheel}{\left\{120\left(\dfrac{N_1 - n}{T - t}\right)\right\}^2}$$

because K.E. is proportional to $(\text{speed})^2$, and thus the K.E. at 1 rev. per min. is obtained by dividing by the square of the speed.

The radius of gyration can then be obtained by the solution of the equation

$$\textbf{K.E. at one rev. per min.} = 0{\cdot}00017WK^2N^2$$

for K, the radius of gyration, where W is the weight of the flywheel and N is the speed, one revolution per minute.

Observations and derived results.

W = weight of flywheel and axle in lb.

W_1 = weight of falling weight.

h = distance weight falls in ft.

t = time of fall of weight in sec.

T = time of flywheel to rest in sec.

d = diameter of axle + rope in ft.

$\dfrac{h}{\pi d} = n$ = number of revolutions of flywheel to fall in weight.

N_1 = number of revolutions of flywheel to rest.

$N_1 - n$ = number of revolutions of flywheel during retardation.

$2h/t$ = final velocity of weight $= V$.

$\left(\dfrac{N_1 - n}{T - t}\right)120$ = maximum speed of flywheel in revolutions per minute.

P.E. of weight $= W_1h$ ft. lb.

K.E. of weight in ft. lb. $= \dfrac{W_1V^2}{2g} = \dfrac{W_1 \times 4h^2}{2gt^2}$.

K.E. of flywheel in ft. lb. $= W_1h - \dfrac{W_1V^2}{2g}$ − energy lost in friction.

K.E. of flywheel at 1 rev. per min.

$$=\frac{\text{K.E. of flywheel}}{\left\{\left(\frac{N_1-n}{T-t}\right)120\right\}^2}\text{ ft. lb.}=\text{K.E.}$$

Energy lost per revolution due to friction

$$=\frac{\text{K.E. of flywheel}}{N_1-n}\text{ ft. lb.}$$

Radius of gyration of flywheel = K.

$$\text{K.E.}=0{\cdot}00017WK^2N^2 \text{ where } N=1.$$

$$K \text{ in ft.}=\sqrt{\frac{\text{K.E.}}{0{\cdot}00017W}}.$$

Simple harmonic motion. Many machines and mechanisms, in addition to engines, employ a reciprocating motion, and this motion generally is operating in unison with a rotating part. For example, the steam engine mechanism converts the forward and backward motion of the piston, through the connecting rod, to the rotary motion of the crank. This type of motion is known as harmonic motion, and when certain conditions are fulfilled as **simple harmonic motion.**

In Fig. 339 suppose the point O to move at a uniform speed of v ft. per sec. around the circumference of the circle to P. At the same time imagine another point to start from O and move across the diameter so that when it reaches A_1, O will have reached A vertically above it. Similarly when O reaches B, C, D, E and P respectively, the point moving across the diameter will reach B_1, C_1, D_1, E_1 and P respectively, then this point is moving in simple harmonic motion.

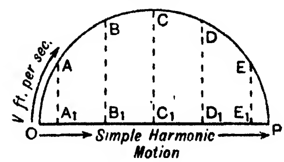


FIG. 339.

The simple pendulum is an example of a close approximation to simple harmonic motion, in which a weight suspended by a light cord is allowed to oscillate in one plane about its point of suspension.

In Fig. 340 let O be the point of suspension and OP the length l of the pendulum in feet, then **one oscillation** is the travel from P to Q and back to P, while **a beat** is the travel from P to Q or Q to P.

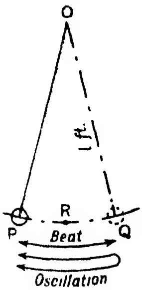


FIG. 340.

The **amplitude** is the name given to the displacement RP from the mid-position R.

EXPT. 36. OBJECT. *To show that the time of swing, or oscillation, of a simple pendulum is independent of its amplitude, providing this is small.*

METHOD OF PROCEDURE. Arrange a simple pendulum of length about 40 in., and set it swinging with a very small amplitude. Time 20 complete oscillations of the pendulum, and calculate the average time of 1 oscillation. Repeat the experiment several times using larger amplitudes, and calculate the time of oscillation in each case.

OBSERVATIONS.

Experiment	Length, ft.	Time of 20 oscillations	Time of 1 oscillation
No. 1	3·25	39 sec.	1·95 sec.
No. 2	3·25	39·4 sec.	1·97 sec.
No. 3	3·25	39·2 sec.	1·96 sec.

CONCLUSION. The time of oscillation is, within practical limits, independent of the amplitude.

The mathematical treatment of the simple pendulum may be better taken at a later stage, but it is possible by performing a series of experiments in which the time of oscillation is found for various lengths to establish a law connecting time of oscillation with length.

From the observations the time of oscillation can be shown to be proportional to the square root of the length, and the law connecting time of oscillation and length can be shown to follow a law, $t = 2\pi\sqrt{\frac{l}{g}}$, where t is the time in sec., l is the length in feet, and π and g have the usual values.

EXPT. 37. The simple pendulum.

OBJECTS. (1) *To compare the time of one oscillation with the length of the pendulum, and to deduce the law connecting time of oscillation with the length.*

(2) *To find the length of a simple pendulum which makes 1 beat per second, or 1 oscillation per 2 seconds.*

(3) *Assuming the law* $t = 2\pi\sqrt{l/g}$, *to find the value of* g.

OBSERVATIONS AND DERIVED RESULTS.

Time, sec., 20 oscillns.	Time, sec., 1 oscilln.	Length, in.	Length, ft.	√length
44	2·2	48	4·00	2·00
39	1·95	39	3·25	1·80
36	1·80	32	2·67	1·63
31	1·55	24	2·00	1·41
27	1·35	18	1·50	1·23
20	1·0	10	0·83	0·91

GRAPHS.

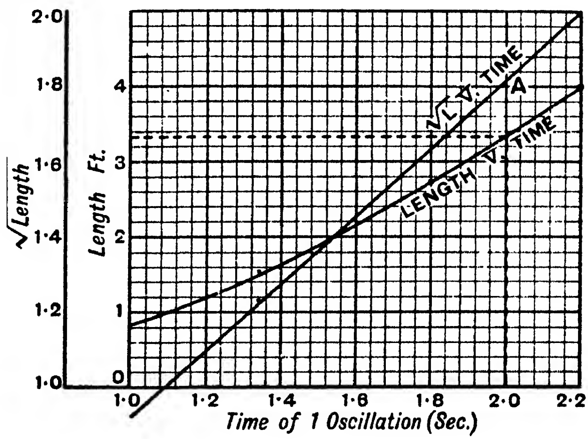


FIG. 341.

Determination of Law.

The law of a simple pendulum is believed to be $t = 2\pi\sqrt{\dfrac{l}{g}}$, *where* t *is the time of* 1 *oscillation,* l *is the length in feet,* g *is the acceleration due to gravity.*

If this is so, $t = \text{constant} \times \sqrt{l}$, and if the graph of $t\ v. \sqrt{l}$ is a straight line (Fig. 341), the constant is equal to the gradient of this graph.

The general law for a straight line passing through the origin is

$$y = mx,$$

so that $$t = mx = m\sqrt{l}.$$

Take a point A on the graph of $t\ v. \sqrt{l}$.

At A, $$t = 2 \text{ sec.}, \quad \sqrt{l} = 1{\cdot}81.$$

Substituting in $t = m\sqrt{l}$,

$$2 = m \times 1{\cdot}81, \quad \mathbf{m = 1{\cdot}105.}$$

$$\textbf{Law is } \mathbf{t = 1{\cdot}105\sqrt{l}}.$$

If the gradient m of this law can now be shown to equal the constant quantity in $t = 2\pi\sqrt{\dfrac{l}{g}}$, the experiment will have verified the correctness of this formula, relating time of oscillation, length and g.

The constant quantity is $\dfrac{2\pi}{\sqrt{g}} = \dfrac{6{\cdot}283}{\sqrt{32{\cdot}2}} = \mathbf{1{\cdot}108}$; thus $t = 2\pi\sqrt{\dfrac{l}{g}}$ is equivalent to $\mathbf{t = 1{\cdot}108\sqrt{l}}$, which very nearly agrees with the obtained law experimentally.

The length of a pendulum to beat once per second, or to oscillate once in two seconds, can be obtained by interpolation of the graph of $t\ v.\ l$, **when t = 2 sec., *l* = 3·3 feet.**

If the formula $t = 2\pi\sqrt{\dfrac{l}{g}}$ is assumed, when

$$t = 2 \text{ sec.}, \quad l = 3{\cdot}3 \text{ feet, and } t = 2\pi\sqrt{\frac{l}{g}} \text{ or } t^2 = 4\pi^2\frac{l}{g},$$

$$4 = 4\pi^2 \times \frac{3{\cdot}3}{g}.$$

$$\mathbf{g = \pi^2 \times 3{\cdot}3 = 32{\cdot}58 \textbf{ ft. per sec. per sec.}}$$

Example 1. *Find the lengths of simple pendulums which would beat seconds (a) at the Equator ($g = 32{\cdot}086$) and (b) at the Poles ($g = 32{\cdot}258$).*

From the formula, $t = 2\pi\sqrt{\dfrac{l}{g}}$, $t^2 = \dfrac{4\pi^2 l}{g}$ or $l = \dfrac{t^2 g}{4\pi^2}$, and $t = 2$ sec.

For case (a), $l = 2^2 \times 32{\cdot}086 \div 4\pi^2 = 3{\cdot}251$ **ft.**

For case (b), $l = 2^2 \times 32{\cdot}258 \div 4\pi^2 = 3{\cdot}268$ **ft.**

EXPT. 38. OBJECT. *To obtain the centre of percussion or centre of oscillation of a cricket bat. The centre of percussion or oscillation will be a point within the blade of the bat such that when the ball is struck over this point no shock or sting will be experienced by the hands.* This point will depend upon the centre or axis of suspension, that is, the point or axis about which the bat is swung. The ball is assumed to strike the bat at right angles to the line containing the centres of suspension, gravity and percussion.

APPARATUS. Clamp two rods, with their axes in line and parallel to a plane containing the blade of the bat, one on each side of the handle to serve as axes of suspension at the desired point. Support the rods on knife edges placed at right angles to the common axis of the rods.

METHOD OF PROCEDURE. Allow the bat to oscillate with small amplitude and arrange alongside a simple pendulum and adjust its length so that it has the same period of oscillation as the bat. Then the length of this **equivalent simple pendulum** will give the distance of the centre of percussion from the centre of suspension measured along the line joining the centres of suspension and gravity.

Example 2. *A portion of a machine that can turn freely in its bearing receives, periodically, an impulsive blow in a plane normal to that containing the axis of the bearing. Find how far from its bearing centre the blow must be given, so that the bearing should not be subjected to shock. Period of vibration for machine part 0·55 sec. when oscillating freely about centre of bearing. $g = 32{\cdot}2$.*

To find the length of the equivalent simple pendulum use $t = 2\pi\sqrt{\dfrac{l}{g}}$.

Then $$l = \frac{gt^2}{4\pi^2} = \frac{32{\cdot}2 \times 0{\cdot}55^2}{4\pi^2} = 0{\cdot}247 \text{ ft.} = 2{\cdot}96 \text{ in.}$$

The required point is 2·96 in. from the bearing centre.

If struck at this point, normal to the line joining the centres of gravity and suspension, purely rotary motion will be given to the machine part.

EXERCISES ON CHAPTER XIII

1. A reservoir contains 100,000 tons of water at an average vertical height of 200 ft. above the inlet of a water turbine generator. What is the potential energy available in ft. tons and H.P. hours if

$$1 \text{ H.P. hour} = 33{,}000 \times 60 \text{ ft. lb. ?}$$

2. What is the kinetic energy of a ball weighing 6 oz. after falling freely through a height of 100 ft. ?

3. Calculate the K.E. of a train of weight 420 tons travelling at 60 m.p.h.

4. The approximate muzzle energy of a 15 inch shell is 80,000 ft. tons. If the shell weighs 1920 lb. find the muzzle velocity.

5. A machine part weighing 1 cwt. slides 100 ft. down a gravity roller conveyor at $7\frac{1}{2}°$ to the horizontal. What is the loss of potential energy and the gain of kinetic energy ? Neglect friction and calculate the velocity at the bottom.

6. What is the kinetic energy of a car weighing 30 cwt. and moving at 30 m.p.h. ? If the car is stopped in 80 ft. by locking the wheels, find the coefficient of friction between the tyres and the road.

7. A weight of 5 lb. is attached to the free end of a string coiled round the shaft of a flywheel. If the weight falls 10 ft. how much potential energy has it lost, and what is the kinetic energy of the weight and flywheel.

8. If the displacement of the liner *Queen Mary* is 83,000 tons and speed 30 knots, what is her kinetic energy ? If 1 lb. of oil fuel gives out 19,000 B.Th.U., how much oil must be burnt theoretically to develop this kinetic energy ? Note that the motion given to the water is not taken into account.

9. A 10 ton truck possesses 54 ft. tons of K.E. as it approaches an incline of 1 in 60. How far will the truck move up the incline, if friction is neglected, before coming to rest ? If the actual distance were 240 ft., what is the track resistance per ton ?

10. What force is required to brake a car of weight 1700 lb. if the speed is to be reduced from 30 to 10 m.p.h. in a distance of 20 ft. ?

11. A hammer weighing $\frac{1}{2}$ ton falls through a distance of 6 ft. on to a forging. Calculate the velocity of striking and the K.E. at this instant. Show that the K.E. at impact theoretically equals the P.E. of the hammer at the top of its fall.

12. What is the energy equation ? A cricket ball weighing $5\frac{5}{8}$ oz. and travelling at 40 ft. per sec. is caught and brought to rest in 1 foot. Calculate (*a*) the K.E. of the ball and (*b*) the average resistance offered while the catch was being made.

13. A 16 inch gun, after it has been fired, commences to recoil with a velocity of 21 ft. per sec. The recoiling mass is 110 tons, and it is brought to rest in 3·8 ft. Calculate (a) the average retardation, (b) the average force exerted by the buffering.

14. Find the cost of running a 5 H.P. motor for 8 hours, if its efficiency is 75%, at ½d. per B.T.U. of electricity.

15. Find the H.P. of a motor which receives 24·3 amperes at a voltage of 440.

16. The main drive belt of a motor has a difference in tensions of 320 lb. The pulley on the main shaft driven by the motor is 3 ft. in diameter and makes 210 r.p.m. Find the amperage received by the motor on a 440 volt supply if the motor efficiency is 80% and the belt slip 3%.

17. The car of an electric elevator weighs 2 tons and is drawn up an incline of 1 in 40. Taking the track resistance as 25 lb. per ton, find the K.E. of the car at 15 m.p.h. and the H.P. of the motor if this speed is uniform. If the voltage on supply to the motor is 440, find the current taken.

18. A lorry weighing 20 tons is descending an incline of 1 in 5. When the brakes are first applied the speed is 30 m.p.h. Calculate the total energy which has to be dissipated and the average braking force required to bring the lorry to rest in 40 ft. How long would the dissipated energy keep the lamps of the lorry alight if the wattage is 20 and if the energy could be so utilised? The wattage of a lamp is the number of watts required to operate it, that is, the amperes taken multiplied by the voltage.

19. Calculate the K.E. of a flywheel of weight 720 lb., radius of gyration 10 in. and speed 140 r.p.m.

20. Find the necessary weight of a flywheel which will give out 20,000 ft. lb. of energy during a speed reduction from 100 to 97 r.p.m. Radius of gyration = 3·5 ft.

21. The stroke of a punching machine is 2 in. and its flywheel weighs 285 lb. and has a radius of gyration 12 in. Find the average force on the punch if the flywheel speed is reduced from 120 to 110 r.p.m. during the stroke.

22. A train is rounding a curve of 600 ft. radius at 45 m.p.h. What is its energy of rotation if the train weighs 500 tons?

23. Obtain the H.P. output which causes a flywheel of weight 4 tons and radius of gyration 3 ft. to suffer a speed reduction of 70 to 65 r.p.m. in 2 seconds.

24. The tup of a pile driver weighs 7 cwt. and strikes the head of a pile at a speed of 14 ft. per sec. Calculate the K.E. of the tup, and the common velocity of the tup and pile immediately after impact, if the latter weighs 1¼ tons. What is the K.E. of the tup and pile just after impact, and how much energy has been lost in making the blow. If

the pile and tup come to rest after a travel of $1\frac{1}{2}$ in., estimate the average resistance of the ground.

25. Find the time of oscillation of a simple pendulum 20 in. in length. $g=32 \cdot 2$.

26. In a certain advertising mechanism the advertisement is to be shown once in every $2\frac{1}{2}$ sec. If the apparatus is to be operated by a pendulum, calculate the length of the simple pendulum to be employed. $g=32 \cdot 2$.

27. The pendulum of a grandfather's clock is 3 ft. 3 in. in length. Treating the pendulum as a simple one, calculate the time for one complete oscillation. $g=32 \cdot 2$.

28. A compound air valve is to be opened at intervals of $1\frac{1}{2}$ sec. Before an automatic device is installed, the engineer decides to guide the operator by installing a simple pendulum. Of what length should the pendulum be made? $g=32 \cdot 2$.

29. At a certain instant a truck weighing 5 tons is moving along a level road at a speed of 6 m.p.h. Find the number of ft. lb. of kinetic energy stored in the truck. Assuming all power shut off, how far would the truck move before coming to rest if the resistances were 12 lb. per ton? (U.L.C.I.)

30. What do you understand by " kinetic energy " and " potential energy " ? Give an illustration of each kind.

A flywheel weighs 8 tons and has a mean diameter of 10 ft. 10 in. How many ft. lb. of energy will be stored in it when it is rotating at 90 r.p.m.? If the speed drops to 80 r.p.m., how many ft. lb. of energy have been taken from it? (U.L.C.I.)

31. An engine of 100 B.H.P. drives a dynamo of 90% efficiency. The output of the dynamo is used for power and lighting. The power load consists of five 10 B.H.P. motors of 85% efficiency and six 2 kW. electric fires. What is the maximum number of 100 watt lamps that must be used to give full load on the dynamo? (U.L.C.I.)

32. Find the kinetic energy of a rolling-mill flywheel weighing 10 tons, having a radius of gyration of 4·5 ft., and revolving at a speed of 180 r.p.m. What will be the speed of the wheel when 250,000 ft. lb. of energy have been taken from it? (U.L.C.I.)

CHAPTER XIV

HYDRAULIC POWER PLANT—HYDRAULIC MACHINES AND PUMPS—HEAD, WORK AND POWER—MECHANICS OF FLUIDS

Hydraulics is the study of the mechanics of liquids and its applications to machines. The principal medium for the transmission of hydraulic power is water, and many of the problems which a hydraulic engineer has to meet concern the conveyance, storage and utilisation of water. This scope is further extended to hydroelectric plants, presses, cranes, accumulators, pumps and other machines in which the energy possessed by water can be employed, and even such matters as the conveyance of sewage, stability of ships and the establishment of pipe lines for oil call for the service of an engineer with a knowledge of hydraulics.

It is interesting, a this stage, to consider the equipment of a factory employing hydraulic power, and to obtain some knowledge of the types of machines employed, the methods used to maintain the hydraulic pressure, and the particular service arrangements necessary to such a plant.

In many respects hydraulic power can be likened to electric power, and it is usual to maintain a definite hydraulic pressure in much the same way as the electrical engineer aims at maintaining a steady voltage on his supply.

Hydraulic pressure. The pressure in a hydraulic service is maintained at a fairly constant figure, often 700 lb. per sq. in. This pressure may be reduced, in some cases, to about 400 lb. per sq. in., and in other instances a factory employs two service pressures, such as (*a*) a high main at 700 lb. per sq. in. or more, and (*b*) a low main at some lower pressure. In order to keep these pressures constant the pumps are used to serve a **hydraulic accumulator.** This is a piece of apparatus in which a ram, with a very heavy load, is pumped to a height, when the valve connecting the accumulator to the pumps is closed, and all the water which is underneath the ram of the

accumulator is subjected to a pressure due to the weight of the ram and its load above it.

Water is practically incompressible, and because of its mobile nature will transmit any load it receives, through itself, to its container, whatever the shape of that container.

Nature of fluid pressure. If a closed vessel of any shape (Fig. 342) be filled with liquid and provided with a piston or some other means of increasing the pressure of the liquid by p lb. per sq. in., then the pressure at every point of the surface of the closed vessel will be increased by this amount. In addition, the pressure acts normally, that is at right angles, to the surface of the container at all points.

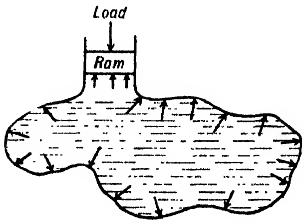


FIG. 342. Pressure normal to surface of container at all points.

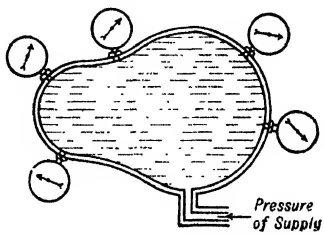


FIG. 343. Equal pressures in a container.

This property of the transmissibility of pressure throughout the whole mass of the liquid is utilised in hydraulic machines.

Actually the pressure increases from the highest to the lowest point of the container due to the weight of the water, but this variation can usually be ignored in comparison with the high pressure transmitted to the liquid from external sources.

Thus all points on the surface of the container will be almost at the same pressure: a fact which can be illustrated by an apparatus such as that shown in the diagram (Fig. 343), in which all the pressure gauges will register the same pressure if they are in the same horizontal plane.

The hydraulic accumulator. This, in common with other hydraulic machines and apparatus, depends for its effective action on the **principle of transmissibility of fluid pressure.** The accumulator (Fig. 344) consists of a ram, working freely in a thick cylinder, with a gland and stuffing box at the cylinder head. To the top of

the ram is attached a framework, or a sheet steel container, to support or contain, as the case may be, either a number of heavy weights or a quantity of scrap iron, concrete or heavy scrap. The weights, or container, rise and fall with the ram, and are guided by a suitable rolled steel external frame. The cylinder is set in a sole

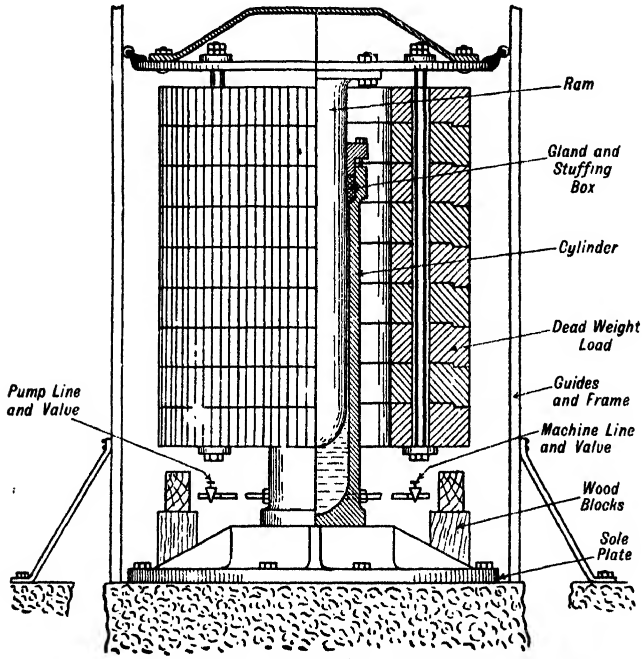


FIG. 344. Hydraulic accumulator.

plate which is securely anchored to a good foundation, and blocks of heavy timber are placed across this foundation to support the dead load when it is lowered. The base of the cylinder is fitted with two pipe junctions leading through valves, in one case to the pumps, and in the other case to the machines. Arrangements are made to close the pump valve automatically when the accumulator is at the top of its travel.

Action of the accumulator. When the pumps are started and the machine line valve closed, water is pumped under the ram,

which is then lifted by the water pressure, carrying with it the attached dead load. Thus at any time when the ram is afloat, the water underneath it is under a pressure due to the weight of the ram and its dead load. This pressure will remain constant during the time the ram is afloat, and will be equally transmitted as a steady pressure to the machines as soon as the machine line valve is opened. Thus by means of the accumulator, water is provided to the machines at a known steady pressure, which could not be obtained by direct pump service because of the irregularity of the pump delivery speed. The efficiency of an accumulator may reach as much as 98%.

Accumulators are often very large, and to maintain an adequate supply of water at 700 lb. per sq. in., the container may support 120 to 150 tons of scrap iron.

Example. *Calculate the ram diameter and storage capacity for a hydraulic accumulator which has a dead weight, including the ram, of* 150 *tons, and is to deliver water at a pressure of* 700 *lb. per sq. in. Lift* = 20 *ft.*

Let diameter of ram be d in.

$$\text{Area of ram} = \frac{\pi d^2}{4} \text{ sq. in.}$$

$$\text{Total load on ram} = \text{dead load} = \text{area of ram} \times 700 \text{ lb.}$$

$$\frac{700\pi d^2}{4} = 150 \times 2240.$$

$$d = \sqrt{\frac{4 \times 150 \times 2240}{700\pi}} = 24{\cdot}73.$$

$$\therefore \textbf{Diameter} = \textbf{24·73 in.}$$

$$\text{Storage capacity} = 150 \times 2240 \times 20 \text{ ft. lb.}$$

$$= \frac{150 \times 2240 \times 20}{33000 \times 60} \text{ or } \textbf{3·4 H.P. hours.}$$

This is the theoretical ram diameter, but in practice an accumulator suffers similar losses to any other machine. These losses are due principally to friction and leakage, and, in consequence, the actual ram diameter would have to be rather smaller. This would ensure the required supply pressure by taking into account the inefficiency of the accumulator.

Hydraulic pumps. These may be divided into three classes : (*a*) suction pumps, (*b*) reciprocating force pumps, (*c*) centrifugal pumps.

The suction or bucket pump. This type of pump (Fig. 345) has a limited performance, inasmuch that it is only capable of lifting water from a depth, which is always less than the height of the water barometer, that is 34 ft. It is used extensively for pumping water from shallow sumps, streams, ponds or wells, and to a certain extent for shallow pumping in small craft.

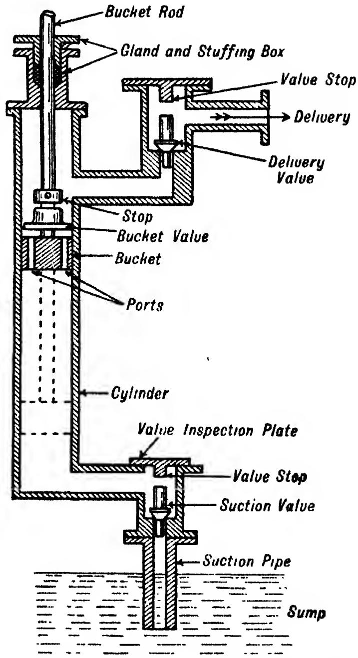


FIG. 345. Suction or bucket pump.

Action. Consider the bucket when it is at the bottom of its stroke; then during its upward motion water will be driven by atmospheric pressure past the suction valve from the sump into the cylinder. During the downward stroke, the suction valve will close and the water in the cylinder will pass through the ports to the top of the bucket, and at the bottom of its stroke the weight of water will close the bucket valve. During the next upward stroke the water above the bucket will be delivered through the delivery valve, and at the same time a further charge will be drawn into the cylinder. It can be readily seen that the depth from which water can be drawn is limited by the atmospheric pressure which supports the column of water in the pipe between the sump and the pump. Owing to leakage of air into the pipe and friction losses, the lift rarely exceeds 25 ft. as compared with a theoretical 34 ft.

Single acting force pump. Force pumps, as a class, are not limited in their performance by atmospheric conditions of pressure. On the delivery side the pump can raise water to a height limited only

by the amount of kinetic energy which can be given to the water by the pump plunger, and the internal design conditions which prevent pumps operating at very high speeds. Force pumps are reciprocating, and generally driven by a prime mover such as a

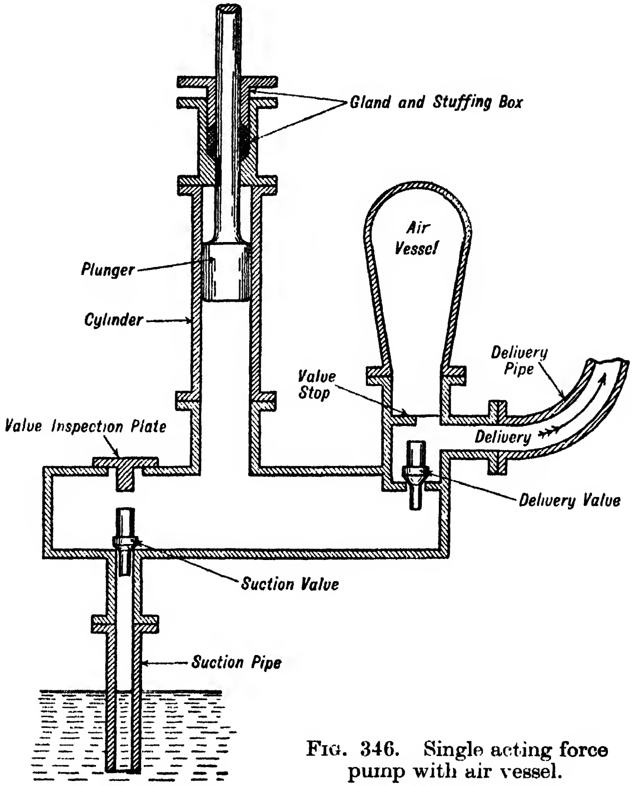


FIG. 346. Single acting force pump with air vessel.

steam or internal combustion engine, or by an electric motor through a reduction gear.

Action. Consider the plunger (Fig. 346) at the bottom of its stroke; then during the upward stroke water will be drawn through the suction valve from the sump into the cylinder. The downward stroke will close the suction valve and force the water content of the cylinder through the delivery valve to the delivery pipe. The

next upward stroke will draw a fresh charge into the cylinder and at the same time close the delivery valve. This completes the cycle, which is made up of two strokes: (*a*) suction, (*b*) delivery.

Function of the air vessel. Since this type of pump is generally driven by a crank mechanism the speed of the plunger is variable throughout the stroke. The column of water on the delivery side will not in consequence constantly be in contact with the plunger; for example, if the plunger is retarded the water ahead of it will obey the first law of motion and continue to move at the faster speed. Thus separation will occur between the plunger and the water column. When the speed of the plunger again increases, impact will occur with the column of water as the separation ceases to exist. This separation and impact is destructive to both the efficiency and life of the pump, so that a preventative in the shape of an air vessel is fitted in the vicinity of the delivery valve. This air vessel contains a quantity of air under pressure, and this air serves to force the water column back on to the plunger during retardation, thus preventing separation. An air vessel is often fitted on the suction side of the pump for a similar reason.

Hydraulic valves and packing. The valves employed are usually of the mushroom or the ball type. In the mushroom type the valve

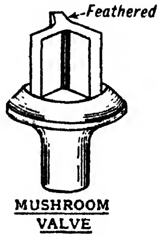


FIG. 347.

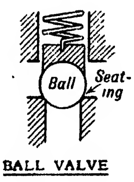


FIG. 348.

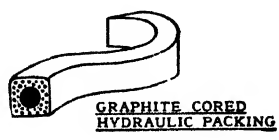


FIG. 349.

seats on to a conical seating, and the stem is feathered to permit the ready passage of water. The ball type seats on to a seating which is a portion of a sphere in shape, and this valve is often held into position by a return spring of such a stiffness that the water pressure will cause it to compress sufficiently to open the valve.

Hydraulic packing is frequently made in the form of a rope, from which lengths can be cut as required (Fig. 349). Cotton or some

similar material is employed for the rope, which may be square or circular in section. The packing is impregnated with tallow, and often a core of graphite is inserted, which under pressure is forced to the outer surface and forms, with the tallow, an effective lubricant in addition to a water-tight joint.

Double-acting force pumps. This type of pump is designed to admit a charge behind the plunger at the same time as delivery is taking place ahead of the plunger. During the return stroke the water behind the plunger is delivered and a fresh charge indrawn on the front of the plunger at the same time. Delivery is effected through two branch pipes, which join to form the main delivery pipe. An air vessel is not required because a balance is established between the water delivered at each stroke, and there is no danger of extensive separation at moderate speeds.

Multiple throw pumps. This is the name given to pumps in which two or more cranks operate separate plungers from the same crank shaft. There is a common delivery pipe, and if the cranks are set at various angles on the crank shaft no air vessel is required, because delivery is taking place at approximately a steady speed.

Centrifugal pumps. The design of this type of pump (Fig. 350) depends upon a jet of water, in rotation, being thrown by centrifugal force away from the centre of rotation. Essentially such a pump must operate at a high speed, and centrifugal pumps are extensively used not only for ordinary pumping purposes, but for forced lubrication systems and the supply of cutting lubricants to machines. Probably their greatest advantage is the absence of valves, which permits the pump to be used for dirty water, heavy liquids and liquids containing sediment.

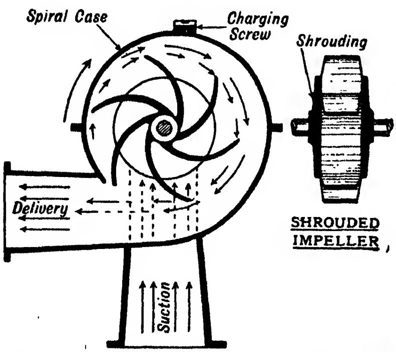


FIG. 350. Centrifugal pump with spiral case front shrouding removed.

Action. The water is drawn through the suction pipe direct into the eye or centre of the impeller, which may or may not be shrouded.

The vanes of the impeller are scientifically designed to produce a streamline flow of water from the eye towards the case of the pump. This case is generally spiral in shape in order that the water, thrown towards the delivery pipe by centrifugal force, may flow into a larger passage as its pressure tends to increase. The greater the speed of rotation of the impeller the greater the centrifugal force driving the water towards the case, so that the height to which the water may be raised, or the pressure overcome by it, is dependent on the speed of rotation.

Necessity for charging. This type of pump will not commence pumping until the interior of the case is full of water, so that a charging screw is provided, which closes a hole through which water may be poured when the pump is first used. This process of filling the interior of the pump before using is spoken of as **charging.** It is often necessary to charge suction and force pumps before using, or after prolonged disuse, and this is generally done through the inspection cover of a valve.

Hydraulic presses. The natural incompressibility of water lends its use to the exertion of very great forces, and hydraulic presses are in extensive use in many of the processes of heavy engineering, in addition to more general uses such as compressing soft materials into bales for shipment, the manufacture of coal briquettes, brick manufacture and stone and ore crushing.

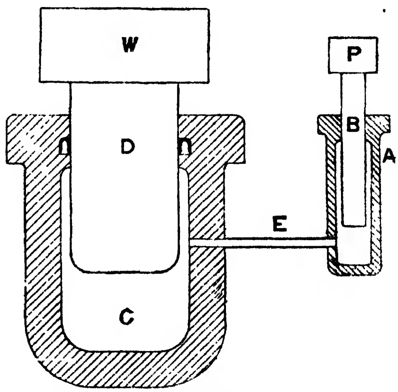


FIG. 351. Hydraulic press.

Principle of the hydraulic press. The press consists in its simplest form of a container A fitted with a plunger B and a ram D (Fig. 351), the ram offering a much greater surface to water pressure than that offered by the plunger. The effort is applied to the plunger; and the load, or resistance, is overcome by the ram.

Consider an effort of E lb. applied to a plunger of area A sq. in.

Then the pressure set up in the container will be

$$=\frac{\text{effort}}{\text{area of plunger}}=\frac{E}{A}\text{ lb. per sq. in.}=\mathbf{p}\textbf{ lb. per sq. in.}$$

Now by the principle of transmissibility, this pressure will act normally to the ram, which has an area of NA sq. in., and the load on the ram will be

$$\mathbf{NA \times p}\textbf{ lb., or }\mathbf{NA \times \frac{E}{A}}\textbf{ lb.,}$$

which is $$\mathbf{NE}\textbf{ lb. where }\mathbf{N}\textbf{ is the ratio }\frac{\textbf{area of ram}}{\textbf{area of plunger}}$$

Work done by a hydraulic press, and its efficiency.

Suppose the plunger moves 1 foot.

Then the $$\textbf{work input}=\mathbf{E \times 1}\textbf{ ft. lb.}=\mathbf{E}\textbf{ ft. lb}$$

Also the ram will move $\frac{1}{N}$ ft. in the time taken for the plunger to move 1 foot, that is, the

$$\textbf{velocity ratio}=\frac{\text{distance moved by effort}}{\text{distance moved by load}}=\frac{N}{1},$$

and the $$\textbf{work output}=\text{load}\times\frac{1}{N}\text{ ft. lb.}=NE\times\frac{1}{N}\text{ or }\mathbf{E}\textbf{ ft. lb.}$$

NOTE.—*This is the theoretical performance; in actual practice the load is not $N\times$effort but, due to frictional losses, a quantity less than NE.*

Therefore $$\text{efficiency}=\frac{\text{actual work output}}{\text{work input}}$$

$$=\frac{\text{actual load}\times\frac{1}{N}}{\text{effort}\times 1}.$$

$$\textbf{Efficiency}=\frac{\textbf{actual load}}{\mathbf{N}\times\textbf{effort}}.$$

Example 1. *A hydraulic press has a plunger* 8 *in. in diameter and a ram* 26 *in. in diameter. The plunger is operated by a main having a*

pressure of 700 lb. per sq. in. Find the resistance overcome by the ram if the efficiency is 90%.

$$\text{Effort} = \pi \times 4^2 \times 700 \text{ lb.} = 35{,}200 \text{ lb.}$$

$$\text{Ratio of areas} = \frac{\text{area of ram}}{\text{area of plunger}} = \frac{\pi \times 13^2}{\pi \times 4^2} = \frac{169}{16}.$$

$$\text{Theoretical load on ram} = \frac{169}{16} \times 35{,}200 \text{ lb.}$$

$$= 371{,}800 \text{ lb.} = 166 \text{ tons.}$$

$$\textbf{Actual load on ram} = \frac{90}{100} \times 166 = \mathbf{149{\cdot}4 \text{ tons.}}$$

NOTE.—Alternatively, the main pressure can be applied direct to the ram.

Example 2. *Find the efficiency of a press which exerts a ram force of 24·4 tons using an effort of 4·7 tons. Diameters of plunger and ram respectively 10 in. and 24 in.*

$$\text{Ratio of areas} = \frac{\text{area of ram}}{\text{area of plunger}} = \frac{\pi \times 144}{\pi \times 25} = \frac{144}{25}.$$

$$\text{Theoretical load} = \frac{144}{25} \times \text{effort} = \frac{144 \times 4{\cdot}7}{25} \text{ tons} = 27{\cdot}07 \text{ tons.}$$

$$\text{Efficiency} = \frac{\text{actual load}}{\text{theoretical load}} = \frac{24{\cdot}4}{27{\cdot}07} = \mathbf{90{\cdot}1\%}.$$

Example 3. *A press has a plunger 5 in. in diameter which employs an effort of 720 lb. What must be the diameter of the ram in order that it may exert a pressure of 5000 lb. with an efficiency of 88%?*

$$\text{Actual load} = 5000 \text{ lb.}$$

$$\text{Theoretical load} = \frac{100}{88} \times 5000 \text{ lb.}$$

$$\text{Effort} = 720 \text{ lb.}$$

$$\text{Ratio of areas} = \frac{\text{area of ram}}{\text{area of plunger}} = \frac{\text{theoretical load}}{\text{effort}}$$

$$= \frac{500{,}000}{88 \times 720} = 7{\cdot}893.$$

Therefore area of ram $= 7{\cdot}893 \times$ area of plunger

$$= 7{\cdot}893 \times \pi \times 2\tfrac{1}{2}^2 = 49{\cdot}4\pi \text{ sq. in.}$$

$$\textbf{Diameter of ram} = 2\sqrt{\frac{49{\cdot}4\pi}{\pi}} = \mathbf{14{\cdot}05 \text{ in.}}$$

Example 4. *Show that the theoretical mechanical advantage of a hydraulic press is equal to the ratio of the squares of the diameters of the ram and plunger. Find the mechanical advantage and velocity ratio for a press where the diameters of the ram and plunger are respectively* 20 *in. and* 5 *in. If the efficiency of the press is* 80%, *what is the true mechanical advantage?*

In a perfect machine the velocity ratio equals the mechanical advantage.

Therefore the theoretical mechanical advantage

$$= \frac{\text{area of ram}}{\text{area of plunger}} = \frac{\frac{1}{4}\pi D^2}{\frac{1}{4}\pi d^2} = \frac{D^2}{d^2}.$$

Theoretical mechanical advantage = velocity ratio $= \frac{D^2}{d^2} = \frac{20^2}{5^2} = 16.$

True mechanical advantage $= 0{\cdot}8 \times 16 = 80\%$ of $16 = 12{\cdot}8.$

Water-tight joints for hydraulic presses. In the operation of hydraulic presses it is necessary to maintain water-tight joints between the plungers or rams and the cylinders. In its simplest form this joint (Fig. 352) is obtained by grooving the ram and tightly packing the groove with hydraulic packing (Fig. 349) well impregnated with tallow. This packing fits tightly into the cylinder, and will hold water in the case of small plungers and rams with moderate pressures.

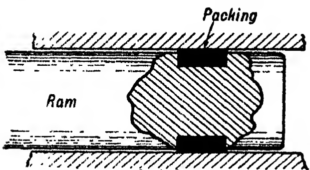


FIG. 352.

For use in larger presses, with greater pressures, it is usual to employ leather joints, either "U" or "cup" leathers (see Figs. 353, 354, 355). These leathers are attached to the ram, and under water pressure they expand and tighten on to the inside of the

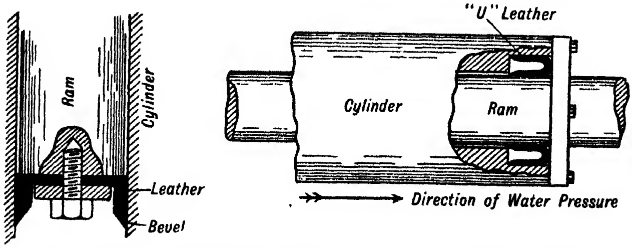


FIG. 353. Cup leather (left) and "U" leather (right).

cylinder. The greater the pressure the more the leather is forced into contact with the cylinder interior. The leathers are shaped, while wet, in a suitable mould, and after drying are soaked in tallow before use. The edges are bevelled to ensure good contact

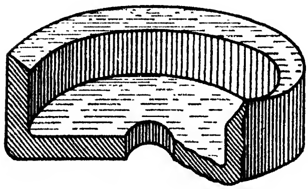


FIG. 354. Cup leather.

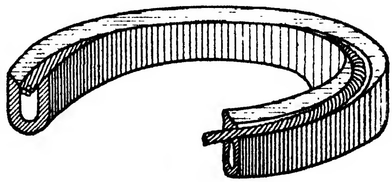


FIG. 355. U leather.

with the cylinder walls and to provide a lead to direct the water inside the leather.

The hydraulic crane. In factories, on wharves, docks and similar undertakings, where hydraulic mains are maintained, the cranes are often served by hydraulic power. A cylinder and ram is installed (Fig. 356), either in a hydraulic pit beneath the crane, or alternatively the cylinder may form the crane post. The ends of cylinder and ram are fitted with mountings which carry the pulleys designed to give the necessary velocity ratio to the crane. The number of pulleys actually employed depends on the speed at which the load has to be raised when compared with that of the ram.

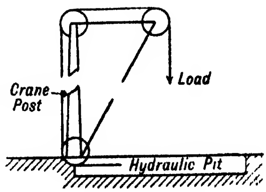


FIG. 356.

The example shown (Fig. 357) has three pulleys A, C and E at the cylinder end, and two pulleys D and B at the ram end. The rope, or chain, is attached to a lug on the cylinder and

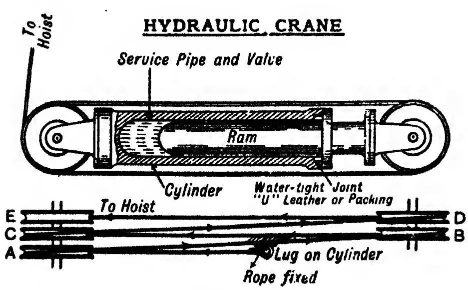


FIG. 357.

brought over the pulleys A, B, C, D and E in turn to the crane hoist. The ram receives water, at pressure, through a service pipe and control valve, and the ram is driven along the cylinder, thus providing the effort.

In the example, if the ram moves 1 foot, it can be seen that each of the ropes between the pulleys will lengthen 1 foot, or, as there are four lengths of rope, the load will be lifted 4 feet. The **velocity ratio** of this crane is 4.

Example 1. *The ram of a hydraulic crane is* 10 *in. in diameter and the velocity ratio is* 7. *Find the speed of lifting of the load, and the load which can be raised, if the hydraulic efficiency is* 92%, *and the mechanical efficiency of the pulleys* 70%. *Pressure on supply* = 750 *lb. per sq. in. Speed of ram,* $1\frac{1}{4}$ *feet per minute.*

Load on ram $= \pi \times 25 \times 750$ lb.

Speed of load in ft. per min. $= 7 \times 1\frac{1}{4} = 8\frac{3}{4}$.

Theoretical pull on hoist $= \dfrac{\pi \times 25 \times 750}{7}$ lb. $= P$.

Actual pull on hoist = hydraulic efficiency $\times$ mechanical efficiency $\times P$

$$= \frac{\pi \times 25 \times 750}{7} \times \frac{92}{100} \times \frac{70}{100} \text{ lb.} = \mathbf{5419\ lb.}$$

Example 2. *Find the necessary diameter of the ram for a hydraulic crane to raise* 5 *tons, through a velocity ratio of* 5 *and overall efficiency* 60%. *Pressure on supply,* 1120 *lb. per sq. in.*

Load = 5 tons.

Theoretical ram effort $= 5 \times 5$ tons or Load $\times$ velocity ratio.

Actual ram effort in tons $= 5 \times 5 \times \dfrac{1}{\text{efficiency}} = \dfrac{2500}{60} = 41{\cdot}66$.

Load on ram $= \pi r^2 \times 120$ lb. $= 41{\cdot}66 \times 2240$ lb.

$\pi r^2 \times 1120 = 41{\cdot}66 \times 2240$.

$r = 5{\cdot}15$.

$\therefore$ **Diameter of ram = 10·3 in.**

Hydraulic intensifier. The operation of hydraulic machines often calls for a pressure, to finish a process, which is greater than the service pressure. This result is obtained by the use of a hydraulic intensifier (Fig. 358), consisting of a ram turned to two diameters

working in a cylinder which is bored to cater for the differential ram. Water at the lower pressure is admitted to the larger cylinder and operates the ram A. At the same time the small cylinder is charged with water and the valve closed, after which it is compressed by the ram B. It follows that the water in the small cylinder will be at a much higher pressure than that supplied to the larger cylinder, and this water at intensified pressure may be used to service a machine with water at a higher pressure.

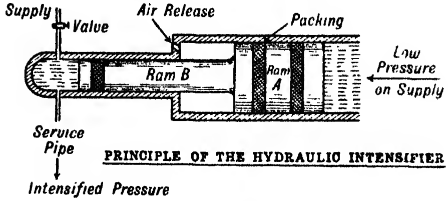


FIG. 358.

Example. *If the diameter of the ram* A (Fig. 358) *is* 12 *in. and that of the ram* B 4 *in., find the ratio of intensification, and the pressure in the smaller cylinder when the supply to the larger cylinder is at* 400 *lb. per sq. in.*

$$\text{Ratio of intensification} = \frac{\text{area of B}}{\text{area of A}} = \frac{\pi \times 36}{\pi \times 4} = 9.$$

$$\text{Pressure in smaller cylinder} = 9 \times 400 = 3600.$$

Ans. 9 and 3600 lb. per sq. in.

The hydraulic jack. The principle of operation of the hydraulic jack (Fig. 359) is similar to that of a hydraulic press, combined with that of a force pump. It contains a reservoir from which water is drawn, through a suction valve, into a cylinder fitted with a plunger of small diameter. This plunger is operated by a hand lever, and delivers water to the cylinder of the lifting ram through a delivery valve. The force applied to this ram serves to raise the load, which can be lifted either on top of the ram or by means of a claw lift attached to the ram.

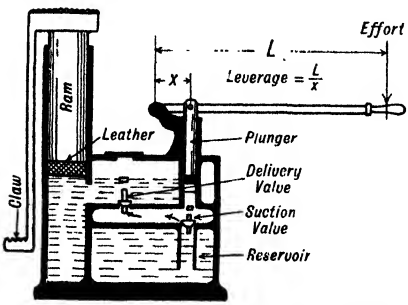


FIG. 359. Diagram to show action of hydraulic jack.

Example. *A hydraulic jack has a plunger 0·4 in. in diameter and a ram 4 in. in diameter. What force may be exerted by the ram if the leverage is 18, and the effort at the end of the lever 20 lb.? Efficiency = 80%.*

$$\text{Ratio of areas} = \frac{\text{area of ram}}{\text{area of plunger}} = \frac{\pi \times 2^2}{\pi \times 0{\cdot}2^2} = \frac{100}{1}.$$

$$\text{Force on plunger} = \text{effort} \times \text{leverage} = 20 \times 18 = 360 \text{ lb.}$$

$$\text{Theoretical force on ram} = 360 \times \text{ratio of areas}$$

$$= 360 \times 100 = 36{,}000 \text{ lb.}$$

$$\text{Actual force on ram} = 36{,}000 \times \text{efficiency} = 0{\cdot}8 \times 36{,}000$$

$$= 28{,}800 \text{ lb.} = \mathbf{12{\cdot}86 \text{ tons.}}$$

Performance of hydraulic machinery. A surface which has superimposed water to a certain height is said to be acted upon by a **head of water.** For example, the bottom of a dock (Fig. 360) containing water to a depth of 30 feet is said to be under a head of water of 30 feet. Similarly a pump which pumps water to a height of 80 feet above its delivery valve is pumping against a head of 80 feet.

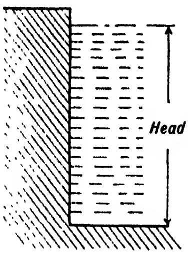


Fig. 360.

The density of fresh water is generally taken as 62·3 lb. per cubic foot and that of salt water 64 lb. per cubic foot.

If a surface 1 sq. ft. in area is acted upon by a head of water of 1 ft., this surface is supporting a weight of water, if fresh, of 62·3 lb. and the pressure is 62·3 lb. per sq. ft. If the head h is 30 ft. the pressure p becomes $62{\cdot}3 \times 30 = 1869$ lb. per sq. ft. or $p = 62{\cdot}3h$.

Energy available per lb. of water. Let 1 cu. ft. of water weigh w lb. Then if w lb. of water enters a hydraulic cylinder 1 sq. ft. in cross-sectional area the ram will move 1 ft. If the pressure of the water is p lb. per sq. ft. the work done will be p ft. lb.

$$\text{Work done per lb.} = \frac{p}{w} \text{ ft. lb.}$$

Hence the **head of water** is a measure of the **energy available per lb. of water** since $p = wh$ or $h = p/w$.

Example 1. *Find the load on a blank at the bottom of a pipe and the maximum pressure in the pipe if it is* 1 *in. in diameter and the head is* 100 *ft. A blank is a closed socket, fitted to the bottom of a pipe to prevent the flow of water through the pipe end.*

$$\text{Area of pipe} = \frac{\pi \times 1^2}{4 \times 144} \text{ sq. ft.}$$

$$\text{Load on blank} = 100 \times \frac{\pi \times 1^2}{4 \times 144} \times 62{\cdot}3 \text{ lb.} = \mathbf{34\ lb.}$$

$$\begin{aligned} \text{Pressure in the pipe} &= 100 \times 62{\cdot}3 \text{ lb. per sq. ft.} \\ &= 6230 \text{ lb. per sq. ft.} \\ &= \mathbf{43{\cdot}3\ lb.\ per\ sq.\ in.} \end{aligned}$$

Example 2. *Find the head corresponding to a pressure of* 300 *lb. per sq. in., and the energy available per lb. of water subjected to this head.*

$$\text{Pressure} = 300 \times 144 \text{ lb. per sq. ft.} = p.$$

$$\text{Head} = \frac{\text{pressure in lb. per sq. ft.}}{\text{wt. of 1 cu. ft.}} = \frac{p}{w} \text{ ft.}$$

where w = density of fresh water

$$\begin{aligned} &= \frac{300 \times 144}{62{\cdot}3} \text{ ft.} \\ &= \mathbf{693{\cdot}4\ ft.} \end{aligned}$$

$$\text{Energy available per lb.} = 1 \text{ lb.} \times \text{head} = \mathbf{693{\cdot}4\ ft.\ lb.}$$

Example 3. *Find the value of the head due to atmospheric pressure, that is* 14·7 *lb. per sq. in.*

$$\text{Pressure} = 14{\cdot}7 \times 144 \text{ lb. per sq. ft.}$$

$$\text{Head} = \frac{14{\cdot}7 \times 144}{62{\cdot}3} = \mathbf{33{\cdot}98\ ft.}$$

NOTE.—*This is the height of the water barometer, and is generally taken as* 34 *ft.*

Example 4. *A turbine bearing is supplied with oil of specific gravity* 0·915 *under a head of* 10 *feet above atmospheric. Find the pressure due to this head in lb. per sq. in.*

$$\begin{aligned} \text{Pressure} &= 10 \times 62{\cdot}3 \times 0{\cdot}915 \text{ lb. per sq. ft.} \\ &= \frac{10 \times 62{\cdot}3 \times 0{\cdot}915}{144} \text{ lb. per sq. in.} \\ &= \mathbf{3{\cdot}96\ lb.\ per\ sq.\ in.\ above\ atmospheric.} \end{aligned}$$

NOTE.—*This is the pressure which would be registered on a pressure gauge, and is called a gauge pressure.*

Datum level is the level from which heads are measured (Fig. 361).

Free surface is the surface of the liquid exposed to the atmosphere.

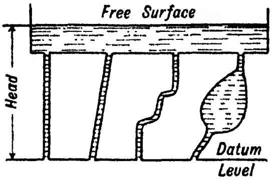


FIG. 361. Head at datum level constant.

The total head is independent of the shape or slope of the container and is always the vertical distance between datum level and free surface.

Performance of hydraulic pumps. The action of a pump is to draw water from a supply and raise it to a height. Thus the pump is working against a head of water, and this head is often called the **pumping head.** If water has to be raised to a head of h ft. it will require $h \times 62 \cdot 3$ ft. lb. of work to raise every cubic foot of the water to the level h, that is, the head $\times$ weight of 1 cu. ft. of water. This quantity is the pump output; and its input, generally supplied by hand or a prime mover, must be *greater than the output* to the extent of the pump's inefficiency.

Suppose a pump to raise 1 gallon of water to a height of 200 ft., since 1 gallon of water weighs 10 lb.

$$\textbf{Work output} = 10 \times 200 \text{ ft. lb.} = \textbf{2000 ft. lb.}$$

Then, *at least*, 2000 ft. lb. of work must be put into the pump by some mechanical means.

Suppose this work input is 2200 ft. lb., then

$$\textbf{pump efficiency} = \frac{\textbf{work output}}{\textbf{work input}} = \frac{\textbf{2000}}{\textbf{2200}} = \textbf{90·9\%}.$$

The ratio $\dfrac{\textbf{actual discharge}}{\textbf{theoretical or ideal discharge}}$ is called the **coefficient of discharge** of the pump. The difference between the theoretical and actual discharge is called the **slip** of the pump. This slip may be **positive or negative**; it is positive when water slips back past the valves before they have had time to close, so that the actual discharge is less than the theoretical. It is negative when the water rushing through the inlet or foot valve increases the pressure sufficiently to lift the delivery valve, and allow some of the water to

pass continuously through the pump. The slip should not exceed 5% if the valves are in good condition.

NOTE.—**Any hydraulic machine is similar to a mechanical machine ; its efficiency $= \dfrac{\text{work output}}{\text{work input}}$, and this is the measure of its performance.**

Examples on the performance of pumps.

Example 1. *The barrel of a suction pump is 7 in. in diameter, and its stroke is 14 in. Find the number of gallons of water it will raise per minute if the speed is 20 strokes per minute. If the coefficient of discharge is 95%, what is the actual discharge and slip of the pump ?*

No. of effective strokes per minute = 10.

$$\text{Volume swept out by the bucket} = \pi \times \frac{7^2}{144 \times 4} \times \frac{14}{12} \text{ cu. ft.}$$

$$= \frac{539}{1728} \text{ cu. ft. per stroke.}$$

$$\text{Weight of water raised per minute} = \frac{5390 \times 62 \cdot 3}{1728} \text{ lb.}$$

$$\text{No. of gallons per minute} = \frac{5390 \times 62 \cdot 3}{1728 \times 10} = \mathbf{19 \cdot 44}.$$

$$\text{Actual discharge} = \frac{95}{100} \times 19 \cdot 44 = \mathbf{18 \cdot 47 \text{ gall.}}; \text{ and}$$

$$\textbf{slip} = \mathbf{0 \cdot 97 \text{ gall.}} = \mathbf{9 \cdot 7 \text{ lb.}}$$

Example 2. *A single acting force pump delivers water against a head of 200 ft. The plunger is 4 in. in diameter, and the stroke is 10 in. Find (a) the quantity of water delivered per hour, neglecting slip, (b) the horse power required to drive it at 140 revolutions per minute, (c) the horse power if the efficiency is 83%.*

(*a*) Quantity = no. of delivery strokes per hour × vol. of cylinder

$$= \frac{140 \times 60 \times \pi \times 2^2 \times 10}{1728} \text{ cu. ft. per hr.}$$

$$= 611 \text{ cu. ft. per hour} = 611 \times 6 \cdot 23 \text{ gall. per hour}$$

$$= \mathbf{3807 \text{ gall. per hour.}}$$

$$(b) \text{ Theoretical horse power} = \frac{\text{work done per minute}}{33{,}000}$$

$$= \frac{3807 \times 10 \times \text{head}}{60 \times 33{,}000} = \frac{38{,}070 \times 200}{60 \times 33{,}000}$$

$$= \mathbf{3 \cdot 85 \text{ H.P.}}$$

$$(c) \text{ Actual horse power} = 3 \cdot 85 \times \frac{100}{83} = \mathbf{4 \cdot 63}$$

Example 3. *A pump, of the double-acting force type, is driven by an electric motor working at 230 volts and taking 25 amp. Find the maximum head to which water can be pumped, if the pump and the motor have efficiencies of 82% and 86% respectively, and the delivery is at the rate of 120 gallons per minute.*

$$\text{Electrical horse power} = \frac{230 \times 25}{746} = 7{\cdot}707.$$

Output electrical horse power $= 7{\cdot}707 \times 0{\cdot}86 =$ **pump input.**

Pump H.P. output $= 7{\cdot}707 \times 0{\cdot}86 \times 0{\cdot}82 = 5{\cdot}436.$

Let the head be h ft.

$$\text{Pump output} = 120 \times 10 \times h \text{ ft. lb. per min.}$$

$$\textbf{Pump output H.P.} = \frac{1200h}{33000} = 5{\cdot}436.$$

$$12h = 330 \times 5{\cdot}436,$$

$$h = 149{\cdot}5. \quad \textbf{Ans. 149·5 feet.}$$

Example 4. *If this pump (Example 3) were converted to steam engine operation with an overall efficiency of 60%, find the necessary engine cylinder diameter, given the mean effective pressure is 41 lb. per sq. in., the engine is double-acting, stroke 1 foot, and speed 90 revs. per minute. Piston and tail rod, 1½ in. dia.*

$$\text{Pump output} = 5{\cdot}436 \text{ H.P.}$$

$$\text{Engine I.H.P.} = \frac{100}{60} \times 5{\cdot}436 = 9{\cdot}06 \text{ H.P.}$$

$$\text{I.H.P.} = \frac{P.L.A.N}{33{,}000} = \frac{41 \times 1 \times A \times 180}{33{,}000}.$$

$$\frac{41 \times 180}{33{,}000} A = 9{\cdot}06 \text{ where } A = \text{piston area.}$$

$$A = \frac{9{\cdot}06 \times 33{,}000}{41 \times 180} = 40{\cdot}51 \text{ sq. in.}$$

$$\text{Area of piston rod} = \pi \times 0{\cdot}75^2 = 1{\cdot}767 \text{ sq. in.}$$

$$\text{Total piston area} = 40{\cdot}51 + 1{\cdot}767 = 42{\cdot}277 \text{ sq. in.}$$

$$\text{Radius of piston} = \sqrt{\frac{42{\cdot}277}{\pi}} = 3{\cdot}67 \text{ in.}$$

Diameter of piston $= 7{\cdot}34$ **in.**

Example 5. *An accumulator has a ram 20 in. diameter, and is to deliver water at 700 lb. per sq. in. pressure. If the stroke, or travel, is 10 ft., find the H.P. of the pumps, allowing for an overall efficiency of 55% together with a slip in the pumps of 4%. Speed of accumulator ram. 2 feet per minute.*

Equivalent pumping head $= \dfrac{700 \times 144}{62 \cdot 3} = 1618$ ft.

Quantity of water per minute (speed 2 ft. per min.) $= \dfrac{\pi \times 10^2 \times 10}{144 \times 5}$ cu. ft. $= 4 \cdot 363$ cu. ft.

Work output per minute $= 4 \cdot 363 \times 62 \cdot 3 \times 1618$ ft. lb.

or $= 4 \cdot 363 \times 700 \times 144$ ft. lb.

H.P. output per minute $= \dfrac{4 \cdot 363 \times 62 \cdot 3 \times 1618}{33000} = 13 \cdot 32.$

Pump input H.P. $= \dfrac{100}{55} \times 13 \cdot 32 = 24 \cdot 22.$

Allowance of 4% for slip, H.P. $= \dfrac{100}{96} \times 24 \cdot 22 = \mathbf{25 \cdot 2}$ **H.P.**

Example 6. *A centrifugal pump has to raise 800,000 gall. of sea water per hour to a height of 20 ft. Find its H.P. if the efficiency is 60%.*

If the pump is operated by a motor at 440 volts, find the current taken by the motor.

Head $= 20$ ft.

$$\text{Wt. of water per minute} = \frac{800{,}000}{60} \times 10 \times \frac{64}{62 \cdot 3} \text{ lb.}$$

NOTE.—1 gall. of sea water weighs $\dfrac{64}{62 \cdot 3} \times 10$ lb.

Work done per minute $= \dfrac{800{,}000 \times 10 \times 64 \times 20}{60 \times 62 \cdot 3}$ ft. lb.

Calculated H.P. $= \dfrac{800{,}000 \times 10 \times 64 \times 20}{33{,}000 \times 60 \times 62 \cdot 3} = \mathbf{83 \cdot 01}.$

Actual H.P. required $= \dfrac{10}{6} \times 83 \cdot 01 = \mathbf{138 \cdot 4}.$

$$138 \cdot 4 \text{ H.P.} = 138 \cdot 4 \times 746 \text{ watts;}$$

$$\therefore \text{ current} = \frac{138 \cdot 4 \times 746}{440} = \mathbf{234 \cdot 6 \text{ amp.}}$$

Example 7. *A hydraulic lift takes 8 gallons of water per lift, and works with a supply pressure of 1120 lb. per sq. in. The charge is 1d. per 100 gallons of water.*

Calculate (a) the energy available per 1 lb. of water, (b) the total amount of work done per lift, (c) the cost in pence to raise the lift, (d) the cost in pence per horse power hour.

(*a*) Work, or energy available per lb. $= \dfrac{1120 \times 144}{62 \cdot 3} =$ **2589 ft. lb.**

(*b*) Work done per lift = wt. of water used × energy per lb.
$= 80 \times 2589 =$ **207,120 ft. lb.**

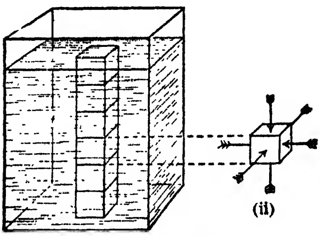
(*c*) Cost to raise lift $= \dfrac{8}{100} \times 1\text{d.} =$ **0·08 pence.**

(*d*) 1 H.P. hr. = 33,000 × 60 ft. lb.

Now 207,120 ft. lb. cost 0·08d., therefore

$$33{,}000 \times 60 \text{ ft. lb. cost } \frac{0 \cdot 08 \times 60 \times 33{,}000}{207{,}120} = \textbf{0·76 pence.}$$

Pressure at a depth. The pressure on an immersed solid is always normal to the surface considered (Fig. 362 (ii)), and this pressure depends upon the depth at which the solid is immersed. The bottom surface of the lowest cube in Fig. 362 (i) has superimposed on it 6 cubes, and if this surface has an area of 1 sq. ft., the water above it would have a volume of 6 cu. ft. or a weight of 6 × 62·3 lb.: thus the pressure on this surface would be 373·8 lb. per sq. ft. Therefore the pressure of water on any immersed surface is the weight of the superimposed liquid or the area of the surface × depth of its centre of area × density of the liquid = **A × h × w lb. per sq. ft.**



(i) Fig. 362. (ii)

The **density** of a substance is its mass per unit volume, but the **specific gravity** of a substance is the ratio of the masses of equal volumes of the substance and pure water.

Inclined surfaces. The pressure on an inclined surface is the same as that on a horizontal surface, *but is acting normally to the surface.*

Example 1. *Find approximately the average pressure upon the hull of the "Titanic", a vessel which sank in mid-Atlantic in a depth estimated at 2 miles.*

Pressure in lb. per sq. ft. $= 1 \times 2 \times 5280 \times 64{\cdot}0$, assuming the density of sea-water 64 lb. per cu. ft.

$$= \frac{5280 \times 128}{2240} \text{ tons per sq. ft.} = \mathbf{301{\cdot}7 \text{ tons per sq. ft.}}$$

Example 2. *Find the total load or whole pressure upon a lock gate 20 ft. span, if the water depth on the flood side is 25 ft. and on the slack side 10 ft. Fresh water.*

Average pressure on the flood side $= 12\frac{1}{2} \times 62{\cdot}3$ lb. per sq. ft.

Load on the flood side $= 20 \times 25 \times 12\frac{1}{2} \times 62{\cdot}3$ lb. $= 389{,}400$ lb.

Average pressure on the slack side $= 5 \times 62{\cdot}3$ lb. per sq. ft.

Load on the slack side $= 20 \times 10 \times 5 \times 62{\cdot}3$ lb. $= 62{,}300$ lb.

The load on the slack side neutralises to some extent that on the flood side, so that:

Total load on the gate $= 389{,}400 - 62{,}300 =$ **327,100 lb.**

Example 3. *A hole in the base of a tank is 2 ft. in diameter. Find the stress in each of 6 rivets, 1 inch diameter, which secure the plate closing the hole if the tank contains oil to a depth of 30 ft. Specific gravity of the oil = 0·86.*

Total pressure on the closing plate

$$= \text{area} \times \text{depth} \times 0{\cdot}86 \times 62{\cdot}3 \text{ lb.}$$
$$= \pi \times 1^2 \times 0{\cdot}86 \times 62{\cdot}3 \times 30 = 5049 \text{ lb.}$$

Area of each rivet section $= \pi \times \frac{1}{2}^2 = 0{\cdot}785$ sq. in.

$$\text{Stress} = \frac{\text{load}}{\text{area}} = \frac{5049}{6 \times 0{\cdot}785} = \mathbf{1072 \text{ lb. per sq. in.}}$$

In practice, the area taken would probably be that of the rivet circle diameter.

Water in motion. When water is allowed to flow from a higher to a lower level, its type of energy changes in accordance with the principle of conservation of energy; that is, it gives up its potential energy at the higher level to form kinetic energy at the lower level.

Consider 1 lb. of water at a height of h ft., where its potential energy is wh or h ft. lb., since w is 1 lb. This potential energy is converted to kinetic energy during the fall, and a velocity V ft. per

second is acquired. The kinetic energy at the lower level is $\frac{wV^2}{2g}=\frac{V^2}{2g}$, if w is 1 lb., so that $\frac{V^2}{2g}=h$ and $V=\sqrt{2gh}$ ft. per sec.

Head of water. It is usual to express the kinetic energy of water as an equivalent head, that is, the equivalent height from which the water would have to fall in order to obtain the velocity which it possesses. Thus water falling from a height of h ft. would acquire a velocity of $\sqrt{2gh}$ ft. per sec., and the **velocity head** equivalent to this velocity is $h=\frac{V^2}{2g}$ ft.

Example. *Find the velocity head possessed by a stream of water flowing at 25 ft. per sec.*

$$h=\frac{V^2}{2g}=\frac{25^2}{64\cdot4} \text{ or } \mathbf{9\cdot7\ ft.}$$

Atmospheric head. If water was connected between two chambers at the same level, one of which was maintained at atmospheric pressure, and the other a vacuum, then the water would flow into the vacuum under the action of atmospheric pressure. The difference of pressure between the two chambers would be equivalent to a head, or difference of water level, and this head would be expressed as

$$\frac{\text{atmospheric pressure in lb. per sq. ft.}}{\text{weight of 1 cu. ft. of water}}.$$

See example and note on p. 313.

It follows from this work that the energy of water may be expressed in the form of equivalent head, or the height at which the water must be stored in order to acquire the given energy at the level considered. This total head is made up of (a) the head due to atmospheric pressure, (b) the head corresponding to the velocity, (c) the head due to the height at which the water is considered above datum level.

Example 1. *Find the total head of a stream of water moving at 20 ft. per sec., which is flowing at a height of 10 ft. above datum level.*

Total head in ft.

= atmospheric head + velocity head + pressure or potential head

$$=34+\frac{V^2}{2g}+10=34+\frac{400}{64\cdot4}+10=\mathbf{50\cdot2}.$$

Example 2. *What velocity would this stream acquire if allowed flow to into a vacuum at datum level ?*

$$V = \sqrt{2gh} = \sqrt{64 \cdot 4 \times 50 \cdot 2^2} = 56 \cdot 8.$$

$\therefore$ **Velocity = 56·8 ft. per sec.**

Bernoulli's theorem is a statement of the principle of conservation of energy when applied to a fluid. *In a steady moving stream of fluid, in which there is no loss by friction or other causes, the sum of the atmospheric, pressure and velocity heads is constant for all sections of the stream.*

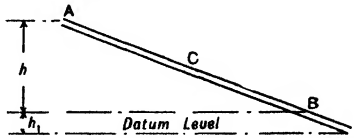


FIG. 363.

Consider a pipe line AB (Fig. 363) in which the velocity of the stream at A is v_1 ft. per sec. and that at B is v_2 ft. per sec.

$$\text{Total head at A} = \frac{p_a}{w} + \frac{v_1^2}{2g} + (h + h_1).$$

$$\text{Total head at B} = \frac{p_a}{w} + \frac{v_2^2}{2g} + h_1.$$

Then from Bernoulli's theorem the total head at A = total head at B = total head at any point C in the pipe line.

Example. *Water is pumped through a pipe AB, in which the area at B, the higher level, is half that at A. The inclination of the pipe is 1 in 4 and its length 100 ft. If the pressure head at A is 40 ft. and the velocity 2·5 ft. per second, find (a) the velocity at B, (b) the pressure head at B. Neglect losses.*

The quantity of water flowing at B is equal to that at A, and since the area at B is half that at A, the velocity at $B = 2 \times 2 \cdot 5$ or **5 ft. per sec.**

The energy equation is then :

at A, $$\frac{p_a}{w} + 40 + \frac{2 \cdot 5^2}{2g} = \text{at } B,\ \frac{p_a}{w} + h + \frac{5^2}{2g} + \frac{100}{4},$$

and $$h = 40 + \frac{1}{2g}(2 \cdot 5^2 - 5^2) - 25$$

$$= 15 - \frac{18 \cdot 75}{2g} = 15 - 0 \cdot 291 = 14 \cdot 71.$$

$\therefore$ **Pressure head = 14·71 ft.**

The Venturi meter. This is a piece of apparatus which applies the Bernoulli theorem to the measurement of the quantity of water flowing in a pipe line. It consists of a short cylindrical throat, at each end of which is fitted a portion or frustum of a cone, the major diameter of each being equal to the diameter of the pipe. The meter is set horizontally in the pipe line, and pressure gauges are fitted at the throat and each of the major cone diameters (Fig. 364). In place of pressure gauges, piezometer tubes may be

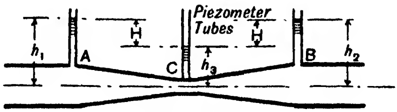


FIG. 364.

fitted at A, B and C. These are small glass tubes fitted into the meter, and the water rises in the tubes to heights h_1, h_2 and h_3, indicating the pressure head at the points where the tubes are fitted and where the cross-sectional areas are respectively a_1, a_2, a_3.

It can be shown that the quantity of water flowing through the pipe is proportional to the square root of the loss of head H at the throat.

Consideration according to the Bernoulli theorem. The heads on the upstream and downstream sides, that is h_1 and h_2, will be theoretically equal, but friction and other causes will make h_2 slightly less than h_1. A loss of pressure head H will occur at the throat, where the pressure head falls to the level h_3, and the loss of head is equal to the additional velocity head acquired at the throat in order to pass the same quantity of water through a smaller area. Thus the velocity at the throat will be $\sqrt{2gH}$, and the quantity of water passing is proportional to $\sqrt{H}$.

Neglecting losses, this can be proved as follows:

Let Q cu. ft. = the discharge per sec., and a_1 and a_3 the cross-sectional areas of the pipe at A and C in sq. ft.

Then $\frac{Q}{a_1}$ and $\frac{Q}{a_3}$ will be the velocities of A and C respectively,

since $$\frac{\text{cu. ft. per sec.}}{\text{sq. ft.}} = \text{ft. per sec.}$$

By Bernoulli's theorem,

$$\frac{Q^2}{2ga_1^2}+h_1=\frac{Q^2}{2ga_3^2}+h_3,\text{ or }\frac{Q^2}{a_3^2}-\frac{Q^2}{a_1^2}=2g(h_1-h_3).$$

$$Q^2\left(\frac{1}{a_3^2}-\frac{1}{a_1^2}\right)=2gH,\text{ since }H=h_1-h_3.$$

$$\therefore\ Q=\sqrt{\frac{2gH}{\dfrac{1}{a_3^2}-\dfrac{1}{a_1^2}}}=\frac{a_1a_3}{\sqrt{a_1^2-a_3^2}}\sqrt{2gH}=\textbf{constant}\times\sqrt{\mathbf{H}}.$$

Flow of fluids through orifices and nozzles. In addition to the general method of allowing water to flow from an orifice in order to empty or extract water from a tank, it is also possible to calibrate the flow from the orifice and thus measure the quantity of water supplied through this source. It must be clearly understood that the speed, or velocity, with which water will flow from an orifice in a tank depends upon three factors: (1) the head, or height of water in the tank, measured above the orifice; (2) the shape of the orifice; (3) the amount of frictional resistance presented to the water during its flow through the orifice or any nozzle attached to the orifice.

Energy aspect of flow through an orifice. Suppose water to be contained in a tank (Fig. 365) and the head above the centre of the

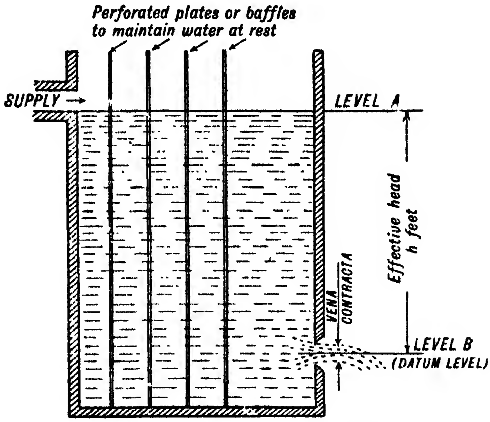


FIG. 365. Sharp-edged orifice.

orifice be maintained at *h* feet, then 1 lb. of water at A will possess a potential energy of *h* ft. lb. when referred to a datum level passing

horizontally through the centre of the orifice. When this water falls to the level of the orifice B, this potential energy will be converted into kinetic energy, and if v ft. per sec. is the velocity at which the water reaches the orifice, this kinetic energy is represented by $v^2/2g$. Thus the energy equation becomes

Potential Energy at A = Kinetic Energy at B,

that is $$h \text{ ft. lb.} = v^2/2g \text{ ft. lb.}$$

and $$v = \sqrt{2gh} \text{ ft. per sec.}$$

NOTE.—This is a theoretical velocity; in actual practice there is a reduction of velocity at the orifice due to the causes shown later in this chapter.

Theoretical height reached by a vertical jet. Fig. 366 shows an orifice arranged to allow the water to flow from the orifice in a vertical direction. According to the reasoning for Fig. 365, the velocity of the water at B will be $\sqrt{2gh}$, a velocity which is obtained by the acceleration of 1 lb. of water falling under gravity a distance of h ft., or from level A to level B. When the water issues from C the jet will be retarded by gravity until at D its velocity will be reduced to zero. Thus the water at D will have a potential energy of h_1 ft. lb. per pound, and this will be equal to its kinetic energy at B or C, which is $v^2/2g$; so that the energy equation becomes $v^2/2g = h_1$, and $v^2/2g = h$ (Fig. 365). Therefore $h_1 = h$, or the water will reach a height equal to the head of water in the supply tank.

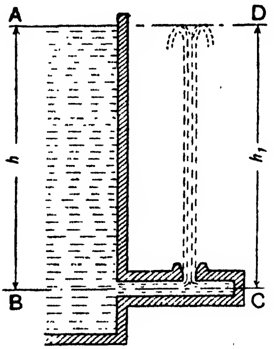


FIG. 366. Vertical jet.

NOTE. Again this makes no allowance for losses in the orifice or during the flow of water to the orifice.

Actual velocity of flow from an orifice. Certain losses occur when water flows from a height h ft. (Fig. 365) and issues from an orifice. These losses may be traced to (1) the fluid friction as the jet of water flows from A to B, (2) the frictional resistance offered by the orifice to the flow of water, (3) the loss of energy due to the change of direction of the jet in its path towards and through the orifice.

Thus the actual velocity of issue is less than the theoretical velocity, and the ratio

$$\frac{\text{actual velocity}}{\text{theoretical velocity}}$$

is called the **coefficient of velocity,** and is known as c_v; thus

$$\text{Actual velocity} = c_v \times \text{theoretical velocity}$$
$$= c_v \times \sqrt{2gh}.$$

Quantity of flow. The quantity of water flowing from an orifice depends upon (1) the velocity of flow, (2) the cross-sectional area of the jet. It follows that the quantity of water flowing from an orifice is given theoretically by the product of velocity in ft. per sec. and the area of the orifice in sq. ft. This product will give the theoretical quantity in cubic feet per second. In practice, the area of the jet is generally reduced in passage through the orifice (Fig. 367), and the actual area is represented by the area at the **vena contracta,** a point in the jet where the area of cross-section produced by the orifice is at a minimum. Thus the actual area is less than the area of the orifice, and the ratio

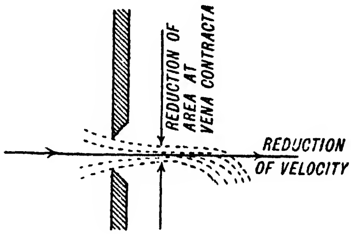


FIG. 367.

$$\frac{\text{area at vena contracta}}{\text{area of orifice}}$$

is known as the **coefficient of contraction of area** and is referred to as c_c. It follows that the actual discharge from the orifice is the product of the actual velocity and the area at the vena contracta, which is

$$c_v \times \sqrt{2gh} \times c_c \times \text{area of orifice};$$

and the ratio

$$\frac{\text{actual discharge}}{\text{theoretical discharge}}$$

is known as the **coefficient of discharge** and referred to as **c**. Thus,

coefficient of discharge, = coefficient of velocity c_v
× coefficient of area contraction c_c,

and **Actual quantity = c × $\sqrt{2gh}$ × area of orifice,**

or $c_v \times c_c \times$ theoretical velocity $\times$ area of orifice.

These coefficients are applicable to both orifices and nozzles, and can be obtained by experiment after the manner described in Expt. 39.

Types of orifices and nozzles. The value of the coefficient of discharge varies with the shape of the orifice and design of nozzle, and these effects are described for the more common examples of mouthpieces and nozzles.

(a) *Projecting pipe* (Fig. 368). In this case the vena contracta is in the pipe, and the stream reaches its full area before ejection from the pipe. Thus the value of c_c is unity. c is about 0·82.

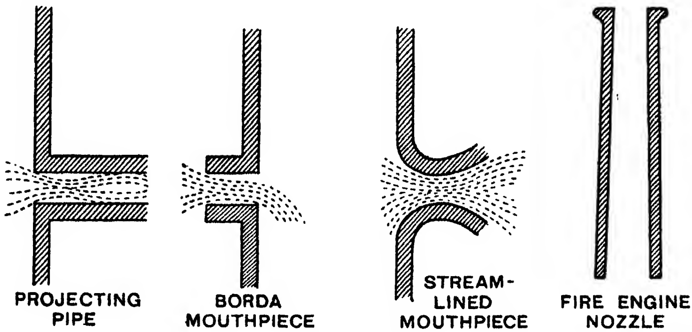


Fig. 368.

(b) *Borda's mouthpiece.* This form of mouthpiece (Fig. 368) has an inward projecting pipe, and the effective area of the discharge stream is that at the vena contracta. c is about 0·48.

(c) *Stream-lined mouthpiece* (Fig. 368). The mouthpiece is shaped to allow the stream to follow a natural stream-line shape, and thus give the maximum efficiency of discharge. c is about 0·99.

(d) *Fire hose nozzle* (Fig. 368). In this case the velocity of the issuing jet is increased by allowing the pressure energy supplied by the pumps to be converted into kinetic energy. Thus according to Bernoulli's Theorem, if the supply jet possesses a pressure energy of p/w, where p is the pressure in lb. per sq. ft. and w the weight of 1 cubic foot of water, then

$$\frac{p}{w}=\frac{V^2}{2g},$$

and
$$V=\sqrt{\frac{2gp}{w}}.$$

It is not possible to absorb the whole of the pressure energy in this way, but in the cases where a fall of pressure is produced the value of p in this argument must be taken as the pressure fall.

NOTE.—This type of nozzle is used extensively in machines in which the energy of a jet of water is the propelling agent, that is, the Pelton wheel type of machine.

EXPT. 39. Sharp-edged circular orifice.

OBJECTS. (1) *To find the coefficient of contraction of area for the orifice.*

(2) *To find the coefficient of discharge.*

(3) *To deduce the coefficient of velocity.*

APPARATUS. (1) A tank fitted with a sharp-edged circular orifice and a ball valve control of the supply to maintain constant head.

(2) A three-point adjustment measuring ring to determine the diameter at the vena contracta (Fig. 369).

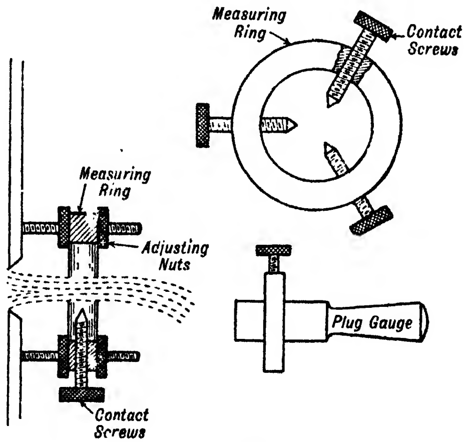


FIG. 369.

METHOD OF PROCEDURE. (1) Allow the water to flow from the orifice and adjust the position of the measuring ring to place it

opposite the narrowest portion of the stream, that is, at the vena contracta. Screw in the contact points until contact is just made with the stream at this point. Remove the ring and obtain the diameter of the vena contracta by fitting a suitable plug gauge between the points of the measuring ring. Calculate the area at the vena contracta, and obtain the ratio

$$c = \frac{\text{area at vena contracta}}{\text{area of orifice}}.$$

(2) Allow the water to flow for a timed period through the orifice into a measuring tank. Then, either by weighing or measuring, obtain the quantity of water discharged in this time. Calculate the theoretical quantity from $\sqrt{2gh} \times \text{area} \times \text{time}$, where h is the head of water above the centre of the orifice.

Then

$$\text{coefficient of discharge} = \frac{\text{actual quantity}}{\text{theoretical quantity}}.$$

(3) To obtain the coefficient of velocity, divide the coefficient of discharge by the coefficient of contraction of area, since

$$c = c_v \times c_c.$$

This experiment may be profitably repeated with orifices of different shapes, and the value of the coefficient of discharge determined for each case.

Example 1. *Calculate the discharge from a sharp-edged circular orifice 2 in. in diameter if the head is 12 ft. and the value of the coefficient of discharge is 0·62.*

Area of orifice $= \pi \times 1^2$ sq. in. $= \dfrac{\pi}{144}$ sq. ft.

Quantity per sec. $= c \times A \times \sqrt{2gh}$

$= 0{\cdot}62 \times \dfrac{\pi}{144} \times \sqrt{64{\cdot}4 \times 12}$

$= 0{\cdot}376$ cu. ft. per sec.

Quantity in gallons per min. $= 0{\cdot}376 \times 6{\cdot}23 \times 60$

$= 140{\cdot}6.$

Example 2. *The pressure in a hydraulic main is 180 lb. per sq. in. Calculate the jet velocity from a nozzle 2 in. in diameter if the coefficient of velocity is 0·95. If the jet is caused to impinge upon a flat plate* (Fig. 370), *calculate the change of momentum per second. If a semicircular vane*

as used in a Pelton wheel be substituted (Fig. 371), *find the change of momentum per second. What does this change of momentum per second, or rate of change of momentum, indicate?*

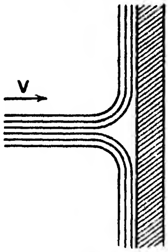


FIG. 370. Impact on a flat plate.

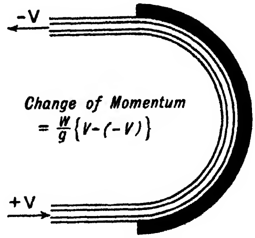


FIG. 371. Impact on a semicircular vane.

NOTE.—This example provides material for the careful consideration of the units employed. The student should study the derivation of units shown on the right.

	Units column.
p = pressure in main = $180 \times 144 = 25{,}920$.	lb. per ft.^{2}
Velocity of jet $= v = 0{\cdot}95\sqrt{\dfrac{2gp}{w}}$	
$= 0{\cdot}95\sqrt{\dfrac{64{\cdot}4 \times 25{,}920}{62{\cdot}3}}$	
$=$ **155·5 ft. per sec.**	$\dfrac{\text{ft.}}{\text{sec.}} = \text{ft. per sec.}$
Weight of water delivered per second = area of nozzle × velocity × density of water	
$= \dfrac{\pi}{144} \times 155{\cdot}5 \times 62{\cdot}3 \text{ lb.} = 211{\cdot}3 \text{ lb.}$	$\dfrac{\text{lb.}}{\text{sec.}} = \text{lb. per sec.}$
(*a*) Original momentum per sec. in the direction of the flow $= \dfrac{211{\cdot}3 \times 155{\cdot}5}{32{\cdot}2}$ lb.	$\dfrac{\text{lb.}}{\text{sec.}} \times \dfrac{\text{ft.}}{\text{sec.}} \div \dfrac{\text{ft.}}{\text{sec.}^2} = \text{lb.}$
Final momentum per sec. = 0 lb.	[$g = 32{\cdot}2$ ft. per sec.^{2}.]
∴ change of momentum per sec. $= \dfrac{211{\cdot}3 \times 155{\cdot}5}{32{\cdot}2}$	
= 1021 lb.	

and this is the force on the plate in pounds.

(*b*) Original momentum per sec. in direction of the flow

$$=\frac{211\cdot 3\times 155\cdot 5}{32\cdot 2}=1021 \text{ lb.}$$

Final momentum per sec. in direction of the flow

$$=-\frac{211\cdot 3\times 155\cdot 5}{32\cdot 2}=-1021 \text{ lb.},$$

where the negative sign indicates a reverse direction.

Hence total change of momentum per sec.

$$=1021-(-1021)\text{ lb.}=1021+1021\text{ lb.}=\mathbf{2042\ lb.}$$

which is the force, neglecting losses, on the semicircular vane.

Example 3. *Water flows through a circular orifice* $\frac{1}{2}$ *in. in diameter under a head of* 40 *feet. Find the time required to fill a 2-gallon can from the source of supply if the coefficient of discharge is* 0·62.

Quantity per second $=0\cdot 62\times\sqrt{64\cdot 4\times 40}\times\pi\times(\frac{1}{4})^2\times\frac{1}{144}$ cu. ft.

$=0\cdot 04292$ cu. ft.

$$\text{Time}=\frac{2}{6\cdot 23\times 0\cdot 04292}\text{ sec.}$$

$=\mathbf{7\cdot 478\ sec.}$

Example 4. *It is required to exert a force of* 100 *lb. on a flat plate by means of a jet* 1 *in. in diameter. Find* (*a*) *the velocity of the jet,* (*b*) *the pressure head required to produce this velocity.*

Area of jet $=\pi\times 1^2\times\frac{1}{4}\times\frac{1}{144}$ sq. ft. $=\frac{\pi}{576}$ sq. ft.

Loss of momentum per sec. = force.

$$\frac{\pi\times V\times 62\cdot 3}{576}\times\frac{V}{g}=100.$$

$$V^2=\frac{57600g}{62\cdot 3\pi}.$$

i.e. Velocity $=\mathbf{97\cdot 34}$ **ft. per sec.**

Let pressure head $=h$ ft.

$$V=\sqrt{2gh}\quad\text{and}\quad h=\frac{V^2}{2g},$$

$$h=\frac{97\cdot 34^2}{64\cdot 4}=\mathbf{147\cdot 1\ ft.}$$

Masonry dams. The storage of water for the development of hydro-electric power and for domestic supply to cities and towns is becoming increasingly important. In the construction of masonry dams care has to be exercised that the resultant force on the base acts within the middle third, that is, the middle of three equal sections of the base, no matter what the level of water supported by the dam. Usually the equilibrium of a foot length of wall is considered. The resultant force R (Fig. 372) on the base of the wall is

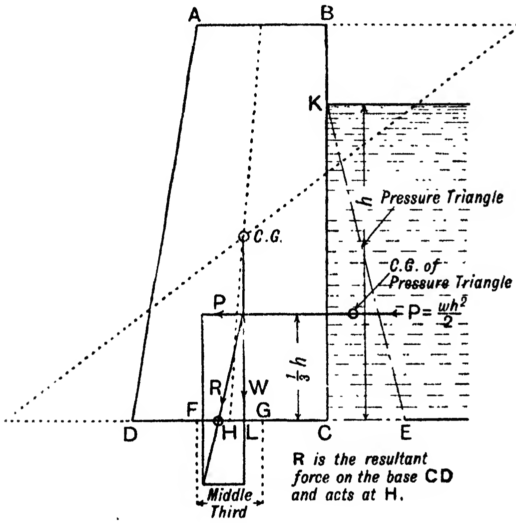


FIG. 372.

then due to the resultant water pressure P acting normally to the wall at the **centre of pressure**, and the resultant weight W of the masonry acting through the centre of gravity.

The **centre of pressure** for any plane surface acted upon by a fluid may be defined as the point of action of the resultant pressure on that surface.

For the rectangular water surface represented by KC (Fig. 372), the centre of pressure occurs at two-thirds of the depth KC measured from K. It may be noticed that the pressure increases uniformly with the depth and the centre of gravity of the pressure triangle

KCE is two-thirds of the depth measured from K. The centre of gravity of the masonry can be found by the methods outlined in Chap. IX, and when the wall is made of material of uniform density the problem becomes that of finding the centroid of the cross-section of the dam.

Retaining walls are employed for supporting earth and rock in the construction of railway cuttings and the sides of natural reservoirs. The method of calculating the forces acting on such walls is beyond the scope of this book, but when known, the resultant of the forces can be found by the rules given in Chap. V.

Example. *In Fig.* 372 ABCD *is the trapezoidal cross-section of a retaining wall of a dam in which* AB = 10 *ft.*, BC = 30 *ft.*, *and* CD = 15 *ft. The depth of water h is* 24 *ft. on one side only, and the wall is of uniform density and weighs* 28 *tons per foot length.*

Calculate per foot length of wall:

(*a*) *the magnitude of the water thrust on the vertical face*;

(*b*) *the magnitude of the resultant thrust on the base of the wall*;

(*c*) *the point in the base where the resultant thrust acts*;

(*d*) *the amount of the overturning moment on the wall due to fluid thrust.*

By a geometrical construction, or by calculation, it will be found that a vertical line through the centre of gravity of the wall cuts the base at L, such that $LC = 6{\cdot}366'$.

Average pressure on water face $= 12 \times 62{\cdot}3$ or $747{\cdot}6$ lb. per sq. ft.

(*a*) Total pressure on wall $= P = 747{\cdot}6 \times 24 =$ **17,942 lb.**

This acts at a depth of 16 ft. normally to the water face of the wall.

(*b*) Resultant of P and $W = \sqrt{P^2 + W^2} = \sqrt{17{,}942^2 + 62{,}720^2}$

$$= \mathbf{65{,}230\ lb.}$$

(*c*) This resultant will cut the base at H, such that,

$$LH = 8 \times \frac{17{,}942}{62{,}720} = 2{\cdot}289;$$

$\therefore$ resultant cuts the base at $2{\cdot}289 + 6{\cdot}366$ or **8·655 ft. from the water face.**

(*d*) **Overturning moment** $= 17{,}942 \times 8$ lb. ft. $=$ **143,536 lb. ft.**

EXERCISES ON CHAPTER XIV

Hydraulic machines.

1. An accumulator ram is 16·4 in. diameter. The dead load, including the weight of the ram, is 24·7 tons. Calculate the pressure in lb. per sq. in. in the machine service main.

2. An accumulator is to produce a service pressure of 800 lb. per sq. in. Find the diameter of the ram if the dead load is limited to 160 tons. If the lift or stroke is 23 ft., find the storage capacity of the accumulator in H.P. hours.

3. Calculate the work done in foot tons, when an accumulator ram weighing, with its dead load, 94 tons is lifted through a travel of 8 feet. If the time of travel is 3 minutes, find the minimum H.P. of the pumps. Would you expect this H.P. to service the accumulator; if not, where would you expect losses?

4. A suction pump of stroke 1 foot, bucket diameter 8 in., and a speed of 60 double strokes per minute, is used to raise water from a pond. Calculate the quantity of water raised in gallons per hour if the coefficient of discharge is 90%.

5. A lift pump, barrel diameter 7 in., stroke 11 in., is to raise water to a point 20 ft. above the lowest position of the pump bucket. Find the maximum tension in the bucket rod, if the rod is 1 in. in diameter.

6. A hydraulic press has a plunger 4 in. in diameter and a ram 20 in. diameter. The plunger is operated by a toggle which exerts a force of 1120 lb. Find the load capacity of the ram if the efficiency is 90%.

7. A small hand press has a plunger 1 in. in diameter, and a ram 4 in. diameter. The plunger is operated by a lever having a leverage of 20. Calculate the force exerted by the ram for a lever effort of 25 lb. when the overall efficiency is 75%.

8. The effort on the plunger of a hydraulic press is limited to a force of 420 lb. It is required to exert a ram force of 5 tons on a ram 6 in. diameter. Find the diameter of the plunger, allowing for an efficiency of 85%.

9. A press is operated by a plunger $1\frac{1}{2}$ in. diameter, stroke 5 in. The plunger effort is 700 lb., and the ram, which is 6 in. in diameter, exerts a force of 10,000 lb. Find the efficiency of the press and the work done during one travel of the ram.

10. A hydraulic jack is fitted with a plunger $\frac{3}{4}$ in. in diameter. An effort of 28 lb. is applied through a lever of leverage 18. Calculate the lifting capacity of the jack if the ram is 4 in. diameter and the overall efficiency is 70%.

11. A hydraulic jack is to have a lifting capacity of 40 tons. Find the diameter of the ram if the plunger diameter is $\frac{1}{2}$ in., effort 25 lb., leverage 22 and efficiency 70%.

12. The ram of a large press weighs 7·6 tons. It is lifted, for purposes of releathering, by two jack rams, one on either side of its cross-head. Calculate the minimum diameter of the jack rams if they are serviced from a main at 400 lb. per sq. in.

13. Intensification is carried out in a press plant by the use of a differential ram of diameters 6 in. and 12 in. Calculate (*a*) the ratio of intensification, (*b*) the maximum pressure obtained if the larger end of the ram is serviced at 400 lb. per sq. in.

14. It is required to intensify from 200 lb. per sq. in. to 1600 lb. per sq. in. The larger diameter cylinder is restricted to a diameter of 16 in. Find the necessary diameter of the smaller cylinder, allowing for a 93% efficiency in the process of intensification.

15. During the process of testing pressure gauges a small intensifier is employed (Fig. 373). Find the maximum pressure available if the screw is of pitch $\frac{3}{16}$ in. and its efficiency is 30%.

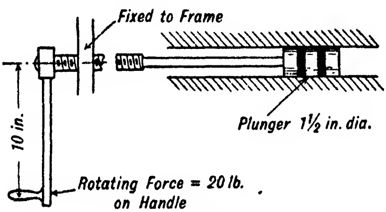


Fig. 373.

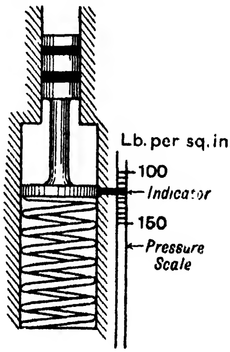


Fig. 374.

16. A diagram (Fig. 374) is shown of a type of pressure gauge in which the compression of a spring is employed to measure hydraulic pressure. Calculate the stiffness of the spring required if the pressure scale is to be 1 in. to 100 lb. per sq. in. Plunger diameter = 0·75 in.

17. A hydraulic crane has a ram 8 in. in diameter, and the lifting mechanism a velocity ratio of 9. Find the load which can be raised by the crane, employing water at 700 lb. per sq. in., if the efficiency of the hydraulic portion is 85% and that of the lifting mechanism 55%.

18. A crane is to raise 4 tons with a lifting mechanism having a velocity ratio of 7. What diameter of ram is required if the water pressure available is at 400 lb. per sq. in. and the overall efficiency 50% ?

19. A hydraulic press is used for the indentation of hot metal bars. Find the diameter of the ram if the indenting force has an average value of 2·3 tons per square inch of indentation, and the area of the indentation is 28 sq. in. Pressure on supply 400 lb. per sq. in., efficiency 80%.

20. Find the ram diameter of the press of a hydraulic shearing machine if it is to be designed to shear plates 8 in. in width, $\frac{3}{8}$ in. in thickness, of ultimate shear stress 27 tons per sq. in. Allow for a working efficiency of 78% and a service of 700 lb. per sq. in.

21. A press is to be employed for forming and shearing. The force required for forming is 11 tons per sq. in., and for shearing 28 tons per sq. in. Forming is done on a service pressure of 400 lb. per sq. in. Find the ratio of intensification required for the shearing operation, if the area resisting shear is $\frac{3}{4}$ of that resisting forming. What will be the water pressure during shearing ?

22. A lift weighs 3·7 tons, and is to raise a load of 14 tons. If the operation is direct from a hydraulic ram, find the necessary ram diameter for a service pressure of 550 lb. per sq. in. and a working efficiency of 75%. If the lift is raised 20 ft., find the cost at $1\frac{1}{2}$d. per horse-power hour.

Hydraulic pumps.

23. The feed pump supplying water to a boiler producing steam at a gauge pressure of 200 lb. per sq. in. is situated 20 ft. below the boiler. Calculate the total head against which the pump works. What theoretical energy does the pump require per pound of feed water ?

24. A single-acting force pump is installed at a height of 18 ft. above the water level in the sump. It delivers to a height of 50 ft. above its installation level. If the ram is 10 in. diameter and stroke 2 ft. 2 in., find the H.P. required to operate the pump with an efficiency of 70% and no slip. Speed, 75 double strokes per minute.

25. A double-acting force pump is to deliver water to a height of 120 ft. Find the necessary plunger diameter if the stroke is 12 in., speed 160 strokes per minute and available driving H.P. 7·4. Efficiency to be taken as 70%, with no slip.

26. What H.P. motor would you suggest in order to drive a centrifugal pump of efficiency 85% against a pumping head of 250 ft. and delivering 100,000 gallons of sea water per hour ? Take the efficiency of the motor as 88%.

27. A centrifugal pump running at 400 revolutions per minute delivers 300 lb. of water per revolution to a height of 85 ft. The pump is driven by a motor having an efficiency of 90% and rated at 300 kW. Find the efficiency of the pump.

28. What pumping H.P. is required to serve an accumulator storing water at 700 lb. per sq. in. if the ram of the accumulator is 22 in. in diameter and the ram can be raised at $1\frac{1}{2}$ ft. per minute under full pumping service ? Overall efficiency of pumps and accumulator to be taken at 67%. If the efficiency of the motor driving the pump is 90%, estimate the input of the motor in kilowatts, and the cost per hour at $\frac{1}{2}$d. per unit.

29. A hydraulic machine requires an input of 5 H.P. to operate it. Find the number of gallons required per minute and the cost per minute at 1d. per 80 gallons if the supply pressure is 750 lb. per sq. in. Would it be cheaper if the supply of power were at the rate of $1\frac{1}{2}$d. per H.P. per hour? Neglect all losses in the machine.

Mechanics of fluids.

30. What is the total or absolute pressure in lb. per sq. in. and in lb. per sq. ft. to which a diver is subjected when working at a depth of 275 ft. in sea water? (This is the greatest depth at which useful work has been accomplished.) Wt. of 1 cu. ft. of sea water = 64 lb.

31. A tank full of fresh water for cooling a gas engine is 6 ft. high, 6 ft. long and 4 ft. wide. Calculate the total pressure on each of the sides and on the bottom of the tank.

32. Obtain the total thrust in tons on the water face of a caisson for a dry dock if the dock entrance is 80 ft. wide and the water 40 ft. deep. Weight of sea water, 64 lb. per cu. ft.

33. A watertight door in the conning tower of a submarine has an area of 6 sq. ft. Calculate the resultant force in tons acting on this door at a depth of 40 fathoms. (1 fathom = 6 ft.)

34. Find the total thrust in tons on a vertical strip 1 ft. wide of a retaining wall for a reservoir containing fresh water. Depth of water, 45 ft.

35. What is the velocity head corresponding to that of a jet issuing at 120 ft. per sec. from the nozzle of a fireman's hose?

36. Find (*a*) the theoretical velocity due to a head of 200 ft. of fresh water, and (*b*) the pressure head and total head corresponding to a depth of 20 ft. of fresh water with a free surface.

37. A petrol storage tank has a small hole 15 ft. below the surface. Assuming no losses, what would the velocity of discharge be? Specific gravity of petrol, 0·7. What is the pressure energy per lb. 15 ft. below the surface?

38. Find (*a*) the total head, (*b*) the energy per lb. of water, (*c*) the horse power available from a waterfall, if the height of the fall is 80 ft., and just above the fall the cross-section of the stream is 12 sq. ft. and mean velocity 2 ft. per second.

39. A submarine oil fuel tank is 8 ft. deep, and as the oil is consumed it is replaced by sea water. Find the difference in pressure in lb. per sq. ft. between the top and bottom of the tank when the depth of oil has been reduced to 3 ft. Specific gravity of oil, 0·9.

40. In a Venturi meter the diameters of the pipe and throat are 2 in. and $\frac{1}{2}$ in. The discharge was found to be 300 gallons in 20 min. Find (*a*) the velocity of the water in ft. per sec. in the throat and pipe,

and (*b*) the theoretical reading of the meter, or difference in pressure head.

41. The constant of a Venturi meter is 0·92 and the difference in heads 0·25 ft. What is the discharge in cubic ft. per sec. ?

Orifices and nozzles.

42. Explain the terms, coefficient of contraction of area, coefficient of velocity and coefficient of discharge, when applied to the flow of water through orifices and nozzles.

43. Draw a diagram to show the formation of the vena contracta in each of the following, (*a*) sharp-edged orifice, (*b*) Borda mouthpiece, (*c*) projecting pipe mouthpiece.

44. The water supply to a small washing plant is measured by passing it through a tank, with a constant head of 20 feet, and discharging through a circular sharp-edged orifice 3 in. in diameter. Find the quantity of water in gallons per hour supplied if the coefficient of discharge is 0·62.

45. Find the required diameter of a sharp-edged circular orifice to discharge 10 cu. ft. of water per minute under a head of 11 ft. Coefficient of discharge = 0·62.

46. A jet of water under a head of 100 ft. is discharged through a nozzle $1\frac{1}{2}$ in. in diameter of coefficient of discharge 0·62. If the jet impinges on a flat plate, find the force exerted if the whole of the momentum is destroyed. Take c_v as 0·97.

MISCELLANEOUS EXERCISES AND EXAMINATION QUESTIONS.

47. A vertical cylindrical water tank is 4 ft. in diameter and 8 ft. high. Find the total water pressure on the bottom and on the curved surface when the depth of water in the tank is 6 ft. (U.L.C.I.)

48. A pump delivers 1,080,000 gall. of water per hour against a total head of 41·5 ft. The pump is driven by an electric motor which uses 435 amperes at 500 volts. Calculate the overall efficiency of the plant. (U.L.C.I.)

49. The gate of a dry dock, which is 40 ft. wide, has water outside it to a depth of 26 ft. What is the total water pressure on the gate, and what is the intensity of the water pressure at the bottom of the gate ? (One cubic foot of salt water weighs 64 lb.) (U.L.C.I.)

50. A jet of water 1 in. diameter issues from a nozzle under a head of 40 ft. Find (*a*) the velocity of the water leaving the nozzle, (*b*) the weight of water issuing per second, and (*c*) the amount of kinetic energy leaving the nozzle per second. (U.L.C.I.)

51. Explain the meaning of the expression "Coefficient of discharge of an orifice." An orifice 1 in. diameter discharges 19·6 gallons of

water per minute under a head of 4 ft. Calculate the coefficient of discharge of the orifice. State how you would measure the discharge when making an experiment. (U.L.C.I.)

52. A jet of water $\frac{1}{4}$ in. in diameter issuing from a nozzle under a head of 20 ft. impinges normally on a flat stationary disc. Find (*a*) the weight of water issuing from the nozzle per second, (*b*) the momentum of the water issuing from the nozzle per second, and (*c*) the pressure exerted by the water on the disc. (U.L.C.I.)

MISCELLANEOUS EXERCISES

SECTION A

The abbreviations indicated in the brackets after each question are :

(U.E.I.) for questions set for the Senior First Year National Certificate by the Union of Educational Institutions.

(U.L.C.I.) for questions set for the Preparatory and Senior First Year National Certificate by the Union of Lancashire and Cheshire Institutes.

Moments.

1. A uniform wrought-iron tube 16 ft. long and weighing 30 lb. is used as a beam and is freely supported at each end. A weight of 15 lb. is attached to the tube $3\frac{1}{2}$ ft. from the left support, and one of 32 lb. at 4 ft. from the right support. Determine the magnitude of the reactions at the supports. (U.L.C.I.)

2. A light rod, AB, 3 ft. long, is suspended by a string attached to it at F.

(*a*) If F is 10 inches from A, find what weight must be hung from B to keep the rod horizontal when a weight of 4 lb. is hung from A.

(*b*) What must be the position of F if a weight of 4 lb. hung at A, and 6 lb. hung at B, keeps the rod in equilibrium ? What would be the pull in the string in this case and how would you show by experiment that your answer is correct ? (U.E.I.)

(NOTE.—The weight of the rod is to be neglected.)

3. Four parallel forces act vertically downwards and are at equal distances apart. If the magnitude of the forces, in order, are 4, 5, 6, and 7 lb. respectively, calculate the magnitude and position of the resultant force. Without further calculation state the magnitude, position, and direction of a force which would neutralise the effect of the four given forces on any body on which they were acting. (U.E.I.)

4. Obtain the turning moment on the crank shaft of an engine when the crank has moved through an angle of 45° from the inner dead centre in a clockwise direction, the total pressure on the piston being 1000 lb. The radius of the crank is 6 in. and the distance between the connecting rod centres, 2 ft. 6 in. (U.E.I.)

5. A light rod AB, 20 in. long, is pivoted at A and hangs vertically. Forces are applied simultaneously by means of strings attached to the rod. A force *CD*, of 4 lb., is applied at C, 16 in. below A, the angle BCD being 45°, D being to the left of the rod, and a horizontal force *BE* of

3 lb. is applied at B, E being to the right of the rod. Find the resultant turning moment on the rod. Where must a horizontal force of 2 lb. be applied to prevent rotation of the rod? (U.E.I.)

6. A uniform lever is 10 ft. long and weighs 100 lb. It is placed with one end resting on a beam and a weight of 60 lb. is suspended from the lever at a distance of 2 ft. from the beam. Make a sketch of the arrangement showing clearly the direction of the vertical force required at the free end of the lever to keep it at rest and horizontal. What is the amount ofthis force and what is the amount of the reaction between the lever and the beam? (U.E.I.)

7. The horizontal arm, OA, of a cranked lever is 12 in. long. The other arm, OB, is 8 in. long and the angle between the arms is 135°, B being above OA. A vertical downward force of 20 lb. acts at A and horizontal force, P, acts at B and keeps the lever balanced. Find the magnitude of the force P, also the magnitude and direction of the reaction at the pivot O. (The forces and lever are all in the same plane.) (U.E.I.)

8. In an experiment on parallel forces a uniform beam 40 in. long and weighing 2 lb., was suspended by means of spring balances at the ends A and B. A weight of 5 lb. was suspended at C, 12 in. from the left balance A. The reading of the spring balance A was observed to be 4·4 lb., and that of B 2·6 lb. Are these readings correct? If they are incorrect, what is the amount of error in each of the readings? (U.E.I.)

9. Draw a horizontal line AE 5 in. long and on it mark off AB, BC, and CD 2 in., 1 in., and ½ in. respectively. AE is a lever pivoted at C. Vertical forces of 2, 3, 4, and 4 lb. act at A, B, D, and E respectively. State what will happen to the lever under the action of these forces and why? What additional force will be required at A to produce equilibrium? You may neglect the weight of the lever. (U.E.I.)

10. A body is pivoted about a point O in a vertical line BOA, the point B being above A. The lengths of BO and OA are 4 and 5 in. respectively. At A a force CA of 10 lb. weight acts towards A, the angle CAO being 60°. Determine the magnitude of the moment tending to rotate the body about the point O. At B a force DB is applied acting towards B, the angle DBO being 45°. Find the magnitude of this force if the body is in equilibrium.

The points D and C are on the right-hand side of the line BOA. (U.E.I.)

11. A uniform bar 10 feet long, weighing 2 lb. per foot, is supported on knife edges at its ends. Loads of 10, 4, and 16 lb. are placed at distances of 1, 6 and 8 ft. respectively from the left-hand support. Find the magnitude of the reactions at the knife edges. (U.E.I.)

Work and power.

12. A traction engine working against a total resistance of 480 lb. does 330,000 ft.-lb. of work per minute. How far does it travel in one minute and what is its speed in miles per hour? (U.E.I.)

13. What do you understand by the terms "force" and "work"? In what units are they generally measured?

A man pushed a truck a distance of one quarter of a mile, in 12 minutes, by exerting a constant force of 30 lb. How much work has he performed per minute? (U.E.I.)

14. The following table gives the forces F lb. acting on a moving body at different distances x ft. from an initial position.

F lb. - -	10	17	25	38	45	50
x ft. - -	0	7	15	28	35	40

Draw to scale the diagram of work and find the number of foot-pounds of work the diagram represents. (U.L.C.I.)

15. A spring balance was inserted between a motor and truck in order to measure the pull exerted by the motor. The readings, P lb., of the balance were taken at distances S yards from the starting point and were as follows:

P lb. - -	180	130	104	88	86	96	108	195·5	125	125	115
S yards -	0	100	200	300	350	400	450	500	550	600	700

Plot a curve connecting P and S, plotting P vertically, and using this curve, find the average pull exerted. What was the work done in foot-tons in hauling the truck over this 700 yards? (U.E.I.)

16. Explain what you understand by "a foot-pound of work?"

A man performed 21 inch-tons of work in raising a lorry by means of a jack. How many foot-pounds of work were done? If the man applied a force of 32 lb. throughout each working stroke of length 15 in., assuming no lost work, how many working strokes did he make? (U.E.I.)

17. An indicator diagram taken from a certain gas engine on being measured shows that the effective pressures, in pounds per square inch in the cylinder at equal intervals of the stroke are 440, 270, 175, 130, 95, 70, 50, 35·5, 30, 25 and 22, the first mentioned being exactly at the commencement and the last at the end of the stroke. The diameter of the piston is 9 in. and the length of the stroke is 16 in. Plot a curve connecting the pressure and stroke (plotting stroke horizontally) and using it, find the average effective pressure, in pounds per square inch,

acting on the piston, also the work done, in foot-pounds, during the working stroke. (U.E.I.)

18. A machine casting weighs 480 lb. How many foot-pounds of work would be expended :

(*a*) In raising it vertically through a height of 66 in.

(*b*) In moving it 5 ft. over a horizontal bed plate, the resisting force being 0·24 lb. per pound weight of casting ? (U.E.I.)

19. A body is being acted upon by a variable lifting force. When the body is lifted S feet, the force P lb. weight is observed :

S - -	0	10	20	30	40	50	60	70
P- -	850	810	720	605	495	390	300	250

Find the average lifting force and the work done by P in lifting the body 70 ft. (U.E.I.)

20. Explain the statement "the power of a machine is one horse-power".

A weight of 1 ton is raised 15 ft. by a crane. What work is done ? If the lifting takes half a minute, what is the effective horse-power of the crane motor ? (U.L.C.I.)

21. A cyclist, moving against an average total resistance of 15 lb., travels a distance of 10 miles in one hour. Calculate the amount of work done in one minute in foot-pounds. (U.E.I.)

22. 500 lb. of material contained in a cage are lifted from a shaft 600 feet deep by means of a rope weighing 1·2 lb. per foot length. Show by means of a carefully drawn diagram the work done in lifting the cage to the surface of the shaft, stating the total amount of work done. If the cage is lifted to the surface in five minutes at uniform speed, what horse-power has been exerted ? (U.E.I.)

23. An electric motor weighs 3 cwt. and is standing on the shop floor. It has to be lifted to permit a low trolley to be slid underneath it. A man passes a long bar through an eye-bolt on the top of the motor, rests one end of the bar on a trestle and lifts up the other end, thus raising the motor 8 in. Neglecting the weight of the bar, how much work, in foot-pounds, is done by the man in actually raising the motor ? If the eye-bolt is 2 ft. from the end of the bar resting on the trestle, and the man applies a force of 112 lb. to the end of the bar, find, neglecting the weight of the bar, its length. (U.E.I.)

Co-planar forces and simple frames.

24. How would you verify by experiment the principle of the parallelogram of forces ?

What is the resultant of two forces of 30 lb. weight and 50 lb. weight respectively, acting at an angle of 45 degrees ? (U.L.C.I.)

25. State carefully how a straight line may be used to represent the various features of a force. What force acting at right angles to one of 4 lb. will give a resultant force of magnitude 5 lb. ? State the direction of the resultant force with reference to that of the 4 lb. force. (U.E.I.)

26. Three forces *A*, *B*, and *C*, in the same plane, meet at a point D. *A*, of magnitude 5 lb. acts in a direction north-west, *B*, of magnitude 4 lb., in a direction due east, and *C*, of magnitude 3 lb., in a direction due south.

(*a*) Is the point D in equilibrium ?
(*b*) If not, what alteration in magnitude and direction must be made to *C* in order that D may remain in equilibrium ? (U.E.I.)

27. Explain fully how a force may be completely represented by a straight line.

A barge is being towed along the centre line of a canal. The horse exerts a pull of 200 lb. on a horizontal rope attached to the barge, the rope being inclined at 30° to the centre line of the canal. Find, using a graphical solution, the force tending to urge the boat along the centre line of the canal and the force tending to send the boat into the side. (U.E.I.)

28. Draw a triangle ABC, AB (of any convenient length) being horizontal and the angles CAB and CBA being 60° and 30° respectively. This triangle represents a roof truss supported at A and B and carrying a vertical load of 2000 lb. at C. Find graphically the forces in each of the sloping members, then, by resolving the force in the member CB, find the vertical reaction at the support B. (U.E.I.)

29. Draw a vertical line YO and from O set out an angle YOA, of 30°, to the right of YO and an angle YOB, of 45° to the left of YO. OA and OB represent the directions of two forces of 5 lb. and 6 lb. respectively acting from O. Determine the magnitude and direction of a third force in order that the point O shall be maintained in equilibrium. (U.E.I.)

30. Two pulling forces acting at a point are applied to a body, one of 50 lb. acting north-east, and the other 30 lb. acting 30° west of north. Represent these two forces graphically and find their resultant. Also find graphically the horizontal and vertical components of the resultant force. (U.E.I.)

31. Forces are applied along four cords, OA, OB, OC, and OD, these cords being knotted together at O. The forces act as follows :

Force in OA acts horizontally from left to right, the force being 6 lb.
Force in OB acts horizontally from right to left, the force being 2 lb.
Force in OC acts vertically upwards, the force being 8 lb.
Force in OD acts vertically downwards, the force being 3 lb.

What are the horizontal and vertical resultants of these forces ? Find, by means of a graphical solution, the magnitude of the resultant of the above four forces. What is its inclination to the horizontal ? (U.E.I.)

32. Two ropes, AB, AC, which make angles of 30 degrees and 45 degrees on opposite sides of a vertical line through A, support a load of 100 lb. attached to them at the point A. Determine the tensions in the ropes. (U.L.C.I.)

33. A string is threaded through the ring of a weight of 8 lb. and the ends brought together so that the parts of the string are vertical. What will be the pull in each part of the string ? If the ends of the string are now moved apart horizontally, what will be the pull in each part of the string when the angle between them is 100° ? If a pull of 10 lb. will break the string, what will be the angle between the parts of the string when failure occurs ? (U.E.I.)

34. A body weighing 20 lb. rests on a plane inclined at 30° to the horizontal. Neglecting friction, find graphically the force which will maintain equilibrium when :

(*a*) it acts horizontally ;
(*b*) it acts upwards at 45° to the horizontal.

NOTE.—The two angles are measured anti-clockwise from the horizontal. (U.E.I.)

35. What quantities are required to completely specify a force ? From your answer show how a force may be completely represented by a straight line. (U.E.I.)

36. Two forces act at a point on a body ; one is horizontal acting from left to right, the other is inclined at 60° to the horizontal acting upwards. Their resultant is a force of 50 lb. inclined at 20° to the horizontal also acting upwards. Determine the magnitude of each of the forces. Show clearly on a diagram, the additional force required to keep the body in equilibrium, denoting the magnitude and direction. (U.E.I.)

37. The piston of a horizontal steam-engine is 12 in. in diameter and the piston rod is $1\frac{1}{2}$ in. diameter. The effective steam pressure on the piston-rod side of the piston is 100 lb. per sq. in., calculate the total pull in the piston rod. The connecting rod of the engine is 5 ft. long and the crank arm 1 ft. long. Find, by means of a scale drawing, the pull in the connecting rod and the normal reaction between the crosshead and the guide bar when the crank arm is 60° from the inner dead centre, the piston moving away from the crank shaft. (U.E.I.)

38. A force, of magnitude 20 lb. weight, acts away from a point and is inclined at an angle of 30° to the horizontal, the angle being measured in an anti-clockwise direction. Show clearly, by means of a diagram, how this force may be represented by a straight line ; then determine, by means of a graphical construction, the horizontal and vertical components of the force. (U.E.I.)

39. What do you understand by the triangle of forces ? A cylinder cover, weighing 3 cwt., has eye-bolts fitted on a diameter 4 ft. 6 in.

apart and equidistant from the centre. It is slung by two chains, each 6 ft. long, from a hook, each chain being attached to an eye-bolt. Find graphically, the pull in each chain. (U.E.I.)

40. Draw a triangle ABC such that AB is vertical, A being above B, the angles BAC and ABC each being 30°. This is a line diagram of a toggle joint, B being pinned to the frame, and A being attached to a block moving vertically between guides. A horizontal force of 50 lb. being applied at C raises a load at A. Find, using a graphical construction, the thrust in the link CA and the magnitude of the load lifted, no account being taken of frictional resistance. (U.E.I.)

Friction and machines.

41. You are required to find how the force of friction varies with the normal force between the surfaces in contact. Give a sketch of the apparatus you would use and describe carefully the method of conducting the experiment. Your answer should include the observations to be made and the relationship you would expect to obtain. (U.E.I.)

42. State clearly what you understand by the terms: Force of Friction and Coefficient of Friction.

A steel block weighing 20 lb. is drawn slowly along a horizontal cast-iron plate by a force parallel to the plate. If the coefficient of friction between the two surfaces is 0·2 find the magnitude of the force. (U.E.I.)

43. A horizontal force of 1·5 lb. acts on a body weighing 18 lb. and is just sufficient to keep it moving slowly over a horizontal table.

(*a*) What is the value of the coefficient of friction ?

(*b*) The brake blocks acting on the rim of a bicycle wheel 26 in. diameter exert a radial force of 20 lb. The coefficient of friction between the brake blocks and the rim is 0·4. If the brake is applied while the wheel makes 10 revolutions, what work is done against friction ? (U.E.I.)

44. A machine and platform, together weighing 2000 lb., are to be moved along a shop floor by horizontal hauling. A ring bolt is fixed to the platform for this purpose and a rope, 0·75 in. diameter, is attached to the bolt. If the coefficient of friction between the platform and floor is 0·3, find the pull required. What would be the stress intensity in the rope if its effective cross-sectional area is 0·9 times the area of a circle 0·75 in. diameter. (U.E.I.)

45. Describe any experiment you have conducted to find how the force of friction varies with the normal pressure between the surfaces in contact. Describe carefully the method of conducting the experiment, giving a sketch of the apparatus used and the observations to be made. State the relationship you would expect to obtain. (U.E.I.)

46. The table of a planing machine and the job fixed to it together weigh 2 tons. It makes six forward and six backward strokes per minute. Each stroke is 5 ft. long and the coefficient of friction between

the sliding surfaces is 0·08. What is the work done in foot-pounds per minute in moving the table ? What horse-power does this represent ? (U.E.I.)

47. The piston of a hydraulic press has an area of 100 sq. in. The water is fed to it from a reservoir 23 ft. above the level of the press. Given that a column of water 2·3 ft. high exerts a pressure of 1 lb. per sq. in., find the total force in pounds exerted by the press. (U.L.C.I.)

48. Give a diagrammatic sketch of an ordinary suction pump, and explain how it works. Is there any limit to the length of the vertical suction pipe which may be employed in this type of pump ? If so, state what factor governs the depth from which water may be drawn. (U.L.C.I.)

49. What is meant by the moment of a force ?

Describe an experiment to show that a rod is in equilibrium when the sum of the moments tending to turn it in one direction is equal to the sum of the moments tending to turn it in the other direction.

A winch (wheel and axle) is used for raising water from a well. The diameter of the axle is 10 in., and the length of the crank is 2 ft. Find what force must be exerted at the end of the crank just to raise a load of 100 lb. (Neglect the weight and diameter of the rope.) (U.L.C.I.)

50. In a lifting block and tackle the velocity ratio is 12.

(*a*) What distance would the load be raised if the effort were applied tangentially at the circumference of a pulley of 28 in. diameter and the pulley made one revolution ?

(*b*) What effort would be necessary to raise a load of 180 lb., the mechanical efficiency of the pulley at this load being 40 per cent ?

(*c*) How would you determine experimentally the velocity ratio of pulley blocks such as the above ? (U.E.I)

51. The following results were obtained in an experiment on a set of pulleys.

W denotes the the weight raised and E the effort applied. Plot these results on squared paper and obtain the law connecting E and W.

W lb. - -	14	21	28	35	42
E lb. - -	2·5	3·3	4·25	5·1	6

(U.E.I.)

52. The following results were obtained during an experiment with a lifting machine having a velocity ratio of 8.

Load raised lb.	6	10	20	30	40
Effort lb. -	4·8	6	10	14	18

Assuming ideal conditions for the machine, calculate the loads that would be raised by the various efforts. Set out on an effort base graphs

of actual and ideal loads raised, and explain the reason for the differences between them. (U.L.C.I.)

53. Explain carefully what is meant by the terms velocity ratio, mechanical advantage, and mechanical efficiency as applied to a lifting machine.

Describe how you would determine, by experiment, the velocity ratio of any lifting machine with which you are familiar. Using symbols to denote the observed quantities, show how the velocity ratio would be calculated.

A lifting machine has a velocity ratio of 72. It is found by experiment that an effort of 70 lb. is required to lift a load of 1800 lb. What is the efficiency of the machine at this load? (U.E.I.)

54. A helical spring is compressed between the jaws of a vice. A boy finds that his full weight on the end of the vice handle, when in a horizontal position, will just close the spring hard up. The boy weighs 100 lb., the vice screw pitch is two threads per inch and the screw efficiency is 0·3. If the vice handle is 20 in. long, find the actual force compressing the spring. (U.E.I.)

55. A worm and worm-wheel used for experimental purposes consists of a single threaded worm A, on the shaft of which is keyed a wheel B of effective circumference 14 in. The worm gears with a worm-wheel C having 80 teeth. On the worm-wheel shaft is keyed a wheel D of effective circumference 16 inches. Around the wheel B is coiled a cord to which the effort is applied and around the wheel D a cord is coiled to which is attached the weight to be lifted. Make a line diagram of the machine and determine its velocity ratio.

In an experiment with the above machine it was found that when the load lifted was 60 lb. the mechanical efficiency was 0·28. What was the magnitude of the applied effort? (U.E.I.)

56. A screw-jack is operated by a bar pushed through a hole in the spindle. The pitch of the screw is ½-inch. A force of 60 lb. applied at right angles to the bar, at a radius of 14 in. from the axis of the screw, is found to lift a casting weighing 2 tons which rests upon the top of the jack. Find the velocity ratio, the mechanical advantage and the mechanical efficiency of the machine under these conditions. (U.E.I.)

57. The following figures were taken from a student's laboratory book:

Results of a Test on a Screw-jack.

Effort applied lb.	Load lifted, lb.	Mechanical Efficiency
0·74	15	0·405
2·5	60	

Deduce the velocity ratio of the machine and fill in the omitted mechanical efficiency. (U.E.I.)

Centre of gravity.

58. A rectangular sheet of metal 8 in. long by 6 in. wide is divided into 4 equal rectangles by lines through the centre parallel to the sides. One of these small rectangles is cut away. Determine the position of the centre of gravity of the remainder. How would you check your result experimentally? (U.L.C.I.)

59. A piece of sheet metal, ABCDE, of uniform thickness and density is in the shape of a combined square and isosceles triangle. ABDE is the square and BDC the isosceles triangle. BC = DC. The sides of the square are 4 in. long and C is 6 in. from BD and 10 in. from AE. Obtain by calculation, the position of the centre of gravity of ABCDE. (U.E.I.)

60. If you were supplied with a metal bar tapering uniformly from one end to the other, and two spring balances, explain carefully how you would determine the position of the centre of gravity of the bar. If the bar weighs 35 lb., is 28 in. long, and has its centre of gravity 12 in. from one end, what would be the readings of the spring balances? (U.E.I.)

61. What do you understand by the term "centre of gravity of an area?" If you were supplied with a thin flat piece of cardboard of irregular shape, describe carefully how you would proceed to find, by experiment, the position of the centre of gravity of the cardboard. (U.E.I.)

62. A connecting rod, 36 in. long, weighing 50 lb., is supported by resting the small end on knife edges whilst the crank pin end is suspended from a spring balance. The spring balance reading is 35 lb. How far is the centre of gravity of the rod from the centre line of the small end?

If the knife edges at the small end weighed 7 lb., and they were resting on a weighing machine, what reading should the machine record? (U.E.I.)

63. ABCD is a rectangular plate of uniform thickness. The long sides AB and CD are each 4 ft. long and the short sides 2 ft. long. E and F are the mid-points of AD and BC respectively. A circular hole, $1\frac{1}{2}$ ft. in diameter, is cut out of the plate. The centre of the hole is on the line EF and $1\frac{1}{2}$ ft. from F. Calculate the position of the centre of gravity of the remainder of the plate, giving the distance from the point E. (U.E.I.)

64. A bar of metal 2 ft. long has been turned down to three parallel portions as follows:

A.	Length,	6 in.	Diameter,	2 in.	Weight,	5·3 lb.
B.	,,	8 ,,	,,	$1\frac{1}{2}$,,	,,	4·0 lb.
C.	,,	10 ,,	,,	1 ,,	,,	2·2 lb.

Find the position of the centre of gravity of the bar from one end. (U.E.I.)

Strength of materials.

65. Describe carefully any experiment to show how the extension of a vertical spiral spring depends on the weights added. What relation between the weight and the extension would you expect the experiment to show ?

A spring, 8 in. long, becomes 10 in. long when a weight of 4 lb. is added. (*a*) What weight is necessary to stretch the spring 1 in. ? (*b*) What weight must be applied to make the length of the spring 12·5 in. ? (U.L.C.I.)

66. The following data were extracted from a straight line load-extension graph of a test on mild steel. Determine a law connecting load and extension.

Load, in tons - -	2	4
Extension, in inches -	0·005	0·01

What would be the extension of the test piece for a load of 5 tons if the law you have obtained continued to be followed ?

The modulus of elasticity of the steel was known to be 13,000 tons per square inch and the sectional area of the test piece $\frac{1}{2}$ square inch, what was the original length of the test piece ? (U.E.I.)

67. A wire, of cross-sectional area 0·012 sq. in. and 30 in. long, was subjected to tensile loading and the following readings were obtained :

Load in lb. - -	20	40	60	80	100	120
Extension in inches -	0·0030	0·0064	0·0094	0·0124	0·0154	0·0188

Plot the load extension curve and from it determine the extension produced by a load of 50 lb.

Calculate, for this load, the stress intensity in the wire and the extension per inch length, *i.e.* the intensity of strain.

What is the modulus of direct elasticity (Young's modulus) of the material of the wire in pounds per square inch ? (U.E.I.)

68. A tension member in a roof truss is subjected to a pull of 12 tons. It is to be made of mild steel having a breaking strength of 30 tons per square inch. Find the diameter of the member if a factor of safety of 6 is to be used.

How much will a 10 ft. length of the member stretch under the above load if the modulus of direct elasticity (Young's modulus) of the material is 13,500 tons per square inch ? (U.E.I.)

69. State Hooke's Law, *i.e.* state the relationship between the intensity of stress and the intensity of strain.

A hollow column, 10 in. internal diameter, metal 1 in. thick, supports an axial load of 80 tons. What is the average compressive stress in the material ? (U.E.I.)

70. In a tensile test on a wire 5 ft. long and 0·0125 sq. in. cross-sectional area, it was found that the wire stretched 0·0032 in. under a load of 10 lb. Calculate the stress in lb. per square inch, the strain and the modulus of direct elasticity of the material of the wire (Young's modulus) in lb. per square inch. (U.E.I.)

71. A pull of 300 lb. is applied to a wire working a signal. The wire is 210 ft. long and 0·03 sq. in. in cross-sectional area. If the movement at the signal end is to be 6 in., find the movement which must be given to the signal box end of the wire.

Assume the modulus of direct elasticity of the material of the wire (Young's modulus) to be 13,500 tons per sq. in. (U.E.I.)

SECTION B

The abbreviations indicated in the brackets after each question are :

(W.S.) for questions set for Stage I in the Organised Continuation Classes in the West of Scotland.

(U.L.C.I.) for questions set for Senior Second year National Certificate by the Union of Lancashire and Cheshire Institutes.

(U.E.I.) for questions set for the Senior Second Year National Certificate by the Union of Educational Institutions.

(I.Mech.E.) for questions set at the Studentship examinations in Elementary Mechanics by the Institution of Mechanical Engineers.

(I.E.E.) for questions set at the Graduateship examinations in Applied Mechanics and Strength of Materials by the Institution of Electrical Engineers.

Co-planar forces, frames and centre of gravity.

1. The jib of a crane, hinged at one end, is being raised into position by means of a rope attached to the free end and in the same vertical plane as the jib. The jib is 50 ft. long, weighs 4 tons, and its centre of gravity is 20 ft. from the hinged end. When the jib is inclined at 45° to the ground the rope is inclined at 60° to the centre line of the jib. Find, by graphical means, the pull in the rope and the reaction at the hinge in magnitude and direction. (U.E.I.)

2. A ladder rests with one end on rough horizontal ground, the other end resting against a smooth vertical wall. Show by means of a diagram the forces acting on the ladder. Denote carefully the directions of the forces and describe the forces, *e.g.* weight, etc.

If the length of the ladder is 60 ft, and its centre of gravity is 25 ft. from the end resting on the ground, find by means of a scale drawing.

the reaction between the ladder and the wall when the foot of the ladder is 30 ft. from the wall. The weight of the ladder is 100 lb. (U.E.I.)

3. A rectangular trap door ABCD of uniform section is hinged along the edge AB and is held in a position inclined at 30° to the ground by a cord attached to the middle of the edge CD. This cord is inclined at 45° to the partly opened door. Find graphically the pull in the cord and the magnitude and direction of the reaction at the hinge. The weight of the door is 50 lb., AB = CD = 5 ft. and BC = AD = 4 ft. (U.E.I.)

4. A beam AB is hinged at A and maintained in a horizontal position by a rope attached to the end B, the direction of the rope making an angle of 60° with the beam. The beam is 10 ft. long and weighs 160 lb., and its centre of gravity is 6 ft. from A. Determine the tension in the rope and the magnitude and direction of the reaction of the hinge. (U.L.C.I.)

5. What is meant by the Polygon of Forces ?

State the conditions that must obtain in order that a body acted upon by three forces may remain in equilibrium.

Fig. 375 shows a trap door of dimensions 4 ft. 0 in. × 3 ft. 0 in. It is hinged along a 3 ft. edge and can be opened by a rope inclined at 60° to the door when closed. The door weighs 60 lb. and the centre of gravity of the door is 18 in. from the hinged edge. When the door is just on the point of opening, determine :

(*a*) the pull in the rope ;

(*b*) the magnitude and direction of the force on the hinge. (W.S.)

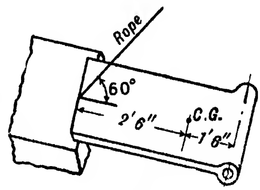


FIG. 375.

6. A uniform bar AB is 30 in. long and weighs 100 lb. It is suspended by two cords AC and BD from points C and D in the same horizontal line, so that A and B are 12 and 6 in. respectively below the horizontal line through CD. The inclination of the cord AC is 45°, *i.e.* the angle DCA is 45°. Find, using a graphical solution, the inclination of the cord BD to the horizontal and the pulls in both cords. (U.E.I.)

7. A ladder weighing 70 lb., with its centre of gravity one-third way up, rests with its tip against a smooth wall 28 ft. up, and its foot on rough ground 12 ft. from the bottom of the wall. Find the pressure of the ladder on the wall if a man weighing 140 lb. is standing at the top of the ladder.

Also find the least coefficient of friction between the ladder and the ground required to maintain equilibrium. (I.Mech.E.)

8. Copper wire of $\frac{1}{4}$ in. diameter is attached to the tops of two standards at the same level and 100 ft. apart. Find the tension in the wire to limit the maximum sag to 2 ft. Copper weighs 0·32 lb. per cubic inch. (I.E.E.)

9. A uniform rod, AB, 6 ft. long, is supported at an angle of 45 degrees to a smooth vertical wall, with its upper end A resting against the wall, and its lower end B attached to a string which is fastened to the wall vertically above A. Find, either by calculation or by a measured diagram, the necessary length of the string. (I.Mech.E.)

10. State the Principle of Moments.

Fig. 376 shows the linkage mechanism of a hand brake. If a force of 50 lb. is applied at A, determine the thrust R on the brake shoe B. (W.S.)

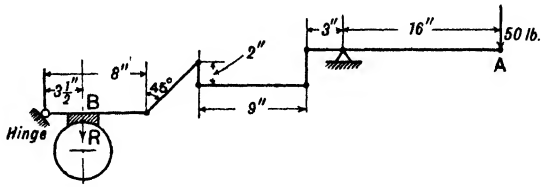


FIG. 376.

11. Show how to resolve a force into two components in given directions.

Fig. 377 shows a horizontal arm on a mast 30 ft. high and a series of stay lines. The arm weighs 20 lb., the mast weighs 300 lb., and the tension in the stays are as indicated. Determine :

(*a*) the vertical thrust on the base of the mast ;

(*b*) the turning moment in magnitude and direction at the base of the mast. (W.S.)

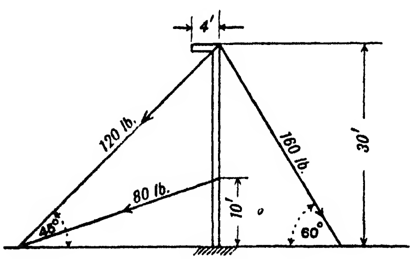


FIG. 377.

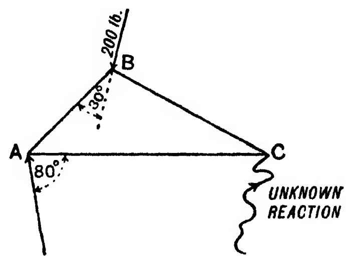


FIG. 378.

12. In the structure shown in the diagram (Fig. 378), AC is horizontal and the lengths of AB, BC, and CA are 10, 15, and 20 ft. respectively. A force of 200 lb. is applied at B. Find, by a graphical construction, the magnitude of the reaction at A and the magnitude and direction of the reaction at C. The reactions are not to be found by finding the forces in the members of the structure. (U.E.I.)

13. Calculate the reactions at the supports A and B of the framework loaded as shown in Fig. 379 and, using these reactions, obtain graphically the forces in the members P and Q of the framework. Distinguish between tensile and compressive stresses. (U.E.I.)

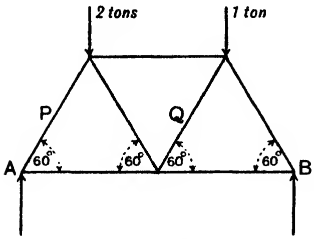


FIG. 379.

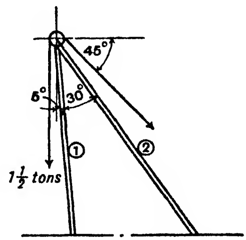


FIG. 380.

14. The head gear for pit-sinking operations is outlined in Fig. 380. The maximum pull in the hoist rope will be $1\frac{1}{2}$ tons, and the weight of the pulley and shaft fittings is 18 cwt.

Determine the loads carried by the supports, and if these be made of timber of 9 in. square section, estimate the stresses in the two members. (W.S.)

15. Fig. 381 shows cantilever framework which supports a load of 3 tons applied at A.

Draw the stress diagram for the frame and tabulate the stresses in the members.

If another vertical load of 5 tons is applied at B, indicate the members affected by this additional load and state the magnitudes and nature of the stress induced in these members. (W.S.)

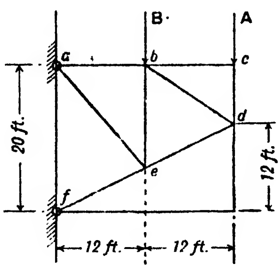


FIG. 381.

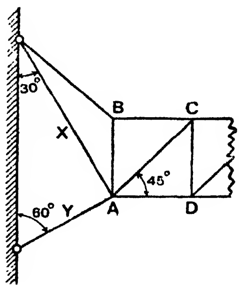


FIG. 382.

16. Fig. 382 shows part of a cantilever truss. At point A the known stresses are: member AB 5 tons tension; member AC 3 tons compression; and in member AD 2 tons compression.

Find graphically and analytically the magnitude and direction of the forces in the members X and Y. (W.S.)

17. State the condition that must obtain in order that a body acted upon by three forces may remain in equilibrium.

Fig. 383 shows a crane structure with a load of 20 tons. The reactions at A and B are vertical. Determine these reactions and the stress in the horizontal and in the vertical members of the structure, giving the kind of stress in each. (W.S.)

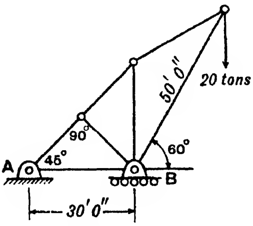


FIG. 383.

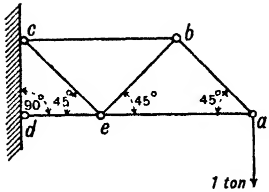


FIG. 384.

18. Determine the load in each member of the pin-jointed frame shown in Fig. 384, stating whether the load is tensile or compressive. Find also the magnitude and direction of the reactions at the wall attachments. (I.E.E.)

19. Determine the forces in each member of the frame shown in Fig. 385. (I.E.E.)

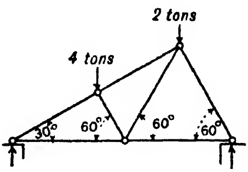


FIG. 385.

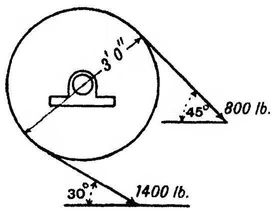


FIG. 386.

20. A pulley, 3 ft. 0 in. in diameter, has belt tensions as shown in Fig. 386. Determine the resultant turning moment on the pulley. If the weight of the pulley and shafting is 2000 lb., determine the resultant load at the bearing and the direction in which it acts. (W.S.)

21. Fig. 387 shows a lever in a link mechanism which at one moment of its movement is subjected to forces of 250 lb. and 180 lb. as shown.

Calculate (*a*) the force in the direction CE which will keep the lever in equilibrium in this position; (*b*) the reaction on the bearing at B. (W.S.)

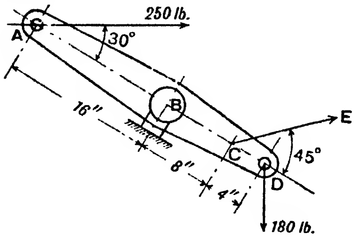


FIG. 387.

22. Forces act along the sides of a square OABC as follows :

19 lb. from O to A ; 8 lb. from A to B ;
10 lb. from O to C ; 1 lb. from B to C.

Find the magnitude and the line of action of their resultant, showing where it cuts OA and AB. (I.Mech.E.)

23. Find the centroid of the section shown in Fig. 388. (W.S.)

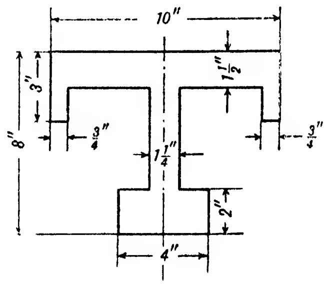


FIG. 388.

24. A square sheet of metal has a square of one-quarter the original area cut from one corner. Calculate the position of the centre of gravity of the remaining portion of the sheet. (I.E.E.)

25. A uniform plate ABCD is in the shape of a trapezium, with the parallel sides AB, DC respectively 6 in. and 10 in. in length, the distance between them being 8 in. Find how far the centre of gravity is from DC. (I.Mech.E.)

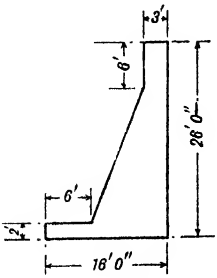


FIG. 389.

26. Find the centre of gravity of a uniform plate in the form of a symmetrical trapezium whose parallel sides are 4 and 8 ft. in length, and 4·5 ft. apart.

If it has a rectangular extension of the same weight per square foot attached to the 4-foot edge, and 4 ft. long, so as just to fit that edge, find what the height of the rectangular piece must be if the centre of gravity of the whole is on the 4-ft. edge of the trapezium. (I.Mech.E.)

27. Fig. 389 shows a section of a masonry wall. Determine the position of the centroid of the section. (W.S.)

28. A uniform frustum of a cone is 4 ft. high, and the radii of its circular top and base are 3 and 5 ft. respectively. Find the distance

of its centre of gravity from its base. (The centre of gravity of a cone is on its axis, a quarter way up.) (I.Mech.E.)

29. Define " Moment of a Force ", " Centre of Gravity of Section ".

Fig. 390 shows an outline of a mobile crane with a swivel boom. The weight of the car is 2 tons, and this weight may be taken to act at its centre of gravity as shown. The minimum inclination of the boom to the ground level is 30°.

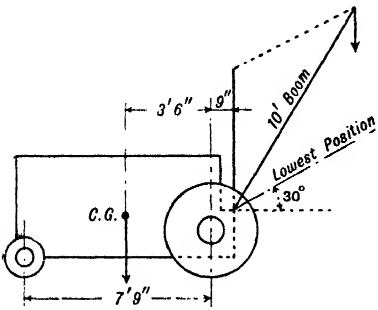


FIG. 390.

What is the maximum lift that can be made when the boom is—

(*a*) at its minimum inclination;

(*b*) inclined at 10° from the vertical? (W.S.)

30. A uniform rectangular lamina ABCD, with AB = 4 ft., AD = 3 ft., is hung up by a cord fastened to a point on AB 1 ft. from A. Find what angle AB will make with the horizontal in the position of equilibrium.

If the lamina weighs 4 lb., what weight hung at D would make AB horizontal? (I.Mech.E.)

31. A horizontal ring weighing 10 lb., and 12 in. in diameter, is suspended by three light chains attached to equidistant points on the rim, and carried to a common point 8 in. above the rim, so that the length of each chain between this point and the rim is 10 in. Calculate the tension in each chain. (I.Mech.E.)

32. A circular disc of 2 ft. diameter weighs 100 lb. and is suspended by three chains, each 20 in. long, which are attached to points equally spaced on the circumference of the disc and are brought together at a point above the disc. Find the tension in each chain. (I.E.E.)

33. A force of 25 lb. acts vertically upwards, and a couple whose moment is 30 lb.-ft. acts in the same plane as the force. Specify completely the resultant of the force and the couple. (I.E.E.)

Friction and machines.

34. A body is in equilibrium on an inclined plane. Show clearly, by means of a diagram, the forces acting on the body, describing the forces, *e.g.* weight, etc.

The inclination of the plane is increased and it is found that uniform motion, once started, continues when the plane is inclined at 15° to the horizontal. What is the coefficient of friction between the body and the plane? What force, parallel to the plane, will be required to keep the body, whose weight is 10 lb., at rest when the plane is inclined at 30° to the horizontal? (U.E.I.)

35. A box and contents weighing 180 lb. is pushed by a horizontal force up a gangway inclined at 20 degrees to the horizontal. The coefficient of friction between the box and the gangway is 0·3. Find the magnitude of the force. (I.E.E.)

36. A stone slab, weighing 50 lb., rests on a horizontal plane. Show in a diagram the forces, in magnitude and direction, acting on the body, describing the forces, *e.g.* weight, etc. A horizontal force of 8 lb., is applied and the body remains at rest, show in a second diagram the forces, in magnitude and direction, acting on the body, describing the forces. If the coefficient of friction between the surfaces in contact is 0·23, find the horizontal force required to maintain steady motion.

If the plane be gradually tilted, to what angle, to the nearest degree, can it be moved through to enable the slab to slide at uniform speed down the plane? (U.E.I.)

37. A 100 lb. mass is dragged by a rope up a 10 degree slope, the rope being parallel to the line of steepest slope. The coefficient of friction is $\frac{1}{2}$. Calculate the tension in the rope. Also draw a force diagram representing the forces acting on the mass. (I.Mech.E.)

38. A metal block which weighs 120 lb. rests on a horizontal plane. The coefficient of friction between the block and plane is 0·2. A horizontal pull of 12 lb. acts on the block. Find the magnitude of an additional horizontal pull which, acting at right angles to the 12 lb., will just cause the block to move. (I.E.E.)

39. A body which weighs 200 lb. rests on a horizontal plane, the coefficient of friction between body and plane being 0·1. Find the force which, acting at 30 degrees to the horizontal, will just move the body. (I.E.E.)

40. A body slides down a plane which is inclined at 20 degrees to the horizontal. The coefficient of friction between the body and the plane is 0·1. Find the acceleration of the body. (I.E.E.)

41. Define Angle of Friction and Coefficient of Friction.

A mass of 100 lb. is maintained in equilibrium on an inclined plane sloping at 40 degrees to the horizontal by a force P acting up the plane. The angle of friction between the mass and the plane is 10 degrees. Show by means of a diagram the greatest and least values of P consistent with equilibrium, and calculate these values. (I.E.E.)

42. State clearly the meaning of the terms Force of Friction and Coefficient of Friction.

An electric motor has four sets of brushes, each set making contact with the commutator over an area of 2·6 sq. in. The brushes are held down on the commutator by springs which exert a pressure of 2·3 lb. per sq. in. of brush-bearing surface. The commutator is 12 in. diameter and runs at 600 rev. per minute. On a no-load test it was found that 0·4 H.P. was required to overcome the friction of the brushes.

Determine the coefficient of friction between the brushes and the commutator. (W.S.)

43. Explain what you understand by "angle of friction" and establish the relationship between the "angle of friction" and the "coefficient of friction".

A rotor weighing 5 tons is mounted in bearings of 4 in. diameter and the torque necessary to make the engine revolve is 20 lb.-ft. Determine the "coefficient of friction" at the bearings and the horse-power absorbed by friction when the speed is 250 rev. per min. (U.L.C.I.)

44. A shaft, 5 in. in diameter, is supported in a bearing and carries a load of 1 ton. If the coefficient of friction between the rubbing surfaces is 0·03, calculate:

(*a*) the horse-power absorbed in friction when the shaft is making 100 rev. per minute; and

(*b*) the amount of heat generated per minute in the bearing assuming that the work done due to friction is converted into heat (U.E.I.)

45. Explain the term "angle of friction"; establish a relationship between the "angle of friction" and the "coefficient of friction". A rotor, weighing 4 tons, is mounted on bearings of $3\frac{1}{2}$-in. diameter and the torque necessary to make the rotor revolve uniformly is 20 lb.-ft. Determine–

(*a*) the coefficient of friction at the bearings;

(*b*) the horse-power absorbed in friction at 300 r.p.m.;

(*c*) the heat generated per minute at the bearings. (W.S.)

46. Define Coefficient of Friction; Angle of Friction.

A slide valve for the low-pressure cylinder of an engine works under the following conditions:

Valve dimensions– 3 ft. broad × 2 ft. 3 in. long.
Valve travel—9 in.
Steam pressure on back of valve—45 lb. per sq. in.
Engine speed—90 rev. per minute.

Taking the coefficient of friction as 0·08, determine the horse-power expended in driving this valve. (W.S.)

47. A test of a hoisting apparatus having a velocity ratio of 30 gave the following results:

No. of experiment	1	2	3	4	5
Load lb.	128	336	544	688	880
Effort lb.	7	15	23	28·5	36

Draw the load *v.* effort graph and the load *v.* efficiency graph (U.L.C.I.)

48. Define velocity-ratio, force-ratio (i.e. mechanical advantage), efficiency, stating the relation between them.

If in a windlass the velocity-ratio is 15 and the efficiency 40 per cent., find the weight that can be raised by an effort of 100 lb. (I.Mech.E.)

49. The following data were obtained for the effort (P) and load (W), when experiments were made with a small winch: $P=5{\cdot}7$ lb., $W=40$ lb., $P=19{\cdot}9$ lb., $W=150$ lb. The effort-load graph is a straight line and the velocity ratio of the winch was 11. Find the effort required for a load of 200 lb. and the efficiencies of the winch for the loads of 40 lb., 150 lb., and 200 lb. Plot an efficiency curve on a load base. (U.L.C.I.)

50. A differential wheel and axle, in which the diameters of the wheel and the two parts of the axle are respectively 40 in., 9 in., 7 in., is being used to raise a weight of 1600 lb. Sketch the apparatus in use; and calculate (1) the velocity ratio of the machine, (2) the effort required if the efficiency is 40 per cent. (I.Mech.E.)

51. A gate of a canal lock is fitted with a sluice gate to permit the outflow of water to be controlled. This sluice gate is just a flat rectangular door 2 ft. wide and 3 ft. deep sliding in guides vertically and being raised by a wire rope passing round a drum. The drum, of diameter 4 in., is turned by a handle of effective length 18 in. The average pressure over the surface of the sluice gate is 200 lb. per square foot and the coefficient of friction between the sluice and lock gate is 0·25. Find the force required to overcome this friction, also find the force exerted by the operator on the handle to raise the gate if the efficiency of the whole lifting apparatus is 0·35 and the dead weight of the parts lifted is 200 lb. (U.E.I.)

52. In a Weston differential pulley block, if the number of teeth in the two pulleys are respectively 59 and 60, find the effort needed to raise a weight of 1000 lb., assuming the efficiency to be 50 per cent. Give a sketch showing how the apparatus is used. (I.Mech.E.)

53. A screw-jack has a velocity ratio of 50. In an experiment it was found that an effort of 0·25 lb. was required to operate the screw when no load was being raised, also the frictional loss increased at the rate of 0·028 lb. effort per pound of load lifted by the machine. Find the effort required to lift a load of 60 lb. and the mechanical efficiency at this load. (U.E.I.)

54. Define the efficiency of a machine.

How may the efficiency of a screw-jack be determined for a given load? An experiment shows that to raise weights of 120, 250, and 390 lb. by means of a lifting appliance, efforts of 44, 72, and 102 lb. weight are required. The velocity ratio of the tackle is 14.

Determine—

(*a*) the law connecting the effort and the load;

(*b*) the effort, the mechanical advantage, and the efficiency when a weight of 320 lb. is being raised. (W.S.)

55. A screw-jack used for heavy work is manipulated by means of a crank handle of 14 in. crank radius, geared in such a way that two turns of the handle are required to raise the load on the jack $\frac{1}{4}$ in. Find the velocity ratio, and the load that can be raised by an effort of 60 lb. applied to the handle if the efficiency is 40 per cent. (Give the load in tons to 1 decimal place.) (I.Mech.E.)

56. Define the term "mechanical advantage".

Calculate the mechanical advantage and the efficiency of a screw-jack in which the screw has 4 turns to the inch, a mean diameter of 2 in., and a coefficient of friction 0·1, the lever being 12 in. long. (I.E.E.)

57. Calculate the velocity ratio when a spanner with an effective length of 15 in. is used to tighten a nut with 8 threads per inch. When the force applied to the spanner is 50 lb., assume a mechanical efficiency of 30 per cent and find the pressure exerted by the nut. (I.E.E.)

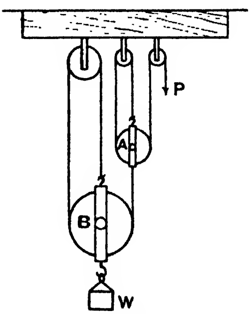


FIG. 391.

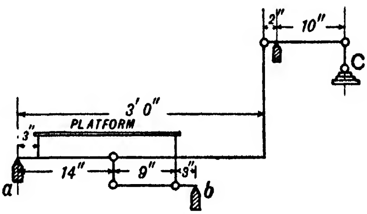


FIG. 392.

58. In the system of pulleys shown in Fig. 391 find what effort would be needed to raise a weight of 540 lb., assuming the efficiency to be 60 per cent, and neglecting the weight of the moving pulleys.

Also find how far it would be necessary for the pulley A to rise in order to raise W a height of 4 ft. (I.Mech.E.)

59. Define, in connection with a simple machine: velocity ratio, mechanical advantage, and efficiency.

Find the velocity ratio of the system of pulleys shown in Fig. 391 and, assuming an efficiency of 45 per cent., calculate the force P required to lift a weight W of 180 lb. (I.E.E.)

60. State the Principle of the Lever and give two simple illustrations.

Fig. 392 shows diagrammatically the arrangement of a platform weighing machine for luggage or heavy parcels. The total weight rests on the knife edges a and b and is balanced by the small weights on the hanger c.

Determine the hanger weights necessary to balance a load of 5 cwt. (W.S.)

61. Fig. 393 shows an outline of the transmission mechanism of a mobile crane to lift a maximum load of 2 tons.

Assuming that the efficiency of the whole gearing is 80 per cent, determine :

(*a*) the force exerted by the driving pinion ;
(*b*) the mechanical advantage of the crane from the driving pinion to the hook ;
(*c*) the hoisting speed of the load when the speed of the engine shaft is 1000 rev. per minute. (W.S.)

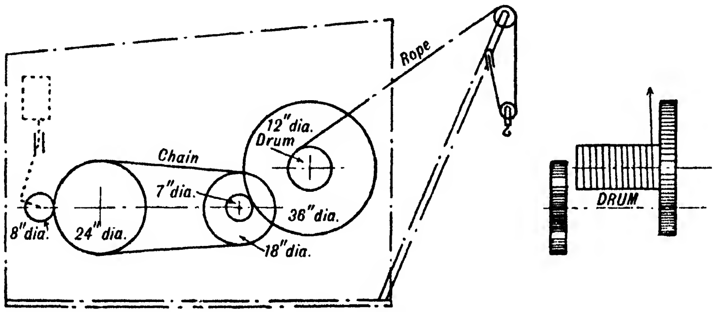


FIG. 393.

62. Explain what i meant by the "Law of a Machine".

The law of a machine is given as $P = 0{\cdot}014W + 15$. The velocity ratio of the machine is 160.

Determine :

(*a*) the effort required to lift 5000 lb. ;
(*b*) the mechanical advantage under condition (*a*) ;
(*c*) the efficiency under condition (*a*) ;
(*d*) the limiting efficiency of the machine. (W.S.)

NOTE.—For (*d*) the efficiency

$$= \frac{\text{actual load}}{\text{theoretical load}} = \frac{W}{P \times \text{velocity ratio}}$$

$$= \frac{W}{160\,(0{\cdot}014W + 15)} = \frac{W}{(2{\cdot}24W + 2400)}.$$

Divide the numerator and denominator by W and

$$\text{then efficiency} = \frac{1}{2{\cdot}24 + \dfrac{2400}{W}}.$$

For large values of W the efficiency approaches the value $\frac{1}{2{\cdot}24}$.

$$\therefore \text{ the limiting efficiency} = \frac{1}{2{\cdot}24} \text{ or } 44{\cdot}64\%.$$

63. A hand-operated travelling crane has a velocity ratio of 45. The efforts required to raise various loads are given in the table :

Load lb. - -	50	100	150	200	300	500
Effort lb. -	4·2	6·3	8·4	10·5	14·7	22·8

Determine the Law of the Machine.
Find an expression for the efficiency of the gear.
State the limiting value of this efficiency. (W.S.)

64. A bridge resting on rollers swings around a point. The weight on the rollers is 610 tons and the mean radius of the roller path is 16 ft. 9 in. The bridge is rotated by means of a rack having a radius of 16 ft. $10\frac{1}{2}$ in. Find the force at the rack radius which is just sufficient to swing the bridge, the coefficient of rolling resistance being 0·005. (U.L.C.I.)

Work, change of energy, power and its transmission.

65. Define the terms " Work " and " Power ".

The table of a small planing machine with its work piece weighs 350 lb. The stroke is $5\frac{1}{2}$ ft. and the machine is arranged to cut both ways. The coefficient of friction between the table and the guide ways is 0·06, and the average cutting force exerted by the tool in removing metal is 420 lb. When the machine is making 90 single strokes per min., determine the H.P. absorbed in overcoming the table friction and in cutting the metal. (W.S.)

66. Define " work " and state the units in which work is usually expressed.

Fig. 394 shows a modified indicator card taken from a double-acting steam engine.

The ordinates represent pressure on the piston to a scale of 1 in. = 160 lb. per sq. in., while the abscissae represents the stroke to a scale of $\frac{1}{12}$ full size. The engine piston is 20-in. diameter and 30-in. stroke.

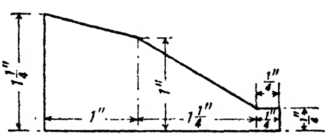


Fig. 394.

Determine :

(*a*) the average steam pressure on the piston in lb./in.^{2} ;

(*b*) the H.P. developed when running at 120 r.p.m. (W.S.)

67. A single cylinder double acting steam-engine drives a dynamo which generates current at 250 volts. The engine cylinder is 12 in. diameter, stroke 20 in., and the speed 120 rev. per minute. The average effective pressure in the cylinder is 80 lb. per sq. in. Assum-

ing that 75 per cent of the engine power is given out at the dynamo terminals, find :

(a) the output of the dynamo in horse-power ;
(b) the output of the dynamo in kilowatts ;
(c) the current in amperes. (U.E.I.)

68. (a) A body weighing 180 lb. is suspended at a distance of 30 ft. above the ground. How much energy is stored in it ? The body is then allowed to fall freely. What is its potential energy after 1·2 seconds and how much kinetic energy does it possess at this instant ?

(b) A steam turbine rotor runs at 3000 rev. per minute. The steam supply is shut off and the rotor comes to rest in 20 sec. Find the angular retardation of the rotor and the number of revolutions made in coming to rest on the assumption that this angular retardation is constant. (U.E.I.)

69. What horse-power is needed to propel a lorry weighing 3 tons at 15 miles per hour up a slope rising 1 in 30 against road resistance of ·2 lb. per ton ? (I.Mech.E.)

70. A truck commences to run down a gradient of 1 in 100, the total resistance being 10 lb. per ton. How far has the truck travelled when its velocity is 20 miles per hour ? (I.E.E.)

71. Find the horse-power needed to propel a 400-ton train at 30 miles per hour up a slope of 1 in 140 against frictional resistances of 10 lb. per ton. Also find what speed (in miles per hour) could be maintained on the level with this same horse-power, assuming that at the greater speed the resistance becomes 15 lb. per ton. (I.Mech.E.)

72. A train of 200 tons total weight is travelling at 60 miles per hour on the level against road resistances of 14 lb. per ton. Find the horse-power exerted by the engine—(1) assuming this speed is maintained ; (2) assuming the train is accelerating at one-fifth of a foot per second per second. (I.Mech.E.)

73. A train weighs 300 tons. Starting from rest with uniform acceleration, it reaches a speed of 30 m.p.h. in 1 minute when travelling up an incline of 1 in 300. The resistance to motion due to friction is 10 lb. per ton. Find the total work done in ft.-lb. in getting up speed, and the tractive horse-power required when the speed is steady at 30 m.p.h. (I.E.E.)

74. It has been stated in *The Times* that a car travelling at 30 miles per hour can be stopped in a distance of 30 ft. Find what retarding force per ton must be exerted in such a case. Also find what distance would be required, with this force, if the initial speed were 45 miles per hour. (I.Mech.E.)

75. Find the speed which a cyclist can maintain on an uphill gradient of 1 in 25, if the weight of the cyclist and bicycle is 200 lb., and frictional

resistances amount to 3 lb.; assuming that he can exert one-tenth of a horse-power. (I.Mech.E.)

76. A train travelling at 30 miles per hour is approaching a station which stands at the top of a long slope rising 1 in 160. Find how far from the station steam should be shut off so that the train may come to rest at the station, frictional resistances being 10 lb. per ton. (I.Mech.E.)

77. An electric tram-car, weighing 12 tons, is being supplied with a current of 25 amp. at a voltage of 500. If the efficiency of the motor and gearing be 68 per cent and the car is maintained at a uniform speed of 12 miles per hour on the level, determine the tractive resistance in pounds per ton weight of the car.

What would be the maximum gradient the car could ascend at the same speed if the maximum current which can be supplied to the motor is 65 amp. and the frictional resistances be assumed unchanged? (U.E.I.)

78. A car for a scenic railway, when occupied, weighs 10 cwt. During its run it ascends an incline 90 ft. long, which makes an angle of 15° with the horizontal. If the speed of the car at the commencement of the incline is 60 ft. per sec. and the rail friction amounts to $1\frac{1}{2}$ lb. per cwt., determine:

(*a*) the kinetic energy of the car at the commencement of the incline;
(*b*) the deceleration or retardation of the car on the incline;
(*c*) the speed when the top of the incline is reached. (W.S.)

79. An electric tram-car, weighing 10 tons, is travelling at a steady speed of 10 miles per hour up an incline of 1 in 50. The tractive resistances are equivalent to 12 lb. per ton weight of the tram-car. The mechanical efficiency of the gearing is 85 per cent. and that of the motor 90 per cent. Find the horse-power supplied to the motor. If the voltage of the supply is 450, what current, in amperes, will the motor be taking? (U.E.I.)

80. Calculate the useful horse-power and the efficiency of a motor taking a current of 40 amp. at 220 volts when developing a torque of 52 lb.-ft. at a speed of 1000 rev. per minute. (U.E.I.)

81. (*a*) The average turning moment on a petrol engine crank shaft when running at 1000 rev. per minute is 990 lb. inches. Find the horse-power transmitted by the shaft.

(*b*) A body weighing 60 lb. rests on a horizontal plane. A constant force of 20 lb. inclined at 30° to the horizontal is applied to the body and moves it a distance of 50 ft. along the plane.

Determine:

(1) the amount of work done by the force;
(2) the normal reaction between the body and the plane. (U.E.I.)

82. A rigid framework consisting of bars OA, OB, and OC rotates about a vertical axis through O at 120 rev. per minute. OA, OB,

and OC are 2, $1\frac{1}{2}$, and 3 ft. long respectively and are at right angles to the axis of the shaft. Masses of 4, 6, and 2 lb. are attached to the bars at A, B, and C respectively. Neglecting the weight of the bars, determine the kinetic energy of the system

If the three masses be concentrated at a radius R feet, find the value of R if it be so chosen that the kinetic energy at the same speed as before remains unchanged. What name is given to this radius R ? (U.E.I.)

83. A mass weighing 1000 lb. is moved from rest on a level plane and the magnitude of the force (F) acting upon it varies at different distances (S) from the commencement of the motion as follows :

F lb. - -	200	188	177	155	135	120	122	145	157	148	130
S lb. - -	0	2	4	6	8	10	12	14	16	18	20

Assuming the frictional resistances to motion to be constant and equal to 70 lb., determine the average effective force acting during the 20 ft., and the amount of kinetic energy in the mass after it has moved through the 20 ft. (U.L.C.I.)

84. A flywheel weighing 5 tons has a radius of gyration of 4 ft. Calculate the amount of energy stored in the flywheel when rotating at 120 rev. per minut

The wheel is secured to a horizontal shaft 6 in. in diameter. The driving force is removed when the shaft is rotating at 120 rev. per minute and the energy of the flywheel is absorbed by the friction of the shaft bearings. If the coefficient of friction at the bearing surface is 0·05 at all speeds, calculate the number of revolutions the flywheel will make before coming to rest. (U.E.I.)

85. Calculate the kinetic energy of a flywheel of 6 ft. diameter, weighing half a ton and making 300 rev. per minute, assuming its mass is concentrated on the rim.

If it is subjected to a retarding force of 100 lb. applied tangentially to its rim, how many revolutions would it make before coming to rest ? (I.Mech.E.)

86. The flywheel of an engine driving a rolling mill weighs 50 tons and has a mean radius of gyration of 7 ft. 6 in. It runs at a normal speed of 200 rev. per minute.

Calculate :

(*a*) the time required to attain the speed of 200 r.p.m. when the flywheel is subjected to a uniform torque of 600 lb.-ft.

(*b*) the K.E. in the flywheel when running at 200 r.p.m. ;

(*c*) the percentage reduction in speed from 200 r.p.m. when the flywheel has to supply 10 per cent. of its energy during a sudden demand period. (W.S.)

87. A flywheel, used for experimental purposes, is 16 in. diameter. It is braked by a wooden block pressed against the outside of the rim with a force of 15 lb. and thereby reducing its speed from 300 to 180 rev. per minute. During this speed change 170 ft.-lb. of energy are given up by the wheel.

If the coefficient of friction between the wood and flywheel surface is 0·14, find how many revolutions the wheel makes while slowing down. What amount of energy will be stored in the flywheel at 180 rev. per minute? (U.E.I.)

88. A flywheel weighing 8 tons and having a mean rim radius of 4·5 ft. runs at a speed of 120 r.p.m. How many ft.-lb. of energy are stored in the wheel? The wheel is fitted to a shaft which revolves in bearings 6 in. in diameter. Assuming the driving power suddenly cut off and the energy of the wheel absorbed by the friction of the shaft bearings, find how many revolutions per minute the flywheel would make before coming to rest. Coefficient of bearing friction. $=0{\cdot}07$. (U.L.C.I.)

89. An experimental flywheel is carried on a horizontal spindle mounted in ball bearings. A cord is coiled round the spindle and a weight attached to the free end of the cord. During the descent of the weight the wheel is rotated and on the weight reaching the ground the cord is detached from the spindle.

In an experiment the following observations were made:

Effective circumference of spindle	5 in.
Distance weight is allowed to fall	100 in.
Time of fall	8 sec.
Weight on end of cord required to overcome friction of apparatus	2 lb.
Total weight on end of cord	34 lb.

Assuming the friction remains constant, find, at the instant the weight touches the ground:

(*a*) the revolutions per second made by the wheel;
(*b*) the velocity of the weight in feet per second;
(*c*) the kinetic energy stored in the effective weight, *i.e.* in the 32 lb. weight;
(*d*) the kinetic energy stored in the flywheel.

If the weight of the flywheel is 60 lb., what is its radius of gyration? (U.E.I.)

90. A wheel weighs 160 lb. and its running diameter and radius of gyration are 3 ft. and 15 in. respectively. Determine its kinetic energy when it is travelling at 40 miles per hour. (I.E.E.)

NOTE.—The kinetic energy is the kinetic energy of translation plus the kinetic energy of rotation.

91. Energy is transmitted from an engine shaft by a belt passing over a pulley 30 in. in diameter. The speed of the engine is 120 r.p.m. and the tensions in the tight and slack sides of the belt are respectively 1400

lb. and 750 lb. Determine the effective torque and the horse-power transmitted by the shaft. (U.L.C.I.)

92. A belt is required to transmit 12 horse-power. It passes over a pulley 2 ft. in diameter which is rotating at 250 rev. per minute. Find the effective pull in the belt. If the pull in the tight side of the belt is 2·1 times the pull in the slack side, find the maximum pull in the belt. If one square inch of belt section will safely withstand a pull of 300 lb. what sectional area of belt will be required? (U.E.I.)

93. A machine tool spindle is fitted with a pulley 8 in. in diameter and is belt driven from a countershaft running at 180 rev. per minute. If the pulley on the countershaft is 22 in. in diameter, find the speed of the machine tool spindle assuming that there is no belt slip.

If 4·8 horse-power is required to drive the machine tool spindle and the pull in the driving side of the belt is 2·15 times that in the driven side, what is the pull in the driving side of the belt? (U.E.I.)

94. A pulley 8 ft. in diameter transmits power by means of ten cotton ropes, the effective driving force of each rope being 160 lb. Determine the horse-power transmitted when the pulley is running at a speed of 220 r.p.m. (U.L.C.I.)

95. Define: Work, Power.

The shafting in a textile mill has an 8 ft. 0 in. diameter driving pulley, which is driven by eight $1\frac{1}{4}$ in. diameter cotton ropes, each of which is limited to a pull of 250 lb. per sq. in. If the tension on the tight side of each rope is three times the tension on the slack side, determine for a speed of 220 r.p.m.:

(a) the linear speed of the ropes;
(b) the tension in the tight and slack sides of each rope;
(c) the horse-power transmitted. (W.S.)

96. An electric motor driving a hoisting winch runs at 600 r.p.m. and is supplied with 8000 watts; it converts 88 per cent of this into useful work at the gearing. The gearing has an efficiency of 85 per cent. and a reduction ratio of 20 to 1; it drives a drum of 2 ft. diameter.

Determine:

(a) the torque on the drum;
(b) the speed of lifting of the load;
(c) the load that can be lifted at the drum;
(d) the useful H.P. of the winch. (746 watts = 1 H.P.) (W.S.)

97. The brake horse-power of an engine is 12 at a speed of 240 r.p.m. It is found that half a minute is required after starting to attain full speed. Estimate the moment of inertia of the flywheel in lb.-ft. units. (I.E.E.)

98. An engine develops 10 H.P. at 200 r.p.m. It is fitted with two flywheels 4 ft. 6 in. diameter, each weighing 4 cwt. Assume that the

useful work done by the engine per revolution is constant. Estimate the time in which the engine attains full speed after starting from rest. (I.E.E.)

Velocity, acceleration, force, momentum and energy.

99. A ball is thrown vertically upward with an initial velocity of 64 ft. per sec.; and, 1 sec. later, a second ball is thrown up with such a velocity that it strikes the first ball just as the first ball reaches its highest point. Find the initial velocity of the second ball, and the velocity with which it strikes the first ball. (I.Mech.E.)

100. A ball falls vertically from a height of 40 ft. and rebounds with a velocity which is 0·8 times that at which it strikes the ground. Calculate the interval between the dropping of the ball and the instant when it strikes the ground the second time. (I.E.E.)

101. A train is observed to travel $\frac{1}{4}$ mile in 40 sec, and the next $\frac{1}{4}$ mile in 30 sec. Find the number of seconds it would take to travel each of the next two quarter-miles, assuming that its acceleration is uniform. (Omit fractions of a second in your answers.) (I.Mech.E.)

102. Two ships, A and B, are at a given moment 10 miles apart, A being due west of B. A is steaming north-east at 15 miles per hour, and B is steaming in a direction 30 degrees north of west at 10 miles per hour. Show in a diagram on the scale of a cm. (or half inch) to a mile the path of B relative to A, and the actual positions of the ships (1) when they are nearest to each other; and (2) when A is due north of B. (I.Mech.E.)

HINT.—Fix A and assume B travels along the path of the relative velocity of B to A. Then the ships will be nearest when B reaches the foot of the perpendicular from A on to the line of relative velocity, and when moving at that relative velocity.

103. A steamer travelling due east at 15 miles per hour is to be intercepted by a despatch boat which starts from a point 5 miles due south of the steamer, and travels at 20 miles per hour. Show in a diagram the track which should be taken by the despatch boat if it is to reach the steamer at the earliest moment; and estimate from your diagram the approximate number of minutes it will take. (I.Mech.E.)

104. A ship, the speed of which is 10 knots in still water, is steered to have a true course in a north-westerly direction. There is a current at 3 knots towards the east. In what direction must the ship be steered and what is its true speed? (I.E.E.)

105. Steam issues from a nozzle with a velocity of 1400 ft. per sec. in a direction which makes an angle of 20 degrees with the direction of motion of the blades of an impulse turbine. The blade velocity is 500 ft. per sec. Find the blade angle to allow the steam to enter without shock. (I.E.E.)

106. An electric train starting from rest at one station comes to rest at the next station 7200 ft. away in 3·5 minutes, having first a uniform acceleration, then a uniform speed for 2·5 minutes, then a uniform retardation. The time of acceleration is twice the time of retardation. Find :

(*a*) the uniform speed, in miles per hour, at which the train runs for 2·5 minutes ;
(*b*) the uniform acceleration in feet per second^2 ;
(*c*) the uniform retardation in feet per second^2 ;
(*d*) the space passed over, in feet, during the acceleration period ;
(*e*) the space passed over, in feet, during the time of retardation. (U.E.I.)

107. A car with wheels 30 in. in diameter moving at 15 miles per hour is brought to rest under a uniform retardation in a distance of 200 ft. Find :

(*a*) the retardation of the car ;
(*b*) the initial angular velocity and the angular retardation of the wheels.

State clearly the units in which each answer is expressed. (U.E.I.)

108. A tram-car, moving at 15 miles per hour, is brought to rest by the application of the brakes in a distance of 20 yd. on a level track. The diameter of the wheels is 2 ft. Find :

(*a*) the average retardation of the car in feet and second units ;
(*b*) the time taken to come to rest from time of application of brakes ;
(*c*) the angular retardation of the wheels in radians per second per second. (U.E.I.)

109. Explain how the momentum of a body, moving in a straight line, is calculated and state the units in which momentum is usually expressed.

A machine-casting and the table of a planing machine to which it is fixed together weigh 1120 lb. Calculate :

(*a*) the momentum when the table is moving at 2 ft. per sec. ;
(*b*) the change in momentum when the velocity of the table changes from 2 to 0·5 ft. per sec. due to belt slip as the tool starts to cut ;
(*c*) the change in momentum between the cutting stroke at 2 ft. per sec. and the return stroke at 4 ft. per sec. (U.E.I.)

110. If an electric train gets up a speed of 30 miles per hour in half a minute from rest, find the acceleration, assuming it to be constant ; and find the distance travelled in the interval. (I.Mech.E.)

111. A tram-car at a given instant has a speed of 12 miles per hour. It is given a uniform retardation of 2·5 ft. per sec. per sec. until its speed is reduced to 7 miles per hour. Determine the time taken in reducing the speed and the distance in feet the car has travelled during retardation. (U.L.C.I.)

112. A motor-car weighing with passengers 22 cwt. is accelerated from rest to a speed of 50 m.p.h. in 20 sec. Assuming the acceleration to be uniform, find its magnitude and also the distance the car moves during acceleration. (U.L.C.I.)

113. A truck, starting from rest, runs 1 mile down to the bottom of a slope of 1 in 110 (*i.e.* $\sin\theta = \frac{1}{110}$). Neglecting friction, find the velocity at the foot of the slope, and the time taken. (I.Mech.E.)

114. A motor-car, weighing 30 cwt., is running at a uniform speed of 35 miles per hour. The brakes are applied and they produce a retardation of 6·8 ft. per sec. per sec.

Calculate :

(*a*) the time required to reduce the speed to 10 m.p.h. ;
(*b*) the magnitude of the retarding force ;
(*c*) the distance the car will travel during the decelerating period. (W.S.)

115. An electric passenger lift for a multi-storey building weighs, with its full load, 15 cwt., and has a maximum speed of movement of 320 ft. per minute. When starting upwards from rest it acquires this speed in 3·2 seconds.

Determine :

(*a*) the uniform acceleration in ft. per sec. per sec.
(*b*) the height to which the lift rises during the accelerating period ;
(*c*) the time to reach the top flat at 120 ft. from the basement, if retardation to stop is at 2 ft. per sec. per sec.
(*e*) the maximum H.P. rating of the hoisting motor, neglecting the friction of the guides. (W.S.)

116. Explain the statement : " Impulse = Change of Momentum ". A hammer head weighing 2 lb. is moving with a velocity of 25 ft. per sec. when it strikes the top of a chisel. What is the average force of the blow if the hammer is brought to rest in $\frac{1}{200}$ sec. after striking the chisel ? (U.L.C.I.)

117. A bowl was suspended from a spring balance. Water is falling from a tap in a steady stream straight into the bowl. The sectional area of the stream is 1 sq. in. and the maximum velocity attained by the stream is 10 ft. per sec. Find the weight of water reaching the bowl per second and its momentum, given 1 cu. ft. of water weighs 62·3 lb. Does the spring balance record the correct weight of water in the bowl at any instant ? If not, what is the amount of the error in pounds weight ? (U.E.I.)

118. Describe any experiment you have performed in connection with Atwood's Machine or Fletcher's Trolley. You should include in your answer the object and method of conducting the experiment, the observations made, and show clearly the method of obtaining the required result. (U.E.I.)

119. A lift cage weighing 4 tons is partly counterbalanced by weights of 2 tons connected to the cage by wire ropes which pass over overhead pulleys. What force must be applied to the cage on the upward journey to give an acceleration of 12 ft. per sec. per sec. ? (I.E.E.)

120. A mass weighing 64 lb., on a horizontal table 4 ft. from the edge, is attached by a string to a 5 lb. weight which hangs over the edge. The coefficient of friction between the 64 lb. and the table is $\frac{1}{16}$. Find the acceleration of the system and the time required for the 64 lb. to fall over the edge. (I.Mech.E.)

121. Three weights A, B and C are attached to a disc which revolves about an axis passing through its centre O and at right angles to the plane of the disc. $A = 4$ lb., $B = 6$ lb., $C = 7$ lb.; $AO = 10$ in.; $BO = 9$ in.; $CO = 9$ in.; the angles measured clockwise from A are AOB $= 90°$, AOC $= 210°$. Find the unbalanced force acting when the disc rotates 120 times a minute. (U.L.C.I.)

NOTE.—Since all the weights are revolving at the same speed the necessary centripetal forces are proportional to the products of weight times radius. Obtain the unbalanced force by using the polygon of forces.

122. Two masses A and B, 8 lb. and 12 lb. in weight respectively are attached to a plate which revolves about its centre O. The distance of both A and B from the centre of motion O are 2 ft. and the angle AOB is 120°. The centrifugal forces set up by A and B are to be balanced by attaching a third mass C to the plate at a distance of 2 ft. 6 in. from O. Determine the weight and position of the mass C, assuming that the plate to which weights are attached is in perfect balance. (U.L.C.I.)

123. The radius of curvature of the bend in a road may be taken as 50 ft. Determine the angle at which the road must be banked to eliminate all tendency to side slip for a vehicle travelling round the bend at 30 miles per hour. (I.E.E.)

NOTE.—The resultant of the normal reaction of the banking and the force of gravitation supplies the horizontal, inward, or centripetal, force necessary for circular motion.

124. A weight of 1000 lb. is accelerated from rest by a constant force of 100 lb. Calculate the velocity of the body, the distance travelled, the work done by the force and the kinetic energy of the body, each after 5 sec. (I.E.E.)

125. A car weighing 1200 lb. is moving at 15 miles per hour. What is its kinetic energy and what is its momentum ?

If the speed of the car is reduced to 3 miles per hour in 5 sec., what is the average force acting on the car during these 5 sec. ? (U.E.I.)

126. A body, weighing 480 lb., is moving horizontally with a speed of 20 ft. per sec. What is the momentum and what is the kinetic

energy of the body ? What constant force will bring the body to rest (*a*) in a distance of 30 ft.; (*b*) in 40 sec. ? How far will the body travel in the 40 sec. ? (U.E.I.)

127. Define kinetic energy, momentum, work, and impulse, explaining what connexions there are between them.

A train is travelling at 45 miles per hour, when the steam is shut off and brakes applied which exert a force of 140 lb. per ton on the whole train. Find how far the train will travel, and for how many seconds. (I.Mech.E.)

128. Define momentum, kinetic energy.

During shunting operations a railway wagon weighing 18 tons and travelling with a velocity of 12 m.p.h., collides with another wagon weighing 10 tons moving in the same direction at 7 m.p.h. If immediately after impact the front wagon moves on with a velocity of 11 m.p.h. determine the velocity of the other wagon.

If the resistance to motion of the front wagon is 25 lb. per ton weight, estimate how far it will travel after the impact before it comes to rest. (W.S.)

129. A hammer weighing 5 tons and moving at a speed of 16 ft. per sec. strikes a stationary body of weight 15 tons. Find the resulting velocity after the blow when the two bodies move together. Also find the amount of kinetic energy lost by the impact. Assume perfect freedom of motion and resistances negligible. (U.L.C.I.)

130. A rivetter, during a hand-caulking operation, uses a hand hammer of weight 2 lb. The face or edge of the caulking tool is 1 in. long and 0·03 in. broad.

The hammer strikes the tool with a velocity of 20 ft. per sec. What will be the pressure in lb. per sq. in. on the tool face if it is brought to rest in $\frac{1}{200}$ sec. ?

If the tool weighs $\frac{3}{4}$ lb., what velocity is imparted to it at the instant of the blow, and what is the loss in energy of the hammer ? (W.S.)

131. A flywheel is rotating at a uniform speed of 150 rev. per minute and has stored 45,000 lb.-ft. of energy. Calculate its moment of inertia.

On a decrease in load on the shaft the speed increases 5 rev. per minute in 8 sec. Determine :

(*a*) the change in kinetic energy ;
(*b*) the angular acceleration. (W.S.)

Strength of materials, moments, bending moment and shearing force.

132. Describe how you would determine the modulus of direct elasticity (Young's modulus) of a given wire. Sketch neatly the apparatus you would use and using symbols to denote the observed quantities, explain how the modulus of direct elasticity of the material would be calculated. (U.E.I.)

133. A mild steel bar, of cross-sectional area 0·5 sq. in. was tested and from a load extension graph obtained it was found that a load of 3 tons produced an extension of 0·0036 in. on a gauge length of 8 in.

What would be the intensity of stress in a tie rod of the same material 8 ft. 4 in. long when its elongation was 0·03 in. (U.E.I.)

134. The diameter of the piston of a steam-engine is 15 in., and of the piston rod 2 in.; the crank is 1 ft. long, connecting rod $2\frac{1}{4}$ in. diameter and 5 ft. long. If when the crank is at right angles to the line of stroke the steam pressure round the piston rod is 100 lb. per sq. in., find how much a 4 ft. length of the connecting rod will stretch under load if the modulus of direct elasticity of the material is 13,500 tons per sq. in. (U.E.I.)

135. A rod 12 in. long is $1\frac{1}{2}$ in. diameter over a length of 10 in. and 1 in. diameter over the remaining 2 in. of its length. The smaller portion is subjected to a tensile stress of 9 tons per sq. in. What is the stress intensity in the larger portion? How much will an 8 in. length of the larger portion stretch under the load if the modulus of direct elasticity of the material is 13,500 tons per sq. in.? (U.E.I.)

136. A flat steel tie-bar of section $3 \times \frac{1}{2}$ in. and 14 ft. long when placed into position is found to be $\frac{1}{16}$ in. short. By means of drifts driven into holes in the ends of the bar it is sprung into place.

Determine:

(*a*) the stress in the member;

(*b*) the ultimate stress in the material if the factor of safety is 6;

(*c*) the area that the bar would require to have if it were desired to limit the stress to 4 tons per sq. in.

Take E for steel as 30×10 lb.-sq. in. (W.S.)

137. A wire rope consists of 114 wires each of 0·041 in. diameter with a tensile strength of 35 tons per sq. in. Calculate the strength of the rope, which may be taken as 88 per cent of the aggregate strength of the wires. (I.E.E.)

138. During a tensile test on a carbon steel bar the following observations were made:

Original diameter of test piece - - 0·82 in.
Distance between tests points - - 8 in.

Load, tons	1	3	5	7	9	11	12	12·5	13
Extension, $\frac{1}{1000}$ in.	0·6	2·7	5	7·3	9·5	11·68	13·95	50	60

Plot the load-extension diagram and on it mark the points of elastic limit and yield, and give their values

Also from the graph determine Young's modulus for the material, and, if the maximum load were 17·5 tons, determine the ultimate stress. (W.S.)

139. Define Yield point, Young's modulus.

A flat steel test piece, 1·5 in. wide and 0·4 in. thick, when under a tensile load is found to show a yield point at 9·5 tons and breaks under a maximum load of 16 tons. A length of 6 in. marked on the bar is found to have stretched to 7·3 in. after the fracture.

Determine (*a*) the yield stress, (*b*) the ultimate stress, (*c*) the percentage elongation on 6 in., and (*d*) the value of Young's modulus if the stretch on a 6 in. length at a load of 7 tons is 0·0058 in. (W.S.)

140. State, in tabular form, the quantities you would measure in an ordinary tensile test of a ductile metal, and the deductions you would make; and give approximate values for mild steel. (I.E.E.)

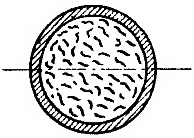


FIG. 395.

141. Define "Compressive Stress" and "Tensile Stress".

Fig. 395 shows the cross-section of a pile used in bridge construction. The outer shell is 12 in. o.d. and 1 in. thick, and is made of cast-iron; the inside is filled with concrete. The compressive load to be carried by the pile is 30 tons.

Find the stresses in the cast-iron and in the concrete.

Take E for cast-iron $= 12 \times 10^6$ lb. per sq. in. $= E_1$,
E for concrete $= 3 \times 10^6$ lb. per sq. in. $= E_c$. (W.S.)

NOTE.—Let the area of the cross-section of the cast-iron $= A_1$ sq. in. and the area of the cross-section of the concrete $= A_c$ sq. in.

Suppose W_1 tons and W_c tons be the loads carried by the iron and concrete respectively.

Then for equal fractional longitudinal strain

$$\frac{W_1}{A_1E_1} = \frac{W_c}{A_cE_c}. \qquad \text{.................................(1)}$$

Also, the total load in tons $= W_1 + W_c = 30$. ....(2)

The solution of these simultaneous equations (1) and (2) will give the values of W_1 and W_c.

142. A vertical concrete column of 15×15 in. square section is reinforced by four steel rods 1 in. in diameter. The load on the column is 50 tons. Calculate the stress in the steel.

The modulus of elasticity of steel should be taken as 12 times that of concrete. (I.E.E.)

143. A compound bar of steel $2 \times \frac{1}{4}$ in. section with brass of $2 \times \frac{3}{8}$ in. section carries a tensile load of 5 tons. How much load is carried by the steel? Give the stress in each metal. (I.E.E.)

[$E_B = 13 \times 10^6$ lb. per sq. in. $E_S = 30 \times 10^6$ lb. per sq. in.]

144. A spur wheel is required to transmit 50 horse-power at 100 rev. per minute. What is the torque transmitted?

The wheel is keyed to a shaft 3 in. diameter by a key of width 0·875 in. and length 5 in. Find the average shear-stress in the key. (U.E.I.)

145. Two lengths of shafting are connected by a coupling which has four $\frac{1}{2}$-in. diameter bolts on a pitch circle 6-in. diameter, that is, the bolt centres are 3 in. from the centre-line of the shafting. Assuming that the bolts are in single shear, and that the working stress is 2 tons per sq. in., calculate the horse-power which can be transmitted at 240 rev. per minute. (I.E.E.)

146. A shaft coupling has four bolts $\frac{5}{8}$ in. diameter set on a pitch circle 6 in. diameter. Find the shearing stress on the bolts when the shaft transmits 50 horse-power at 200 r.p.m. (I.E.E.)

147. Two rods are to be connected by a pin joint, *i.e.* an eye is forged upon the end of one rod, this fits into a fork on the end of the other rod, and a pin passes through the fork and eye. The joint is subjected to a pull of 5 tons applied along the centre line of the rods. If the allowable tensile and shearing stresses of the material used be 4 and 3·2 tons per sq. in. respectively, find the required diameters of the rod and pin. (U.E.I.)

148. Two steel bars each of $2 \times \frac{3}{8}$ in. section, are to be connected in line by means of a lap joint and a single rivet. Take the shearing stress for the rivet as $\frac{3}{4}$ the tensile stress for the plate and calculate the rivet diameter for the strongest joint. (I.E.E.)

149. A beam of wood rests on supports 10 ft. apart. At a point 3 ft. from the left-hand support pulley blocks are suspended in order to raise a machine weighing 2000 lb. The effort, applied vertically downwards, to raise the machine, is 100 lb. Calculate the bending moment and shearing force on the beam at a section 4 ft. from the left-hand support and draw to scale the curves of bending moment and shearing force. (U.E.I.)

150. How does the deflection of a supported beam, centrally loaded, of rectangular cross-section depend upon its dimensions? A timber beam, 0·5 in. broad, 1 in. deep, rests on supports 2 ft. apart. It is found that a central load of 56 lb. produces a deflection of 0·2 in. What would be the probable deflection of a joist of the same timber 4 in. broad, 6 in. deep, resting on supports 8 ft. apart under a centrally applied load of 3600 lb.? (U.E.I.)

151. A shaft 12 ft. long is supported in two bearings 10 ft. apart, the shaft overhanging the left-hand bearing A, $1\frac{1}{2}$ ft. The weight of the shaft, which may be looked upon as a load at its centre, is 300 lb. The shaft carries three pulleys B, C, and D transmitting power to machines below causing three vertical downward loads (including the weights of the pulleys) of 600, 1000 and 800 lb. respectively. B is 1 ft. to the left and C $2\frac{1}{2}$ ft. to the right of bearing A and D is 1 ft. to the left of the second bearing. Calculate the vertical reactions at each bearing. (U.E.I.)

152. A girder, 50 ft. span, supports three side wheels of a six-wheel motor-lorry. The fore wheel is 10 ft. from the left-hand support and

the distances between the axles, in order, are 12 and 14 ft., and the loads transmitted to the girder are, in order, $4\frac{1}{2}$, $3\frac{1}{2}$, and 4 tons respectively. Calculate:

(*a*) the reactions at the supports;
(*b*) the bending moment at a section 12 ft. from the left-hand support;
(*c*) the shearing force at a section 12 ft. from the left-hand support. (U.E.I.)

153. An electric motor weighing 4 tons is carried by two parallel beams which rest on supports 12 ft. apart. The centre line of the motor is at a distance of 8 ft. from the left-hand support. Assuming the load to be a concentrated load, draw curves of bending moment and shearing force for one of the loaded beams, each beam carrying the same load. Each beam is of rectangular section, 4 in. wide and 6 in. deep. Calculate the maximum bending stress in each beam due to the weight of the motor. (U.E.I.)

154. It is required to find how the deflection of a supported centrally loaded beam varies with the load when the distance between the supports remains constant. Describe carefully the method of conducting the experiment giving a sketch of the apparatus used and particulars of the observations to be made. State the relationship you would expect to obtain.

A beam $\frac{1}{2}$ in. broad and 1 in. deep, placed on supports 24 in. apart, deflects 0·2 in. under a central load of $\frac{1}{2}$ cwt. What probable deflection would be produced by a central load of 100 lb.? (U.E.I.)

155. A shaft, 3 in. in diameter, is supported in bearings 10 ft. apart. Vertical downwards loads of 500, 700, and 600 lb. are applied at points 3, 5, and 8 ft. respectively from the left-hand bearing. Calculate the reactions at each bearing. (The weight of the shaft is included in the 700 lb. load.) If the shaft makes 120 rev. per minute and the coefficient of friction between the surface of the shaft and the bearing is 0·025 find the horse-power wasted in friction. (U.E.I.)

156. A rod, 40 in. long, is supported at its ends and carries loads of 10 and 8 lb. at distances of 12 and 30 in. respectively from the left-hand support.

(*a*) Calculate the reactions at the supports and the bending moment at each point of load.
(*b*) Draw the bending moment diagram making use of the values obtained in (*a*).

(Neglect the weight of the rod.) (U.E.I.)

157. A beam is 20 ft. long and is supported at its ends. It carries loads of 200, 100, and 300 lb. at 5, 8 and 12 ft. respectively from the left-hand end. Find the magnitude of the maximum bending moment on the beam and state where this occurs. (I.E.E.)

158. A bar 6 ft. long is inclined at 45 degrees and carries vertical loads of 1 and 2 cwt. at 2 and 4 ft. respectively from the lower end. The bar is supported by forces at its ends, that at the upper end being horizontal. Find the magnitudes of the supporting forces. (I.E.E.)

159. A steel bar of 2×2 in. square section is placed horizontally on two supports 4 ft. apart. What central load will produce a stress of 10 tons per sq. in. in the steel? (I.E.E.)

160. It is required to find how the angle of twist of a round rod varies with the torque applied when the length of the rod remains constant. Describe carefully the method of conducting the experiment, giving a sketch of the apparatus used and particulars of the observations to be made. State, in general form, the relationship you would expect to obtain. (U.E.I.)

Hydraulics.

161. A hydraulic press has a hand-operated pump unit. The ram is 10 in. diameter and the plunger 1 in. diameter. The centre line of the plunger is 2 in. from the fulcrum of the hand lever and an effort of 36 lb. is applied to this lever 30 in. from the fulcrum. Find the force on the ram, in tons, if the efficiency of the press is 85 per cent. (U.E.I.)

162. A 40,000 gal. cistern is being filled from a reservoir 100 ft. lower than the cistern by means of a pump. Find the amount of useful horse-power developed by the pump if the cistern is filled in $\frac{1}{2}$ hour. (A gallon of water weighs 10 lb.) (I.Mech.E.)

163. A tank of rectangular section, 8 ft. long and 5 ft. wide, has an inclined base giving 10 ft. depth at the back and 6 ft. depth at the front. Find the fluid thrusts on the back, *i.e.* the 10 ft. by 5 ft. side, front and the sloping base when the tank is filled with water.
(1 cu. ft. of water weighs 62·3 lb.) (U.E.I.)

164. A rectangular tank is 6 ft. wide, 8 ft. long, and 4 ft. deep. It has a vertical pipe 1 ft. diameter fixed in its upper side. Calculate the total fluid pressure on each side of the tank and on the base of the tank when the level of the water in the pipe is 16 ft. above the base of the tank.
(1 cu. ft. of water weighs 62·3 lb.) (U.E.I.)

165. A tank is of square section, base 4 ft. $\times$ 4 ft., and contains water to a depth of 8 ft. Calculate the total fluid pressure on each side of the tank and on the base of the tank. A vertical pipe leads downwards from the tank and is closed by a plug 1 in. in diameter. What is the pressure on the plug if it is placed 28 ft. below the base of the tank?
(1 cu. ft. of water weighs 62·3 lb.) (U.E.I.)

166. The centre of a circular door 3 ft. diameter in the side of a tank is 40 ft. below the surface of the sea. Determine the intensity of

pressure at this depth in lb. per sq. in. and the total pressure on the door.

(1 cu. ft. of sea-water weighs 64 lb.) (U.L.C.I.)

167. It is required to find the coefficient of discharge of a given orifice. Describe carefully the method of conducting the experiment. Using symbols to denote the observed quantities show how the numerical value of the coefficient would be obtained. (U.E.I.)

168. Water discharges from a sharp-edged circular orifice in the vertical side of a tank. The diameter of the orifice is 0·3 in. and the head of water above the centre of the orifice is kept constant at 4 ft. Determine the theoretical quantity of water discharged in cubic feet per minute. If the actual discharge is 0·28 cu. ft. per minute, what is the coefficient of discharge for the orifice ? (U.E.I.)

169. A sharp edged opening in a tank is 1 sq. in. in area and its centre is 5 ft. below the constant level of water in the tank. Water flows through the opening at the rate of 6 cu. ft. per minute. Find the coefficient of discharge for the opening. (U.E.I.)

170. Calculate the force exerted by a jet of water, 1 in. in diameter, when it strikes a flat surface, normally, with a velocity of 30 ft. per sec. Assume that the velocity of the jet, in direction of motion, is completely destroyed and that 1 cu. ft. of water weighs 62·3 lb. (U.E.I.)

171. Water is discharged through a nozzle of 1 in. diameter with a velocity of 50 ft. per sec. Calculate the reactive force of the jet. (I.E.E.)

172. A tank 10 ft. long, 8 ft. wide, and 6 ft. deep has to be filled with water weighing 62·3 lb. per cu. ft., from a sump 8 ft. in diameter. When the operation of filling the tank commences, the bottom of the tank is 40 ft. above the free surface of the water in the sump. Find the work done, in foot-tons, in filling the tank : no water entering the sump during the operation. (U.E.I.)

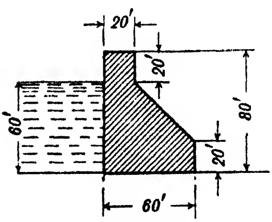


FIG. 396.

173. The water in a dock is 24 ft. deep. Find the total force, in tons, on one foot-length of the wall. What will be the overturning moment on the wall about its base due to this pressure ? (1 cu. ft. of sea-water weighs 64 lb.) (U.E.I.)

174. Fig. 396 shows a cross-section of the retaining wall of a dam. The wall weighs 250 tons per ft. run, and the depth of the water on one side is 60 ft.

Determine :

(a) the magnitude of the water thrust on the vertical face ;
(b) the magnitude of the resultant thrust on the base of the wall ;
(c) the point in the base of the wall where the resultant thrust acts. (W.S.)

MATHEMATICAL TABLES.

TABLE I. USEFUL CONVERSION CONSTANTS.

Linear. 1 inch = 25·4 millimetres, 1 mm. = 0·03937 in.
1 foot = 30·48 centimetres, 1 metre = 39·37 in.

Superficial. 1 sq. in. = 6·452 sq. cm., 1 sq. cm. = 0·1557 sq. in.

Volume. 1 cu. in. = 16·39 cu. cm., 1 cu. cm. = 0·06011 cu. in.

Capacity. 1 gallon = 0·1604 cu. ft. = 10 lb. of water at 62° F.

Weight. 1 pound avoirdupois = 453·6 grammes.

Density. 1 cu. ft. of water weighs 62·3 lb.
1 cu. ft. of air at 0° C. and 1 atmos. weighs 0·0807 lb.
1 cu. ft. of hydrogen at 0° C. and 1 atmos. weighs 0·00559 lb.

Pressure. 1 atmosphere = 14·7 lb. per sq. in. = 2116 lb. per sq. ft. = 760 mm. of mercury.
Gauge Pressure = Absolute Pressure − 14·7 lb. per sq. in.

Velocity. 1 knot = 6080 feet per hour.
60 miles per hour = 88 feet per second.

Power. 1 Horse Power = 33,000 ft. lb. per minute = 550 ft. lb. per second = 746 watts.
1 watt = 1 volt × 1 ampere.

Energy. 1 British Thermal Unit = 778 ft. lb.
1 Centigrade Heat Unit = 1400 ft. lb.

Angular. 1 radian = 57·3 degrees.

Logarithms. To convert common into Napierian Logarithms multiply by 2·3026. Base of Napierian Logarithm = e = 2·7183.

Acceleration due to earth's pull. Value of "g" in London = 32·182 ft. per sec. per sec.

TABLE II. LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE III. ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

TABLE III. ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE IV.

Angle.		Chords.	Sine.	Tangent.	Cotangent.	Cosine.			
Deg.	Radians.								
0°	0	0	0	0	∞	1	1·414	1·5708	**90°**
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3006	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	**85**
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·139	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	**80**
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·217	1·3090	**75**
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	**70**
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	**65**
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	**60**
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	**55**
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	**50**
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	**45**
								Radians.	Deg.
			Cosine.	Cotangent.	Tangent.	Sine.	Chords.	Angle.	

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ANSWERS TO EXERCISES

CHAPTER I (p. 11)

13. 300 lb.

14. 1000 lb., motion would occur in the direction of the greater force.

20. 13·5 lb. per inch of compression. 22. 5·92 lb. per inch.

23. 320 lb. per inch. 24. 0·422 in.

25. (*a*) 5·69 lb. per inch. (*b*) 4 lb. per inch. (*c*) 10·17 lb. per inch. *c, a, b*.

26. (*a*) 3·875 lb. (*b*) 4·4 in.

CHAPTER II (p. 24)

2. (*a*) 2·19 in. (*b*) 1·54 in. (*c*) 0·95 in. 4. 0·419 in.

CHAPTER III (p. 43)

2. $8\frac{3}{4}$ lb. 3. 21 lb. 4. $14\frac{2}{7}$ lb., 72 in., 12 in. 5. $415\frac{5}{9}$ lb.

6. 12·47 lb. 7. $56\frac{2}{3}$ lb. at each pin. 8. $249\frac{1}{3}$ lb. 9. 69·28 lb.

10. 8 in. 11. $21\frac{1}{4}$ lb. 12. 25 lb. per inch. 13. 86·6 lb.

14. 67·2 in. from the fulcrum. 15. $1666\frac{2}{3}$ lb. 16. $6\frac{1}{4}$ lb. ft.

17. 31 lb. in., $3\frac{7}{8}$ lb. 18. 1634 lb. 19. 3 tons. 20. 15,111 lb.

21. 5760 lb. in., 384 lb.

22. (*a*) $2\frac{2}{3}$ cwt., $5\frac{1}{3}$ cwt.; (*b*) 8·12 cwt., 16·38 cwt.

23. L.H. 80 lb. per in., R.H. 40 lb. per in.

24. $60\frac{1}{8}$ lb., 30·6 lb. per in. 25. $58\frac{1}{3}$ lb. 26. 13·44 lb. 27. 8 lb.

28. $65\frac{5}{8}$ ton. 29. 3906 and 3234 lb. 30. $2883\frac{1}{3}$ and $4376\frac{2}{3}$ lb.

31. 5·77 lb. 32. $23\frac{1}{3}$ and $36\frac{2}{3}$ lb.

CHAPTER IV (p. 66)

1. (*a*) 17,500 ft. lb., (*b*) 15,000,000 ft. lb., (*c*) 480,000 ft. lb.

2. (*a*) 557 lb. per sq. in., (*b*) 8750 ft. lb. 3. 770 ft. lb.

4. 700,000 ft. lb. 5. (*a*) 115,500 ft. lb., (*b*) 35,640 ft. lb., (*c*) 151,140 ft. lb.

6. 3·98 H.P. 7. 3·6 H.P. 8. 5·175 H.P.

9. (*a*) 17,880 ft. lb., (*b*) 406·4 ft. 10. (*a*) $83\frac{1}{3}$ ft. lb., (*b*) 1·39 gallons.

11. (*a*) 833,333 ft. lb., (*b*) 25·25 H.P. 12. 20·37 H.P. 13. 365·91 H.P.

14. 2·18 tons. **15.** (*a*) 123,900,000 ft. lb., (*b*) 12·51 hours.

16. *Av.* $47\frac{1}{11}$ H.P. **17.** 485 tons. **18.** 4480 ft. lb. **19.** 0·287 H.P.

20. (*a*) 6·58 ft. tons, (*b*) 9·537 H.P. **21.** (*a*) 5·25 ft. tons, (*b*) 2·85 H.P.

22. (*a*) 82,500 ft. tons, (*b*) 16·3 miles per hr. **23.** 41·8 H.P.

24. (*a*) 750 ft. tons, (*b*) 50·9, (*c*) 45·2 in. **25.** (*a*) 25·6 lb., (*b*) 0·23 H.P.

26. (*a*) 1299 ft. lb., (*b*) 7·87 sec., (*c*) 86·6 lb. **27.** 16·6 H.P.

28. 0·393 H.P. **29.** 2794 in. lb. **30.** (*a*) 16,600,000 ft. lb., (*b*) 335 H.P.

31. 37·72 in. tons. **32.** (*a*) I.H.P. = 19·13, (*b*) B.H.P. = 15·9.

33. I.H.P. = 94·9. **34.** I.H.P. = 20,040. **35.** I.H.P. = 1299.

36. I.H.P. = 59. **37.** B.H.P. = 7·61, I.H.P. = 9·39.

38. B.H.P. = 726·4, efficiency 76·6%. **39.** 10·58 in. diameter.

40. M.E.P. crank end 33·9 lb. per sq. in.
M.E.P. cylinder end 34·44 lb. per sq. in. } Total I.H.P. 90·75.

41. (*a*) 51·5 lb. per sq. in., (*b*) 563·1 ft. lb., (*c*) 2·150 H.P.

42. 510,000 and 1 million ft. lb. **43.** 5,376,000 ft. lb., 163 H.P.

44. 45 ft. lb. **45.** 0·2885 H.P.

CHAPTER V (p. 96)

1. Resultant 104 lb. **2.** Equilibrant 135 lb. at 214° 12′ to *A*.

3. 1699 lb., 581 lb. **4.** 164·5 lb. tie, 251·5 lb. strut.

5. E = 650 lb., moment = 2340 lb. ft.

6. 2 lb. at 26° 40′ anticlockwise from *A*. **7.** 338 lb.

8. AD = 300 lb., DC = 770 lb. **9.** AC = 2200 lb., BC = 3700 lb.

10. 1300 lb., 920 lb. **11.** 14,400 lb.

12. (*a*) AC = 640 lb., CB = 1023 lb.
(*b*) DB = 957 lb. strut, DC = 785·7 lb. tie, AD = 957 lb. strut.
(*c*) AD = 490 lb. strut, BD = 610 lb. strut, CD = 410 lb. tie.
(*d*) Struts, CB = 2460 lb., CD = 1400 lb., BD = 2910 lb.
Tie, AC = 1877 lb.
(*e*) Struts, CB = 860 lb., DB = 1300 lb.
Ties, AC = 340 lb., CD = 860 lb.
(*f*) Tension in rope, 663 lb.
Thrust in pole, 650 lb.
Thrust in strut, 260 lb.
(*g*) Strut, $AD = BF$ = 3200 lb.
Ties, $DC = CF$ = 2550 lb.
$DE = EF$ = 550 lb.
CE = 2300 lb.
(*h*) Struts, CD = 1070 lb., BE = 628 lb.
Ties, AE = 54 lb., ED = 880 lb.

13. (*a*) **Struts,** $DH = 39$ cwt., $GH = 6{\cdot}8$ cwt., $FE = 17$ cwt., $FD = 46{\cdot}3$ cwt.
Ties, $AE = 66$ cwt., $BG = 49{\cdot}5$ cwt., $CH = 44$ cwt., $GF = 4{\cdot}9$ cwt.
(*b*) **Struts,** $EC = CD = CF = 346$ lb.
Ties, $DE = EF = 346$ lb., $DA = FB = 173{\cdot}2$ lb.
(*c*) **Struts,** $JD = AF = 3470$ lb., $CH = BG = 2600$ lb., $FG = JH = 860$ lb.
Ties, $GH = 1000$ lb., $FE = JE = 2850$ lb.

14. $AC = 2960$ lb. tie, $BC = 2800$ lb. strut, moment $= 23{,}650$ lb. ft.

15. AC, 75 lb. per fork, strut; CB, $44\frac{1}{4}$ lb. per fork, tie; AB, $37\frac{1}{2}$ lb., strut; AD, 75 lb., strut.

16. Horizontal 45 lb., vertical 53·6 lb.

17. (*a*) $S = 9$ cwt., $R = 8$ cwt., Resultant 17 cwt., 4·73 ft. from R.
(*b*) $S = 12{\cdot}7$ cwt., $R = 7{\cdot}3$ cwt., Resultant 20 cwt., 8·85 ft. from R.

18. Reaction of wall $= 14{\cdot}5$ lb. Resultant force on the ground $= 67$ lb. at 77° to the horizontal through the foot of the ladder.

19. Equilibrant, a force of 2·646 lb. acting at an angle of 19° 6′ to the 2 lb. force at a perpendicular distance of 1·31 in. from the junction of the lines of action of the 5 and 2 lb. forces.

20. Rope inclined at 36° 52′ to the vertical with its lowest point 3 ft. below B. Tension $1\frac{1}{4}$ cwt.

21. 1610 lb., at 60° 15′ to the direction of the 800 lb. force.

22. Force at A, 5600 lb. at 28° to the horizontal. Force at C, 3790 lb.

23. $T = 4650$ lb. Thrust at the hinge $= 10{,}450$ lb.

24. Force $= 690$ lb., force in guy rope $= 474$ lb. Resultant force on pulley $=$ 1280 lb.

25. (*a*) Force is 36·1 lb. acting at an angle of 213° 42′ to the 90 lb. force and tangent to a circle of 54 ft. radius concentric with capstan.
(*b*) Resultant moment $= 1950$ lb. ft.

26. (*a*) $E = 141$ lb., $R = 169$ lb. (*b*) $E = 143$ lb., $R = 193$ lb. (*c*) $E = 185$ lb., $R = 287$ lb.

27. 3 and 5·196 tons. 28. 9·9 lb. at 221° 40′ with 5 lb. force.

29. 0·868 and 4·924 tons. 30. 4·62 lb. and 4 lb. 31. 7967 and 6044 lb.

CHAPTER VI (p. 120).

1. 191·7 lb. 6. 0·204 and 11° 32′. 7. 14° 2′.

8. Effect of friction 5 ft. lb., or 2·5 lb. on load, 88·9%.

9. Effect of friction 1·1 lb., 93·5%. 10. 23·8 lb. 11. 1·44 lb., 16° 42′.

12. 0·702 lb. 13. 0·9375 H.P. 14. 962 lb., 6·997 H.P.

15. 3·86 ton, 18·35 H.P. 16. 380·9 lb. 17. 6·14 H.P.

18. 10,920 lb. per foot.

19. 702·1 lb. at 108° clockwise from 200 lb. force, 0·1 H.P. 20. 3·44 H.P.

21. 2·58 H.P. 22. 4·58 H.P. 23. 0·26 H.P.

24. (*a*) 270·8 lb., (*b*) 0·009, (*c*) 0·26 H.P. 25. 0·2, 5·5 lb.
26. 24 ft., 24 ft. lb. 27. $13\frac{1}{2}$ lb. 28. 161·3 lb., 362·1 lb.

CHAPTER VII (p. 149)

2. V.R. = 9; M.A. = 7·5; effect of friction = 36 lb.; efficiency $83\frac{1}{3}$%.
3. 58·88 lb. 4. V.R. = $2\frac{1}{3}$ (*a*); V.R. = 14 (*b*). 5. V.R. = 200.
6. 11·80 lb. 7. 138·6 lb. 8. 111·6 lb.; M.A. = 3·1.
9. V.R. = 60; 0·635 H.P. 11. 60 lb. 12. 14,140 lb. 13. 4849 lb.
14. 583·1 lb. 15. $E = 82{\cdot}6$ lb.; $L = 56{\cdot}35$ lb.; 5·635 lb.; 84·59%.
16. $L = 13{\cdot}33E - 40$; efficiency = 23·53%.
17. $L = 1{\cdot}6E - 6$; effect of friction = 26·26 lb.; efficiency 74·09%.
18. 0·205 H.P. 19. 408·4 lb.; 54·8. 20. 30·38 in.
21. 141·7 lb. 22. 9·62 lb.; 19·86%.
24. 75, 85·2, 325, 2250 lb., 2556 lb., 9750 lb.
25. Velocity ratios ($2\frac{1}{2}$°, 370); ($7\frac{1}{2}$°, 67·6); ($12\frac{1}{2}$°, 38); ($17\frac{1}{2}$°, 25·5); ($22\frac{1}{2}$°, 19); ($27\frac{1}{2}$°, 15); ($32\frac{1}{2}$°, 12·5); forces 2030 lb. and 1140 lb. approx.
26. 240; 5 lb. 27. 270; 13·5; 162 lb. 28. 4·28 cwt.
29. V.R. = 14; 56 lb.; 160 lb.; 30%. 30. V.R. = 108; 34·6 lb.; 2618 lb.
31. 14·67 cwt. 32. 620·7 lb.; 1·8; 3·86; 46·7%. 33. 8·4; 6° 48′.
34. 2460 lb.; 8·2. 35. 17,144 lb. 36. 48. 37. $133\frac{1}{3}$ lb.
38. 27·85 lb.

CHAPTER VIII (p. 183)

1. 3·281 and 3·183. 2. 191 r.p.m. 3. 1·78 H.P.
4. 318·4 lb. and 795·8 lb. 5. 4·83 in.—allow a 5 in. belt.
6. 161·9 r.p.m. 7. $1\frac{3}{8}$ and 257·7 r.p.m. 8. 6 and 1320 r.p.m.
9. Driver : Follower :: 10 : 3, diameters 20 and 6 in.
10. 2195 r.p.m. and $11\frac{3}{7}$. 11. 905, 537, 329, 195 r.p.m.
14. 0·333, 1·047 in., 0·333 in., 0·386 in. 15. $B = 150$ teeth; $A = 20$, $B = 60$ in.
16. 120 r.p.m.; $A = 36$, $B = 180$ in. 17. 75 in.; 9·6 teeth per ft.
18. 0·21 in.; 36°. 19. 15, $\frac{1}{15}$, 40 r.p.m. 20. 16, $\frac{1}{16}$, 160 r.p.m.
24. 40 driving 100; drivers 40 and 20, followers 75 and 80; drivers 90 and 80, followers 40 and 60.
25. V.R.; S.P. 36, D.P. 108; efforts, S.P. 14·81 lb., D.P. 6·17 lb.
26. Direct 625, 333, 188, 100 r.p.m.; in gear 105, 56, 32, 17 r.p.m.
27. 1680 r.p.m., 3240 r.p.m.; 2·1, 4·05.
28. 27·96, say, 28 H.P. 29. 191·0 lb. 30. 0·354 H.P.; 64 r.p.m

31. 2269 lb. 32. 77·6 ft. per min., 5·375 amp.

34. 108 and 180 ; or 135 and 144. 35. 453 lb. ; 212·7 r.p.m.

36. $262\frac{1}{2}$ and $85\frac{5}{7}$ r.p.m. 37. 22·5 ; 30 lb. 38. 20 r.p.m.

CHAPTER IX (p. 201)

4. 4 ft. 10 in. from the base, 2 ft. from left-hand face.
5. (*a*) 4·76 ft. from A ; (*b*) 5·63 ft. from A.
6. 15·65 in. from the 14 cwt. load.
7. (*a*) $\bar{y}=2\cdot25$ in. ; (*b*) $\bar{y}=2\cdot36$ in. ; (*c*) $\bar{x}=1\cdot02$, $\bar{y}=2\cdot27$ in. ; (*d*) $\bar{y}=4\cdot5$ in.
8. 0·2 in. from the centre of the large plate on the common diameter.
9. 8·27 in. from A.
10. (*a*) $1\frac{3}{16}$ in. from L.H. end ; (*b*) 1·33 in. from base ; (*c*) 13·98 in. from cone apex.
11. 0·13 in. from the centre of the hexagon.
12. 0·956 in. from the minor axis. 13. 3·64 in. from the base.
14. 69·2 in. from the front axle. 16. 36° 52′.
17. 1 ton 6·37 cwt. and 44·72 in. from the base. 18. 8 ton.
19. 10 ft. 11·8 in. from first driving wheel.
21. 3·79 in. from small end. 22. 71 in. above rails.
23. 1765 lb., 15·38 lb., 34·9%. 24. At mid-pt. ; $1\frac{1}{2}$ ft. from each leg.
25. 19·16 ft. from left end ; 24·52 and 39·48 tons.

CHAPTER X (p. 225)

1. 6·11 tons per sq. in. 2. 21 tons. 3. 70 in.
4. 1·75 tons per sq. in. ; 0·00526 in. 5. 30×10^6 lb. per sq. in.
7. 0·356 in. 8. 40·82 and 5·83 tons per sq. in. 9. 1·37 in.
21. 30 lb. ; 18,400 lb. per sq. in. ; 0·00078. 22. 29,960,000 lb. per sq. in.
23. 11,330 lb. 24. 10·2 tons per sq. in. 25. $30\cdot14\times10^6$ lb. per sq. in.
26. 16,000 lb. per sq. in. ; 0·000625 ; 25,600,000 lb. per sq. in.
27. 4·53 and 2·74 tons per sq. in. 28. 6·47 in. and 2·73 in.

CHAPTER XI (p. 252)

3. (*a*) 6760, (*b*) 4160 lb. ft. 5. Max. S.F. 21 cwt., max. B.M. 137 cwt. ft.
6. Max. B.M. $47\frac{1}{2}$ cwt. ft. ; max. S F. 12 cwt. 7. 10,667 lb. per sq. in.
8. 210 lb. 9. $\bar{y}=3\cdot94$ in. 10. 29 in.^{3} units. 11. 7 tons 4 cwt.
12. Max. B.M. $16\frac{2}{3}$ tons ft. ; max. S.F. $3\frac{5}{8}$ tons. 16. 34·2 tons per sq. in.
17. $1\frac{1}{24}$ ton.

CHAPTER XII (p. 274)

2. 908 ft. 3. 117·7 ft.; 16·8 ft./sec. 4. 0·31 sec.; 64·5 ft./sec./sec.
5. 122·2 sec.; 6276 ft. 6. 50 ft./sec.; 5000 ft./sec./sec.
7. 30·02 ft./sec.; 0·932 sec. 8. 880 ft./sec. 9. 39,750 ft.; 49·7 sec.
10. 29,937 ft. 11. 24·8 sec. 12. 10 rad./sec.; 95·5 r.p.m. 13. 9018 lb.
14. 3·52 ft./sec./sec.; 166·2 lb. 15. 7·14 ft./sec.
16. 2·79 ft./sec./sec.; 266·8 lb.; 308 ft.
17. 2·255 and 1·692 ft./sec./sec.; 164·7 and 123·6 lb.; 26·5 ft./sec./sec.; 1935 lb.
18. 4905 lb. per sq. in. 19. 11·62 ft./sec.
20. 1·342 ft./sec./sec.; 32·8 sec. 21. 102·2 lb. 22. 27·2 lb. 23. 17·2 lb.
24. 248·4 tons. 26. $16\frac{2}{3}$ ft./sec./sec.; 7·76 lb. 27. 12·58 ft./sec.
28. 36° 52′ to the vertical at 25 m.p.h.
29. 27·98 knots, 30° 22′ E. of N.
30. 17·43 ft./sec. at 36° 36′ with the 20 ft./sec. vector. 31. 999·3 ft./sec./sec.
32. 109·2 lb. 33. 22·4 lb. ft. sec. units. 34. 11·7 lb.; 996·5 H.P.
35. 0·11 and 0·31 ft. per sec. per sec. 36. 4·29 ft. per sec. per sec., 4·2 ft.
37. 3·047 tons, 3 ft. per sec., 9 ft.

CHAPTER XIII (p. 294)

1. 2×10^7 ft. tons; 22,626 H.P. hours. 2. $37\frac{1}{2}$ ft. lb.
3. 50,510 ft. tons. 4. 2451 ft./sec. 5. 29 ft./sec.; 1461·6 ft. lb.
6. 101,000 ft. lb.; 0·376. 7. 50 ft. lb. in each case.
8. 3,309,000 ft. tons; 501·2 lb. 9. 324 ft.; 13·1 lb. 10. 2272 lb.
11. 19·66 ft./sec.; 6720 ft. lb. 12. 8·73 ft. lb.; 8·73 lb.
13. 58 ft./sec./sec.; 198·2 tons. 14. $2/11\frac{3}{4}$. 15. 14·33. 16. 41·9 amp.
17. 33,760 ft. lb.; 6·48; 11 amp. 18. 761·3 ft. tons; 19·03 tons; 1928 min.
19. 1665 ft. lb. 20. 16,250 lb. 21. 111·4 ft. lb.; 668·4 lb.
22. 33,820 ft. tons. 23. 8·41.
24. 2386 ft. lb.; 3·06 ft./sec.; 521·9 ft. lb.; 1864·1 ft. lb.; 4175 lb.
25. 1·43 sec. 26. 5·1 ft. 27. 1·996 sec. 28. 1·835 ft.
29. 13,468 ft. lb.; 224·5 ft. 30. 725,200 and 152,000 ft. lb. 31. 112.
32. 2,502,000 ft. lb., 170·7 r.p.m.

CHAPTER XIV (p. 333)

1. 261·9 lb. per sq. in. 2. 23·89 in., 4·16 H.P. hours.
3. 752 ft. tons, 17·01 H.P. 4. 7069 gal. per hr. 5. 326·2 lb.

6. 11·25 ton. 7. 6000 lb. 8. 1·07 in.
9. (*a*) 89·3%, (*b*) 260·4 ft. lb. 10. 10,030 lb. 11. $d=7\cdot63$ in.
12. 5·21 in. 13. (*a*) 4, (*b*) 1600 lb. per sq. in. 14. 5·46 in.
15. 1138 lb. per sq. in. 16. 44·17 lb. per inch compression.
17. 1827 lb. 18. 20 in. 19. 23·95 in. 20. 20·57 in.
21. Ratio of intensification $=\frac{21}{11}$, pressure $=763\cdot7$ lb. per sq. in.
22. 8·3 in., 0·6d. 23. 482·3 ft.; 482·3 ft. lb. 24. 16·26 H.P. 25. 5·12 in.
26. 173·4 H.P. 27. 85·5%. 28. 18·05 H.P., 15 kW., $7\frac{1}{2}$d. per hour.
29. 9·54 gals.; 0·12d.; No. 30. 136·9 lb. per sq. in.; 19,715 lb. per sq. in.
31. Each side 6728 lb.; each end 4486 lb.; bottom 8971 lb. 32. $1828\frac{4}{7}$ tons.
33. $41\frac{1}{7}$ tons. 34. 28·2 tons. 35. 223 ft.
36. 113·4 ft. per sec.; 20 ft.; 54 ft. 37. 31·1 ft. per sec.; 15 ft. lb.
38. 114·1 ft.; 80·1 ft. lb.; 217·6 H.P. 39. 488·2 lb. per sq. ft.
40. 29·44 and 1·84 ft. per sec.; 13·4 ft. 41. 0·46 cu. ft./sec.
44. 24,500 gal. per hr. 45. 1·36 in. 46. 92 lb.
47. 6263 lb., 14,092 lb. 48. 77·64%. 49. 865,280 lb., 1664 lb. per sq. ft.
50. 50·76 ft. per sec., 1?·25 lb. per sec., 690 ft. lb.
51. 0·599. 52. 0·762 lb., 0·85, 0·85 lb.

MISCELLANEOUS EXERCISES

SECTION A (p. 339)

1. L.H. 34·72 lb., R.H. 42·28 lb. 2. $1\frac{7}{13}$ lb.; 21·6 in. from A, 10 lb.
3. 22 lb., $1\frac{8}{11}$ units from 4 lb. force; equal and opposite to R.
4. 404 lb. ft. 5. 14·75 lb. in.; 7·37 in. from A. 6. 62 lb.; 98 lb.
7. $P=42\cdot43$ lb.; $R=46\cdot91$ lb. at 25° 14′ with OA.
8. Errors $A=-0\cdot1$ lb., $B=+0\cdot1$ lb
9. Rotation about C clockwise; $\frac{1}{3}$ lb. 10. 43·3 lb. in.; 15·31 lb.
11. L.H. 23·8 lb.; R.H. 26·2 lb.
12. 687·5 ft. per min.; 7·81 m.p.h. 13. 3300 ft lb. 14. 1200 ft. lb.
15. 122·7 lb.; 115 ft. tons. 16. 3920 ft. lb.; 98.
17. 107·6 lb. per sq. in.; 9128 ft. lb. 18. 2640 ft. lb.; 576 ft. lb.
19. 552 lb.; 38,647 ft. lb. 20. 33,600 ft. lb.; 2·036 H.P.
21. 13,200 ft. lb. 22. 516,000 ft. lb.; 3·13 H.P. 23. 224 ft. lb.; 6 ft.
24. 74·31 lb. at 16° 35′ with the 50 lb. force.

25. 3 lb. at 36° 52′ with 4 lb. force.

26. (*a*) No ; $C = 3{\cdot}567$ lb. at 7° 29′ W. of S.

27. 173 lb. forward ; 100 lb. to side.

28. *CB*, 1000 lb. ; *AC*, 1732 lb. ; $R = 500$ lb.

29. 8·75 lb. at 11° 29′ to *OY* on side of *OA*.

30. 64·63 lb. at 18° 21′ to vertical ; 20·36 lb., 61·33 lb.

31. 4 lb. ; 5 lb. ; $R = 6{\cdot}4$ lb. at 51° 20′ to horizontal.

32. *AB*, 73·21 lb. ; *AC*, 51·77 lb. **33.** 4 lb. ; 6·22 lb. ; 132° 51′.

34. 11·55 lb. ; 10·35 lb. **36.** 37·11 lb. horizontally ; 19·74 lb.

37. Pull, 11,300 lb. ; $R = 1960$ lb. **38.** 17·32 lb. ; 10 lb.

39. 181·2 lb. **40.** 50 lb. ; 43·3 lb. **42.** 4 lb.

43. 0·0833 ; 544·5 ft. lb. **44.** 600 lb. ; 1509 lb. per sq. in.

46. 21,504 ft. lb. ; 0·652 H.P. **47.** 1000 lb. **49.** $20\frac{5}{6}$ lb.

50. (*a*) $7\frac{1}{3}$ in. ; (*b*) 37·5 lb. **51.** $E = \frac{1}{8}W + \frac{3}{4}$.

52. 38·4 lb., 48 lb., 80 lb., 112 lb., 144 lb. **53.** 35·71%.

54. 7540 lb. **55.** 70 ; 3·06 lb. **56.** 176 ; $74\frac{2}{3}$; 42·44%.

57. 50 ; 0·48. **58.** $3\frac{1}{3}$ in. from 6 in. side and $2\frac{1}{2}$ in. from 8 in. side.

59. $3\frac{5}{7}$ in. from *AE*. **60.** 15 lb., 20 lb.

62. 25·2 in. from C.L. of small end ; 22 lb. **63.** 1·859 ft. from *E*.

64. 8·496 in. from end *A*. **65.** (*a*) 2 lb. ; (*b*) 9 lb.

66. $L = 400E$; $\frac{1}{80}$ in. ; $16\frac{1}{4}$ in.

67. 0·0078 in. ; 4166·7 lb. per sq. in., 0·00026 ; 16,030,000 lb. per sq. in.

68. 1·748 in. ; 0·0444 in. **69.** 2·316 tons per sq. in.

70. 800 lb. per sq. in. ; 0·0000533 ; 15,000,000 lb. per sq. in.

71. $6\frac{5}{8}$ in.

SECTION B (p. 350)

1. 1·306 tons ; 3·87 tons at 71° to horizontal. **2.** 24·05 lb.

3. 30·63 lb. ; 51·42 lb. at 54° 54′ to horizontel.

4. 110·8 lb. ; 84·74 lb. at 40° 49′ to the vertical.

5. 25·98 lb. ; 39·65 lb. at 70° 53′ with the trap door.

6. 54° 37′ ; *AC*, 58·65 lb. ; *BD*, 71·57 lb. **7.** 70 lb. ; $\frac{1}{3}$.

8. 118·2 lb. **9.** 9·49 ft. **10.** 2743 lb.

11. 568·7 lb. ; 940·7 lb. ft. anticlockwise.

12. R at $A = 146$ lb. ; R at $C = 90$ lb. at 35° to horizontal.

13. R at $A = 1\frac{1}{4}$ tons ; R at $B = 1\frac{3}{4}$ tons ; *P*, 2·02 tons, strut ; *Q*, 0·289 ton, tie.

14. (1) $4\frac{1}{2}$ tons, (2) 3 tons ; 124·4 and 83 lb. per sq. in.

15.

Ties	A	A and B
ab	2·6	same
bd	3·1	same
ae	1·6	6·1

Struts	A	A and B
cd	3	same
be	1·7	6·7
de	2·9	same
ef	4·03	7·4

Figures in tons

16. $X = 4·553$ tons ; $Y = 2·13$ tons.

17. At A, $16\frac{2}{3}$ tons downwards ; at B, $36\frac{2}{3}$ tons upwards ; $16\frac{2}{3}$ tons compression ; 7·8 tons compression.

18. Struts, *de* 3 tons, *ae* 2 tons, *ad* $\sqrt{2}$ tons. Reaction at d, 3 tons horizontally outwards.
Ties, *ec* $\sqrt{2}$ tons, *bc* 2 tons, *ab* $\sqrt{2}$ tons. Reaction at c, 3·162 tons inwards at 18° 26′ above horizontal.

19. Struts, *ab* 3·46 tons, *bc* 4 tons, *cd* 6 tons, *ce* 3·46 tons.
Ties, *be* 3·46 tons, *ae* 1·73 tons, *de* 5·2 tons.

20. 900 lb. ft. ; 3718 lb. at 28° 34′ with the 2000 lb. load.

21. 684·2 lb. ; 911 lb. at 89° 49′ to vertical.

22. $18\sqrt{2}$ lb. passing from mid-point of OA to that of AB.

23. $\bar{y} = 4·974$ in. 24. $\frac{5}{12}$ of side from each uncut side. 25. $3\frac{2}{3}$ in.

26. 5·81 in. 27. [illegible]·192 ft. from vertical face ; 9·245 ft. above base.

28. $1\frac{23}{48}$ ft. from large end. 29. 0·744 ton ; 3·956 tons.

30. 33° 42′ ; 4 lb. 31. $4\frac{1}{6}$ lb. 32. $41\frac{2}{3}$ lb.

33. 25 lb. in same plane and 1·2 ft. from the original force and parallel to it. On R.H. side for A.C.W. couple and on L.H. side for a C.W. couple.

34. 0·268 ; 2·68 lb. 35. 134·2 lb. 36. 11·5 lb. ; 13°.

37. 66·6 lb. 38. 20·78 lb. 39. 21·85 lb.

40. 7·987 ft. per sec. per sec.

41. 50 lb. ; 100 lb. 42. 0·2927.

43. $\mu = 0·0107$; 0·952 H.P. 44. 0·267 H.P. ; 11·32 B.Th.U.

45. 0·0153 ; 1·14 H.P. ; 48·46 B.Th.U. 46. 14·31 H.P.

48. 600 lb. 49. 26·36 lb. ; 63·8% ; 68·5% ; 69·%.

50. 40 ; 100 lb. 51. 300 lb. ; 158·7 lb. 52. $16\frac{2}{3}$ lb.

53. 3·13 lb. ; 38·3[illegible]%. 54. $E = 0·2225W + 16$; 87·2 lb., 3·67, 26·2%.

55. 704 ; 7·5 tons. 56. 85·51 ; 28·34%. 57. 754 ; 11,310 lb.

58. 100 lb. ; 12 ft. 59. 9 ; $44\frac{4}{9}$ lb. 60. $10\frac{1}{9}$ lb.

61. (*a*) 363 lb. ; (*b*) 12·34 ; (*c*) 135·7 ft. per min.

62. (*a*) 85 lb. ; (*b*) 58·82 ; (*c*) 36·76% ; (*d*) 44·64%.

63. $E = 0·0414W + 2·$[illegible] ; $\dfrac{53·68W}{W + 50·72}$ % ; 53·68%. 64. 6781 lb.

65. 6·615 H.P. 66. 126 lb. per sq. in. ; 2261 H.P.

67. 82·27 H.P. ; 61·37 kW. ; 245·5 amp.

68. (*a*) 5400 ft. lb. ; 1227 ft. lb., 4173 ft. lb. ; (*b*) 15·7 rad. per sec., 500 rev.

69. 14 H.P. 70. 2414 ft. 71. 832 H.P. ; 52 m.p.h.

72. 448 H.P. ; 893·3 H.P. 73. 27,126,000 ft. lb. ; 1644 H.P.

74. 2245 lb. ; $67\frac{1}{2}$ ft. 75. 3·41 m.p.h.

76. 2805 ft. 77. 29·68 lb. per ton ; 1 in 47·2.

78. (*a*) 62,610 ft. lb. ; (*b*) 8·762 ft. per sec.^{2} ; (*c*) 44·98 ft. per sec.

79. 19·8 H.P. ; 32·82 amp. 80. 9·9 H.P. ; 83·93%.

81. (*a*) 18·84 H.P. ; (*b*) 866 ft. lb. ; 50 lb. 82. 116·5 ft. lb. ; 1·99 ft.

83. 151·2 lb. ; 3024 ft. lb. 84. 196·2 ft. tons ; 499·6 rev.

85. 154,400 ft. lb. ; 81·93 rev. 86. 6828 sec. ; 434,700 ft. lb. ; 5·13%.

87. 19·3 rev. ; 95·63 ft. lb. 88. 889,800 ft. lb. ; 451·7 rev.

89. (*a*) 5 r.p.s. ; (*b*) $2\frac{1}{12}$ ft. per sec. ; 2·17 ft. lb. ; 264·5 ft. lb. ; 6·43 in.

90. 14,480 ft. lb. 91. 812·25 lb. ft. ; 18·56 H.P.

92. 252·2 lb. ; 481·3 lb. ; 1·6 sq. in. 93. 495 r.p.m. ; 285·6 lb.

94. 268·1 H.P. 95. 5529 ft. per min. ; 306·8 and 102·3 lb. ; 306·2.

96. 1405 lb. ft. ; 188·5 ft. per min. ; 1405 lb. ; 8·02 H.P.

97. 313·4. 98. $112\frac{1}{3}$ sec. 99. 80 ft. per sec. ; 48 ft. per sec.

100. 4·36 sec. 101. 25 and 22 sec.

102. 20·06 m.p.h. ; after $28\frac{3}{4}$ and 30 min. 103. 48° 36′ E. of N. ; 22·7 min.

104. 32° 45′ N. of W. ; 7·65 knots. 105. 30° 25′.

106. (*a*) 40 ft. per sec. ; (*b*) 1 ft. per sec.^{2} ; (*c*) 2 ft. per sec.^{2} ; (*d*) 800 ft. ; (*e*) 400 ft.

107. 1·21 ft. per sec.^{2} ; 17·6 rad. per sec. ; 0·968 rad. per sec.^{2}

108. (*a*) 4·034 ft. per sec.^{2} ; (*b*) $5\frac{5}{11}$ sec. ; (*c*) 4·03 rad. per sec.^{2}

109. 69·56 ; 52·17 ; 208·68 in Engineers' units.

110. 1·46 ft. per sec.^{2} ; 660 ft. 111. $2\frac{14}{15}$ sec. ; 40·87 ft.

112. $3\frac{2}{3}$ ft. per sec.^{2} ; $733\frac{1}{3}$ ft. 113. 55·6 ft. per sec. ; 190 sec.

114. 5·39 sec. ; 709·6 lb. ; 178 ft.

115. $1\frac{2}{3}$ ft. per sec.^{2} ; $8·5\dot{3}$ ft. ; $25\frac{13}{30}$ sec. ; 17·13 H.P. 116. 310·6 lb.

117. 4·325 lb. ; 1·346 Engineers' units ; No ; 1·346 lb.

119. 2·982 tons. 120. 0·467 ft. per sec.^{2} ; 4·14 sec.

121. 10·95 lb. at 122° 54′ clockwise with OA.

122. 8·466 lb. and angle $AOC = 100°\ 53'$. 123. 50° 15′.

124. 16·1 ft. per sec. ; 40·25 ft. ; 4025 ft. lb. ; 4025 ft. lb.

125. 9018 ft. lb. ; 819·8 Engineers' units ; 131·2 lb.

126. 298·1 Engineers' units ; 2981 ft. lb. ; 99·4 lb. ; 7·45 lb. ; 400 ft.

127. 1082 ft. ; 32·8 sec. **128.** 9·8 m.p.h. ; 231·8 ft.

129. 4 ft. per sec. ; 14·9 ft. tons.

130. 8281 lb. per sq. in. ; $14\frac{6}{11}$ ft. per sec. ; 5·85 ft. lb.

131. 364·8 in Engineers' units ; 3050 ft. lb. ; 0·0654 rad. per sec. per sec.

133. 4 tons per sq. in. **134.** 0·00707 in.

135. 4 tons per sq. in. ; 0·00237 in.

136. 11,161 lb. per sq. in. ; 66,966 lb. per sq. in. ; 1·863 sq. in.

137. 11·26 tons.

138. E.L. 11·4 tons ; Yield Point 23·67 tons per sq. in. ; 13,600 tons per sq. in. ; 33·2 tons per sq. in.

139. 15·83 tons per sq. in. ; $26\frac{2}{3}$ tons per sq. in. ; $21\frac{2}{3}$% ; 12,070 tons per sq. in.

141. 1240 lb. per sq. in. ; 310 lb. per sq. in. **142.** 5177 lb. per sq. in.

143. $3\frac{1}{33}$ tons ; 6·06 tons per sq. in. in steel, 2·63 tons per sq. in. in brass.

144. 2627 lb. ft. ; 4803 lb. per sq. in. **145.** 40·19 H.P.

146. 4280 lb. per sq. in. **147.** Rod $1\frac{5}{16}$ in. ; Pin 1 in.

148. 0·854 in. (a $\frac{7}{8}$ in. hole and $\frac{13}{16}$ in. dia. rivet would probably be used).

149. B.M. 3780 lb. ft. ; 630 lb. **150.** 0·476 in.

151. 1655 lb. ; 1045 lb.

152. L.H., 6·68 tons ; R.H. 5·32 tons ; 71·16 tons ft. ; 2·18 tons.

153. $2\frac{2}{3}$ tons per sq. in. **154.** 0·3571 in.

155. 820 lb., L.H., 980 lb., R.H. ; 0·1285 H.P.

156. 9 lb. each ; 108 lb. in. ; 90 lb. in.

157. 2160 lb. ft. under 300 lb. load.

158. Upper end, $1\frac{2}{3}$ cwt. ; Lower end, 3·433 cwt. at 60° 57′ to horizontal.

159. $1\frac{1}{3}$ tons. **161.** 20·49 tons.

162. 40·4 H.P. **163.** 15,575 lb. ; 5607 lb. ; 22,290 lb.

164. End, 20,933 lb. ; side, 27,910 lb. ; base, 47,846 lb.

165. Base, 7975 lb. ; end, 7975 lb. ; 12·23 lb.

166. $17\frac{7}{9}$ lb. per sq. in. ; 18,090 lb.

168. 0·4729 cu. ft. per min. ; 0·5923. **169.** 0·8026. **170.** 9·495 lb.

171. 26·37 lb. **172.** 637·8 ft. tons. **173.** 8·23 tons ; 65·8 tons ft.

174. 50·06 tons ; 254·96 tons ; $27\frac{1}{3}$ ft. from the water face.

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